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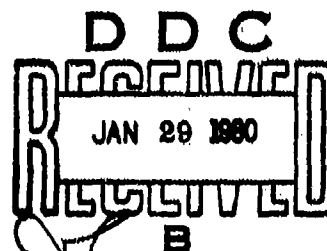USARDC ltr, 10 Apr 1984

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TECHNICAL REPORT ARLCD-TR-79030

# FUZE GEAR-TRAIN ANALYSIS

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This report documents the development of the tools needed to compare the efficiency of fuze-related gear trains designed to operate in a spin environment. Computer models have been developed for two and three pass step-up designs with clock (ogival) and involute tooth shapes. Using appropriate moment input-output relationships, the computer programs develop point and cycle efficiency for each type of gear train. Pivot friction partially caused by centrifugal force on the gear and pinion combinations during spin, and

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20. Abstract (continued)

... tooth-to-tooth contact friction are considered. All models allow a variety of parameter variations.

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# INTRODUCTION

This project provides the computer programs needed to compare the efficiency of fuze-related gear trains operating in a spin environment. Specifically, two and three pass step-up computer models with both involute and ogival (clock) tooth shapes were developed.

By using appropriate moment input-output relationships, the computer programs allow the determination of point and cycle efficiencies. Pivot friction, partly due to the centrifugal forces on the gear and pinion combinations, is considered in addition to tooth-to-tooth contact friction. The models derived allow a wide variety of parameter variations.

The main body of this report consists of nine appendixes, each of which contains a detailed analysis of each combination of tooth forms, the number of passes, and the spin environment. The derivation of moment relationships, pivot friction, gear tooth geometry, and the direct-contact mechanism kinematics are also included. The computer programs used are listed and instructions in their use and in interpreting the results are given.

The point efficiency $\varepsilon_P$ is defined as

$$\varepsilon_P = \frac{M_{oREF}}{M_{in}} \qquad (1)$$

where $M_{in}$ represents the instantaneous input moment to the gear train. $M_{oREF}$ stands for the instantaneous equilibrant output moment $M_o$ after it has been referred to the input shaft by way of the instantaneous angular velocity ratio

$$K_{RATIO} = \left| \frac{\dot{\psi}}{\dot{\phi}} \right| . \qquad (2)$$

In the above,

$\dot{\psi}$ = instantaneous angular velocity of the output gear

$\dot{\phi}$ = instantaneous angular velocity of the input gear

Equation 1 then becomes

$$\varepsilon_P = K_{RATIO} \frac{M_o}{M_{in}} \qquad (3)$$

The cycle efficiency $\varepsilon_C$ represents the ratio of the work available at the output shaft to that done by the input moment during one tooth cycle of the input gear. Thus,

$$\varepsilon_C = \frac{\int M_o \, d\psi}{\int M_{in} \, d\phi} \qquad (4)$$

The quantities $d\psi$ and $d\phi$ represent infinitesimal rotations of the output and input gears, respectively.

DESCRIPTION OF STUDY

## Appendix A

Appendix A furnishes the background, as well as the deriva-
tions, for the moment input-output expressions for two and three
pass step-up gear trains (where in each mesh the gear is the driver)
with involute teeth and unity contact ratio.

Section 1 shows the development of a sign convention for the
direction of the contact point friction force. It is based on the
direction of the relative velocity between the contact points on
the gear and pinion teeth. Section 2 discusses how to deal with
the normal and friction forces at the gear and pinion pivots of
single and multiple mesh trains. Section 3 shows the application
of the above results to the moment input-output analysis of a
single mesh. The frame is assumed stationary for this case, and
the external loads are confined to the driving input moment and the
equilibrating output moment.

The basic geometry of the three pass step-up gear train,
mounted on a rotating fuze body, is formulated in section 4 for
use in the moment input-output analysis. Force and moment equilibria
of the individual component gears, which also account for the cen-
trifugal forces, lead to the desired expression. Section 5
includes a similar derivation for a two pass step-up gear train with

involute teeth. In order to be able to continuously compute the moment relationships for these trains, a method for determining the simultaneous locations of the contact points of all the meshes had to be worked out. Such a method is given in section 6 together with certain angular relationships of the pivot locations on the model fuze body.

The kinematic relationships in involute gear trains are relatively simple compared to those in ogival trains because of the constant transmission ratio and the invariant direction of the line-of-action in each individual mesh.

## Appendix B

To avoid the severe undercutting of pinions which generally is associated with step-up gear meshes, it is necessary to use nonstandard involute gearing. Appendix B shows both the theory and the necessary steps for the design of unequal addendum gears and pinions of unity contact ratio. In addition, a numerical example is given.

## Appendix C

This appendix contains four computer programs which make it possible to determine the point and cycle efficiencies of three gear combinations containing unequal addendum involute meshes with

unity contact ratio.  In each case, the structure of the program is thoroughly discussed and a sample run is used to interpret the results.  The names of these programs and their relationship to work described in the other appendixes are given below:

1.  Program INVOL 1 :  Design of unequal addendum involute gear and pinion set with unity contact ratio.

This program is based on the work in Appendix B.Five sample computations,which are used in other programs,are shown.

2.  Program INVOL 2 :  Point and cycle efficiencies for single pass involute step-up gear mesh with unity contact ratio.

This program is based on the work in section 3 of Appendix A.

3.  Program INVOL 3 :  Point and cycle efficiencies for three pass  involute step-up gear train in spin environment.(All meshes have unity contact ratio).

This program is based on the work in sections 4 and 6 of Appendix A.

4.  Program INVOL 4 :  Point and cycle efficiencies for two pass involute step-up gear train in spin environment.(All meshes have unity contact ratio).

This program is based on the work in sections 5 and 6 of Appendix A.

## Appendix D

This appendix describes the geometry of an ogival tooth in which each side of the tooth profile has a circular arc blending tangentially into a radial straight line flank. The basic tooth nomenclature is defined and methods for determining the required tooth parameters are given.

## Appendix E

Section 1 of Appendix E gives all necessary kinematic derivations for a single step-up mesh with ogival teeth. The motion of an ogival mesh consists of two phases. On first contact, the circular arc portion of the driving gear tooth makes contact with the circular arc portion of the driven pinion tooth. Later in the cycle, and up to the point of final disengagement, the circular arc portion of the gear tooth contacts the straight line portion of the pinion tooth. These phases of motion were named " round on round " and " round on flat, " respectively. Equivalent four-link mechanism models were used for both regimes to obtain expressions for the pinion output angles, for transition angles, for output angular velocities and for contact-point relative velocities. In addition, a sensing expression was developed which allows the computer determination of that position of a given mesh at which the subsequent mesh comes into engagement. (Because of the variable transmission ratio of ogival meshes, there is only one set of teeth in contact at any one time).

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Section 2 of this appendix shows derivations of moment input-output expressions for both phases of contact of a single ogival mesh. Again, while pivot friction is considered in addition to contact friction, the frame is assumed to be stationary for this single mesh.

## Appendix F

Two computer programs which deal with the kinematics and the moment input-output relationships of a single pass step-up gear mesh with clock teeth are given in this appendix. The structure of each of these programs is again discussed in detail and sample runs are used to explain their input and output parameters. The names of these programs and their relationships to work in other sections are given below:

1. Program CLOCK 1 : Kinematics of a single step-up gear mesh with clock teeth.
   This program is based on work in Appendix D as well as on work in section 1 of Appendix E. In addition, it has been used to check the geometry of the ogival meshes which were used in programs CLOCK 2, CLOCK 3 and CLOCK 4 (See Appendix I for the latter two).

2. Program CLOCK 2 : Point and cycle efficiencies for single pass step-up gear mesh with clock teeth.
   This program is based on work in section 2 of Appendix E.

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## Appendix G

When one considers the kinematic relationships of ogival meshes which are mounted on a fuze body as parts of two or three pass step-up trains, it becomes necessary to account for the relative positions of the individual meshes on the fuze body. Appendix G gives the appropriate derivations for each of the three meshes of a three-pass train. The model of the fuze body is identical with that used for involute step-up trains.

## Appendix H

This appendix shows the derivations of moment input-output expressions for two and three pass step-up gear trains with ogival teeth which must operate in a spin environment.

Because of the increase in rotational speed associated with each tooth mesh, increasingly more sets of teeth will come into engagement in the second and third meshes as one set of teeth moves through one complete contact cycle in the first (i.e., the input) mesh. With two phases of motion for each mesh, there will be eight contact combinations in a three-pass train. Section 1 of Appendix H gives a derivation for the moment input-output expression of each of these eight cases.

Section 2 shows similar work for the four contact combinations which are associated with two pass step-up gear trains with ogival teeth.

Both analyses account for the effects of the centrifugal forces.

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## Appendix I

This appendix contains two computer programs which allow
the determination of point and cycle efficiencies for two
and three pass step-up gear meshes with clock teeth. As for
other programs, the origins of their mathematical formulations
are thoroughly discussed. In addition, their input and output
parameters are explained with the help of a sample run. The
names of these programs as well as their relationships to
work in other appendixes are given below:

1.  Program CLOCK 3 : Point and cycle efficiencies for
    three pass clock (ogival) step-up
    gear train in spin environment.
    This program is based on work
    given in Appendix G as well as in
    section 1 of Appendix H. The general
    fuze geometry is that described in
    section 6 of Appendix A.

2.  Program CLOCK 4 : Point and cycle efficiencies for
    two pass clock (ogival) step-up
    gear train in spin environment.
    The kinematics of this program
    is again based on work given in
    Appendix G. The moment input-
    output relationships are from
    section 2 of Appendix H.

# APPENDIX A

## STEP-UP GEAR TRAINS WITH INVOLUTE TEETH

### 1. DIRECTION OF FRICTION FORCE AT TOOTH-TO-TOOTH CONTACT

Figure A-1 uses the base circle and the line-of-action configuration of an involute mesh, in which the gear drives the pinion, to determine the direction of the friction forces at contact point C before and after the contact point passes through pitch point P. Distance d represents the length of the line-of-action between base circle tangent points L and L'. The distance between contact point C and point L along the line-of-action is measured by length a. Further:

$$\theta = \text{actual (rolling) pressure angle}$$

$$R_b = \text{gear base radius}$$

$$r_b = \text{pinion base radius}$$

The friction force of the pinion tooth on the gear tooth, for example, will have the direction of the relative velocity, $\bar{V}_{C_p/C_g}$, of the contact point on the pinion tooth, $C_p$, with respect to the coincident contact point on the gear tooth, $C_g$. (The friction force of the gear tooth on the pinion tooth has the opposite direction.) This relative velocity changes direction at the pitch point, where it becomes instantaneously zero.

A-1

Pinion

Gear

$O_n$

$O_N$

$r_b$

$R_b$

$\theta$

$n_\theta$

Direction of Rotation

d

a

L

C

P

L'

$\overline{V}_{C_g}$

$\overline{V}_{C_p}$

$\overline{V}_{C_p/C_g}$

Direction of Friction Force of Pinion on Gear

Contact Before Pitch Point (Approach)

$O_n$

$O_N$

d

a

L

P

C

L'

$\overline{V}_{C_p}$

$\overline{V}_{C_g}$

$\overline{V}_{C_p/C_g}$

Direction of Friction Force of Pinion on Gear

Contact After Pitch Point (Recess)
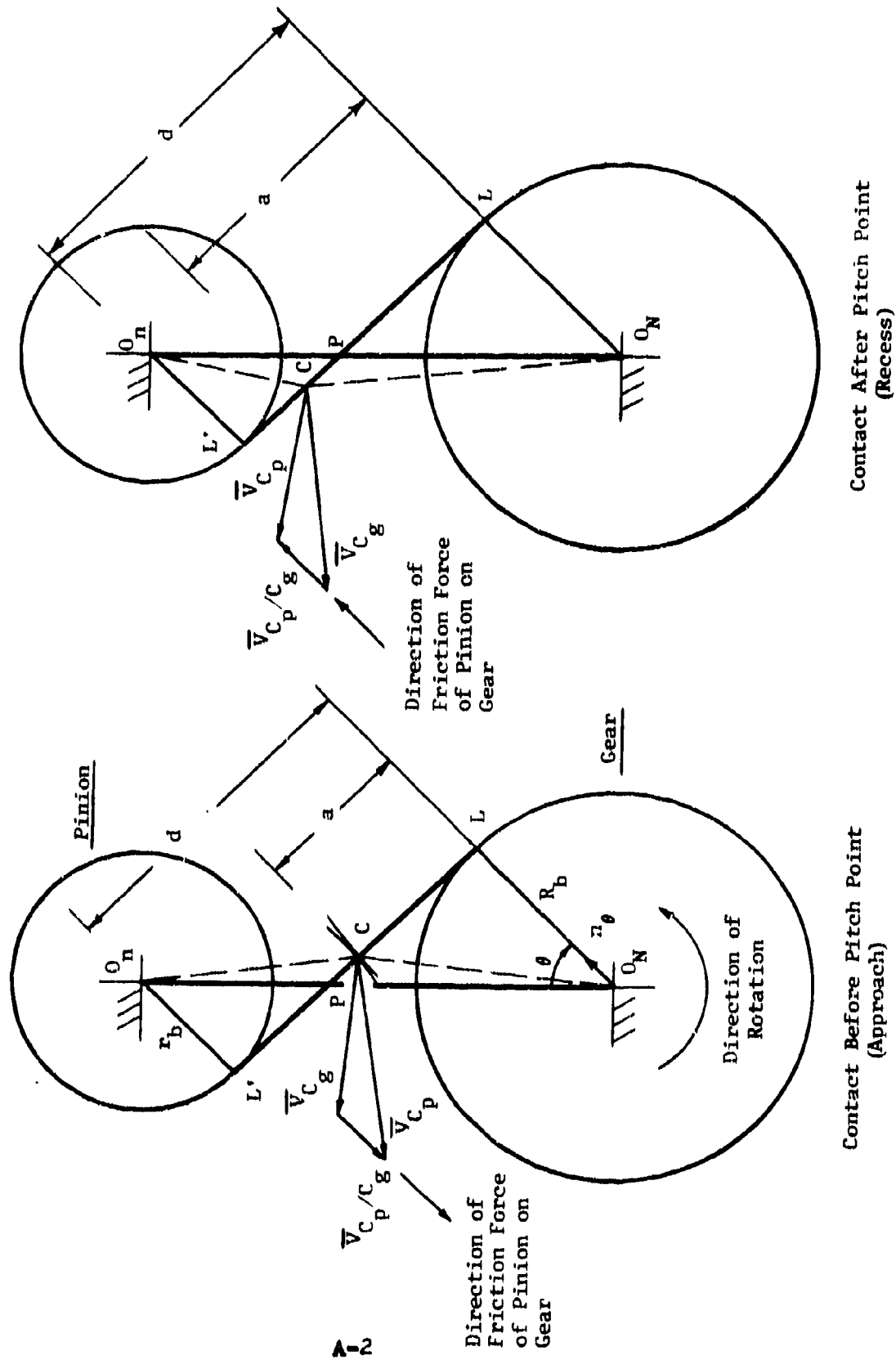
FIGURE A-1. DETERMINATION OF DIRECTION OF CONTACT FRICTION FORCES BY VELOCITY ANALYSIS

A-2

Figure A-1 shows contact before the pitch point (during approach). To obtain the direction of the relative velocity, $V_{C_p/C_g}$, by a graphical analysis, one makes use of the velocity equation

$$V_{C_p} = V_{C_p/C_g} + V_{C_g} \qquad (A-1)$$

where $V_{C_p}$ = velocity of point $C_p$ on pinion tooth with direction normal to line $O_n$- C

$V_{C_p/C_g}$ = relative velocity between point $C_p$ and point $C_g$. The direction of this velocity is normal to the line of action.

$V_{C_g}$ = velocity of point $C_g$ on gear tooth with direction normal to line $O_N$- C. The magnitude of this velocity was arbitrarily chosen.

The graphical construction, according to Eq (A1), shows that $V_{C_p/C_g}$ has the direction opposite to that of the unit vector $\pi_\theta$ shown at point $O_N$. As stated earlier, this represents

the friction force on the gear tooth during approach.

Figure A-1b shows the same graphical analysis for contact during recess. Once the pitch point is passed, both the relative velocity, $\overline{V}_{C_p/C_g}$, and the friction force of the pinion on the gear have the direction of the positive unit vector $\overline{n}_\theta$.

## 2. ASSUMPTIONS CONCERNING NORMAL AND FRICTION
## FORCES AT PIVOTS



a.                                                                b.
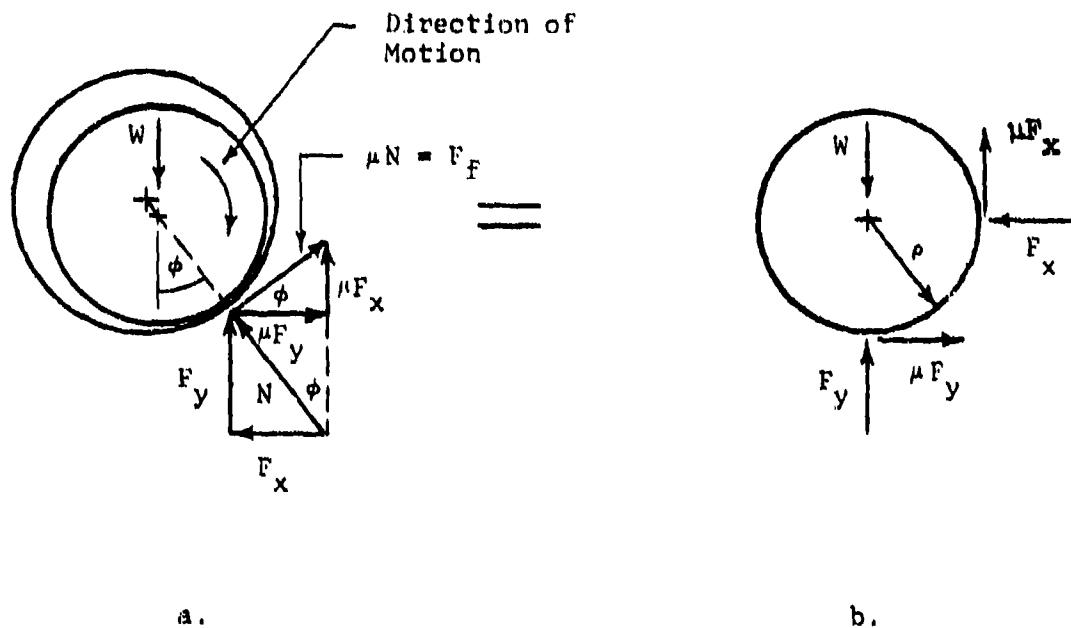
FIGURE A-2   FREE-BODY DIAGRAMS OF PIVOT SHAFTS

Figure A-2a shows a pivot shaft which is loaded by a

known external force, W, and rotates in a clockwise

direction.  Due to friction between the shaft and the bearing,

contact is made at an angle, $\phi = \tan^{-1}\mu$, where $\mu$ is the

associated coefficient of friction.  N is the normal contact

force. The friction force $F_f = \mu N$ opposes the clockwise

rotation by creating a counterclockwise moment.

N may be resolved into the components $F_x$ and $F_y$. The associated

x and y components of the friction force are $\mu F_y$ and $\mu F_x$, respectively.

The directions of the components of N and $F_f$ are drawn

in the same manner in Fig. A-2b in a somewhat more convenient

representation. When the direction of the resultant external

force ,W,is not known, contact is possible anywhere on the

periphery of the bearing and the components $F_x$ and $F_y$ of the

normal contact force cannot be drawn with certainty in the

free body diagram. The direction of the friction components

must still oppose the motion.

Figure A-3 shows two general possibilities of

drawing the free body diagram of a pivot which rotates in a

clockwise direction. In either case,.the moments of the friction

components oppose the rotation while $F_x$ and $F_y$ may be positive

or negative. Assume now, for example, that Fig. A-3a shows

the wrong direction for $F_x$ and that the solution of the

applicable equilibrium equation will reverse the sign of $F_x$.

This will automatically reverse the sign of the friction

component $\mu F_x$ also. Since contact is now made on the opposite

A-6

a.                                    b.
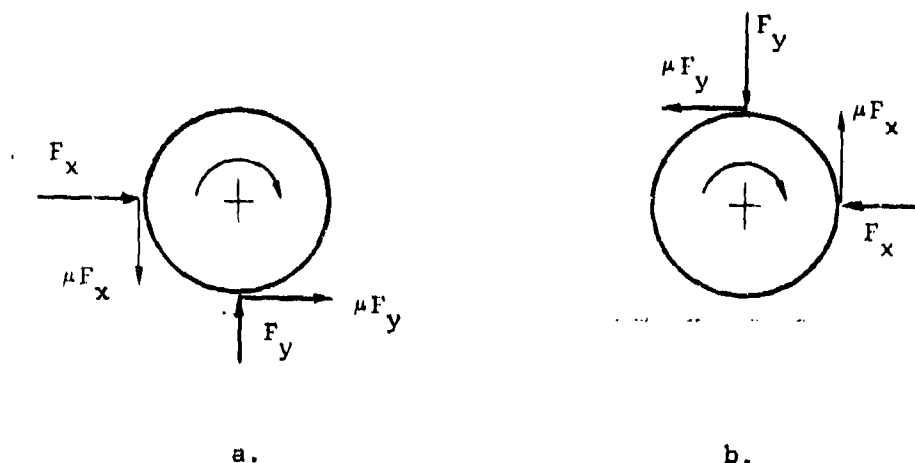
FIGURE A-3    MOMENTS DUE TO FRICTION COMPONENTS
              ALWAYS OPPOSE MOTION

side of the pivot, the correct sign of the friction component

is automatically assured.

The total friction moment then is expressed by

$$M_f = \pm \mu \rho \sqrt{F_x^2 + F_y^2} \qquad \qquad (A-2)$$

where $\rho$ is the pivot radius.  The sign of the above is chosen

so  that the rotation is opposed.        In case   $F_x$ and $F_y$

contain required terms that cannot be factored out of the

square root of equation (A-2), the friction moment is conservatively

overstated by the use of the absolute values of $F_x$ and $F_y$

A-7

$$M_f = \pm \mu \rho \left( \left| F_x \right| + \left| F_y \right| \right) \qquad (A-3a)$$

If the expressions for $F_x$ and $F_y$ consist of sums of positive and negative terms, then $F_x$ and $F_y$ are presented as the sums of the absolute values of these terms. A conservative pivot friction moment becomes, similar to equation (A-3a),

$$M_f = \pm \mu \rho \left( \widetilde{F}_x + \widetilde{F}_y \right) \qquad (A-3b)$$

The tildes represent the sums of the absolute values of the component terms.

## 3. MOMENT INPUT-OUTPUT RELATIONSHIP FOR SINGLE STEP-UP

## GEAR MESH WITH INVOLUTE TEETH

Figure A-4 shows free body diagrams of the gear and the pinion of a single mesh where the gear is driven by a counterclockwise input moment $M_{in}$. It is desired to find the equilibrating output moment $M_o$.

### a. UNIT VECTORS

The unit vector directed from point $O_N$ to point L is given by

$$\bar{n}_\theta = \sin\theta\,\bar{I} + \cos\theta\,\bar{J} \tag{A-4}$$

where $\theta$ represents the actual pressure angle, regardless of tooth modification. The unit vector directed along the line of action from point L to point L' is given by

$$\bar{n}_{\theta T} = -\cos\theta\,\bar{I} + \sin\theta\,\bar{J} \tag{A-5}$$
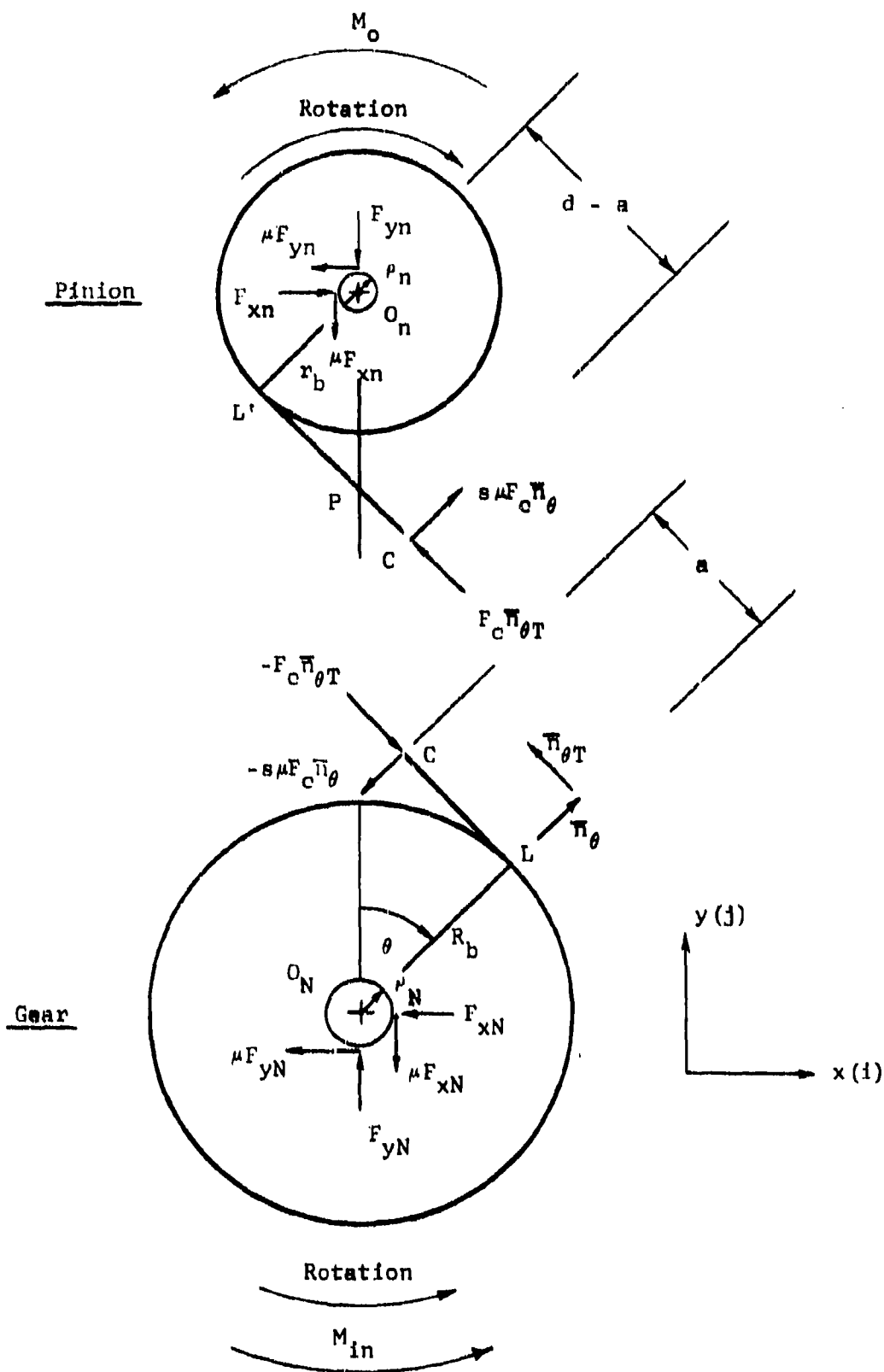
FIGURE A-4. FREE BODY DIAGRAM FOR SINGLE STEP-UP INVOLUTE GEAR MESH

A-10

## 5. NOMENCLATURE AND SIGNUM CONVENTION

$F_{xN}$, $F_{yN}$    =   x and y components of normal force acting on
gear pivot

$\mu F_{xN}$, $\mu F_{yN}$   =   friction force components acting on gear
pivot. Directions chosen to result in friction
moments which oppose motion. (See part 2).

$F_{xn}$, $F_{yn}$    =   x and y components of normal force acting on
pinion pivot

$\mu F_{xn}$, $\mu F_{yn}$   =   friction force components acting on pinion pivot

$\mu$    =   coefficient of friction

$F_c$    =   normal contact force between gear and pinion.
The force of the pinion on the gear is $(-)F_c \overline{n}_{\theta T}$,
while the normal force of the gear on the pinion
becomes $F_c \overline{n}_{\theta T}$.

$\mu F_c$    =   tooth contact friction force. The analysis in
Section 1 shows that the friction force of the
pinion on the gear, before the pitch point, acts
in the direction of $(-)\overline{n}_{\theta}$. Therefore

$-s \mu F_c \overline{n}_{\theta}$   =   friction force on gear with

     $s$    =   +1 for $a < \overline{LP}$ (approach)        (A-6)

and

$$s = -1 \quad \text{for } a > \overline{LP} \quad \text{(recess)} \qquad (A-7)$$

while

$$s = 0 \text{ for } a = \overline{LP} \quad \text{(at pitch point)} \qquad (A-8)$$

Further,

$\rho_N, \ \rho_n$ = gear and pinion pivot radii

$R_b, \ r_b$ = gear and pinion base circle radii

## c. FORCE ANALYSIS OF THE GEAR

By inspecting Figure A-4, one sees that force equilibrium of the gear is expressed by

$$-F_c \bar{n}_{\theta T} - s\mu F_c \bar{n}_\theta - F_{xN}\bar{i} - \mu F_{xN}\bar{j} + F_{yN}\bar{j} - \mu F_{yN}\bar{i} = 0 \qquad (A-9)$$

Similarly, moment equilibrium of the gear is given by

$$M_{1n}\bar{k} - \rho_N \mu \sqrt{F_{xN}^2 + F_{yN}^2}\,\bar{k} + R_b \bar{n}_\theta \times (-)F_c \bar{n}_{\theta T} + (R_b \bar{n}_\theta + a\bar{n}_{\theta T})$$
$$\times (-)s\mu F_c \bar{n}_\theta = 0 \qquad (A-10)$$

Note the use of equation (A-2) to express the friction moment at the gear pivot.

With the help of equations (A-4) and (A-5) one may write equations (A-9) and (A-10) in scalar form. Thus,

$$F_c \cos\theta - \mu s F_c \sin\theta - F_{xN} - \mu F_{yN} = 0 \qquad (A-11)$$

$$-F_c \sin\theta - \mu s F_c \cos\theta + F_{yN} - \mu F_{xN} = 0 \qquad (A-12)$$

Equation (A-10) becomes

$$M_{in} - \rho_N \mu \sqrt{F_{xN}^2 + F_{yN}^2} - R_b F_c + \mu sa F_c = 0 \qquad (A-13)$$

Simultaneous solution of equations (A-11) and (A-12) for

$F_{xN}$ and $F_{yN}$ gives

$$F_{xN} = F_c \frac{(1 - \mu^2 s)\cos\theta - \mu(1 + s)\sin\theta}{1 + \mu^2} \qquad (A-14)$$

and

$$F_{yN} = F_c \frac{(1 - \mu^2 s)\sin\theta + \mu(1 + s)\cos\theta}{1 + \mu^2} \qquad (A-15)$$

When the above expressions are substituted into the moment

equation (A-13) and if one notes that $s^2$ always equals +1,

the following expression for $F_c$ is obtained:

$$F_c = \frac{M_{in}}{R_b + \mu(\rho_N - as)} \qquad (A-16)$$

A-14

## d. FORCE ANALYSIS OF THE PINION

Force equilibrium of the pinion is assured by

$$F_c\bar{n}_{\theta T} + \mu s F_c \bar{n}_\theta + F_{xn}\bar{I} - F_{yn}\bar{J} - \mu F_{xn}\bar{J} - \mu F_{yn}\bar{I} = 0, \qquad (A-17)$$

while moment equilibrium is given by

$$M_o\bar{k} + \rho_n\mu\sqrt{F_{xn}^2 + F_{yn}^2}\,\bar{k} + \left[-r_b\bar{n}_\theta - (d-a)\bar{n}_{\theta T}\right] \times (F_c\bar{n}_{\theta T} + \mu s F_c\bar{n}_\theta) = 0 \qquad (A-18)$$

In scalar form, the above become

$$-F_c\cos\theta + \mu s F_c\sin\theta + F_{xn} - \mu F_{yn} = 0 \qquad (A-19)$$

$$F_c\sin\theta + \mu s F_c\cos\theta - \mu F_{xn} - F_{yn} = 0 \qquad (A-20)$$

$$M_o + \mu\rho_n\sqrt{F_{xn}^2 + F_{yn}^2} - r_b F_c + \mu s(d-a)F_c = 0 \qquad (A-21)$$

Equations (A-19) and (A-20) are now solved simultaneously

for $F_{xn}$ and $F_{yn}$. This gives

$$F_{xn} = F_c \frac{(1 + \mu^2 s)\cos\theta + \mu(1 - s)\sin\theta}{1 + \mu^2} \qquad (A-22)$$

and

$$F_{yn} = F_c \frac{(1 + \mu^2 s)\sin\theta - \mu(1 - s)\cos\theta}{1 + \mu^2} \qquad (A-23)$$

Equations (A-22) and (A-23) are then substituted into

the moment equation (A-21). This furnishes the following

expression for the normal contact force, $F_c$. ( Again, $s^2$ always

equals +1.):

$$F_c = \frac{M_o}{r_b - \mu\left[\rho_n + s(d - a)\right]} \qquad (A-24)$$

e. MOMENT INPUT-OUTPUT RELATIONSHIP

The equilibrant moment, $M_O$, may be expressed as a function
of the input moment, $M_{in}$, after equations (A-16) and (A-24)
have been set equal to each other.  Thus,

$$M_O = \frac{M_{in}\left\{r_b - \mu\left[\rho_n + s(d - a)\right]\right\}}{R_b + \mu\left[\rho_N - sa\right]} \qquad (A\text{-}25)$$

The input-output relationship may also then be expressed in terms of

$$M_O = M_{in}\frac{r_b}{R_b}E_2 \qquad (A\text{-}26)$$

where

$$E_2 = \frac{1 - \dfrac{\mu\left[\rho_n + s(d - a)\right]}{r_b}}{1 + \dfrac{\mu(\rho_N - sa)}{R_b}} \quad,$$

and represents the efficiency of moment transmission of a
single step-up mesh with involute teeth.

## 4. MOMENT INPUT-OUTPUT RELATIONSHIP FOR THREE STEP-UP GEAR TRAIN IN SPIN ENVIRONMENT

Figure A-5 shows the basic configuration of the three step-up gear train for which the relationship between the equilibrant output moment $M_{o4}$, acting on pinion 4, and the input moment $M_{in}$, acting on gear 1, is to be found.

The body-fixed x-y coordinate system has its origin at the spin axis, C, of the fuze body, and its x-axis coincides with the line $C-O_1$, where $O_1$ represents the pivot axis of the input gear-spin rotor combination. Points $O_2$, $O_3$ and $O_4$ represent the pivot axes of gear and pinion no. 2, gear and pinion no. 3, and pinion no. 4, respectively. Further,

$\mathcal{R}_i$ = distance from the spin axis to the various pivot axes

$R_{bi}$ = base radii of gears

$r_{bi}$ = base radii of pinions

$\beta_1$ = angle between positive x-axis and line of centers $O_1-O_2$

$\beta_2$ = angle between positive x-axis and line of centers $O_2-O_3$

$\beta_3$ = angle between positive x-axis and line of centers $O_3-O_4$

$\gamma_i$ = angle between positive x-axis and lines $C-O_1$
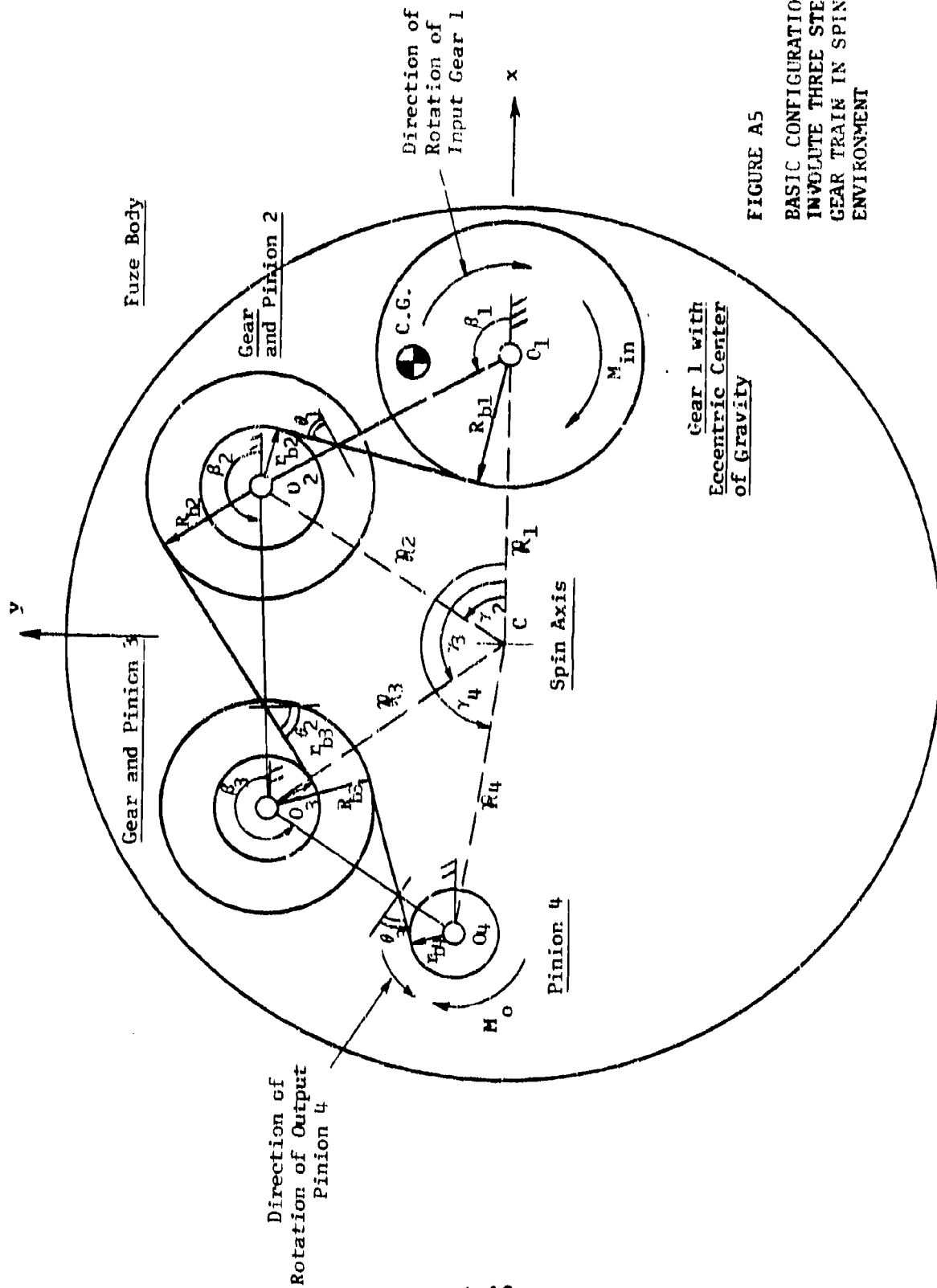
FIGURE A5

BASIC CONFIGURATION FOR
INVOLUTE THREE STEP-UP
GEAR TRAIN IN SPIN
ENVIRONMENT

$\theta_1$ = pressure angle of mesh between gear no. 1 and pinion no. 2

$\theta_2$ = pressure angle of mesh between gear no. 2 and pinion no. 3

$\theta_3$ = pressure angle of mesh between gear no. 3 and pinion no. 4

To obtain the moment input-output relationship of the total train, the input-output relationships of the individual components must first be obtained. The following equilibrium analyses include pivot as well as contact friction forces, in addition to loads due to the centrifugal forces on the individual components. The directions of the tooth-to-tooth friction forces are chosen according to the rules of Section 1 of this appendix, using an appropriate signum convention. The direction of the pivot friction forces is chosen according to Section 2 and, to avoid difficulties with the direction of the associated friction moment, equation (A-3b) will be used.

a. <u>EQUILIBRIUM OF PINION 4</u>

Figure A-6 shows a free body diagram of pinion 4. The contact between gear 3 and pinion 4 is shown before contact point $C_3$ has passed through pitch point $P_3$. The unit vector $\bar{n}_{34}$ is along the line-of-action in the direction of the contact force of the gear on the pinion.
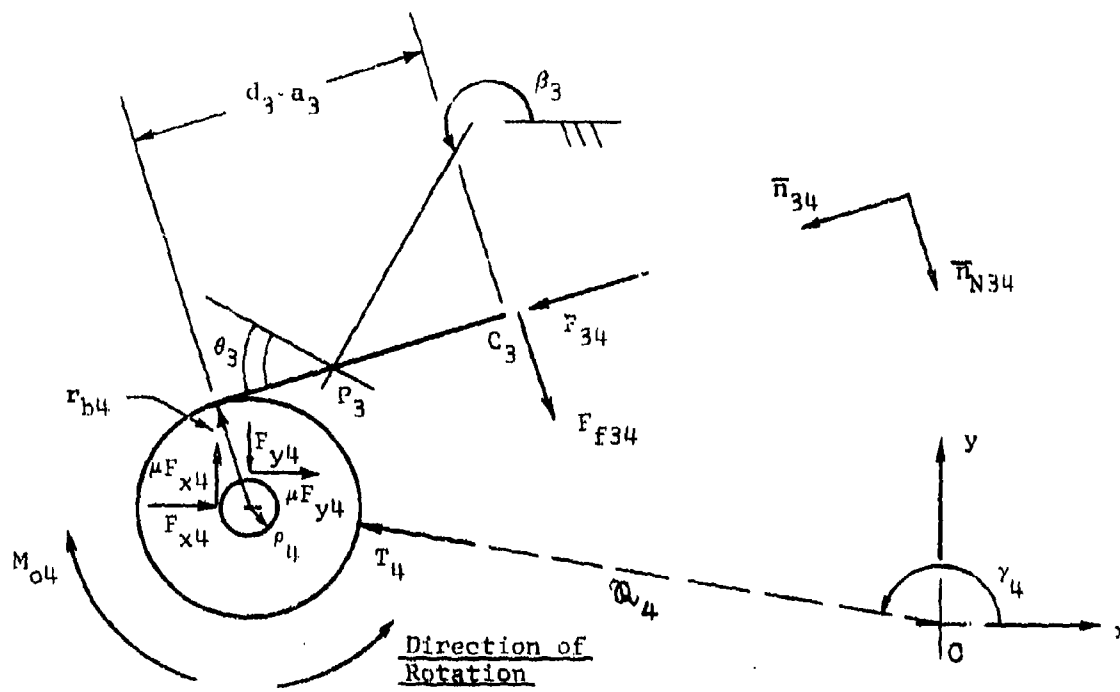
FIGURE A-6.   FREE BODY DIAGRAM OF PINION 4

A-21

Thus,

$$\overline{n}_{34} = \sin(\beta_3 + \theta_3)\overline{I} - \cos(\beta_3 + \theta_3)\overline{J} \qquad (A-27)$$

The unit vector normal to the line-of-action is given by

$$\overline{n}_{N34} = \cos(\beta_3 + \theta_3)\overline{I} + \sin(\beta_3 + \theta_3)\overline{J} \qquad (A-28)$$

The contact force $\overline{F}_{34}$ then becomes

$$\overline{F}_{34} = F_{34}\,\overline{n}_{34} \qquad (A-29)$$

The friction force of gear 3 on pinion 4 is given by

$$\overline{F}_{f34} = \mu s_3\, F_{34}\,\overline{n}_{N34} \qquad (A-30)$$

where   $s_3 = +1$,   for contact before the pitch point

$\qquad\quad s_3 = 0$,   for contact at the pitch point

$\qquad\quad s_3 = -1$,   for contact after the pitch point

(See also Section 1.)

The normal forces on the pivot shaft are given by

$$\overline{F}_{x4} = F_{x4} \overline{I} \tag{A-31}$$

and

$$\overline{F}_{y4} = -F_{y4} \overline{J} \tag{A-32}$$

The associated pivot friction forces are given by $\mu F_{x4} \overline{J}$ and $\mu F_{y4} \overline{I}$ for the indicated direction of rotation. The centrifugal force $T_4$ on the pinion is represented by

$$\overline{T}_4 = T_4(\cos \gamma_4 \overline{I} + \sin \gamma_4 \overline{J}) \tag{A-33}$$

where

$$T_4 = \mathcal{R}_4 \omega^2 m_4 \tag{A-34a}$$

with

$$\omega = \text{spin angular velocity} \tag{A-34b}$$

and

$$m_4 = \text{mass of pinion 4} \tag{A-34c}$$

The force equilibrium equation is given by

$$F_{34}\overline{n}_{34} + \mu s_3 F_{34}\overline{n}_{N34} + T_4(\cos \gamma_4 \overline{I} + \sin \gamma_4 \overline{J}) + F_{x4} \overline{I}$$

$$+ \mu F_{y4} \overline{I} + \mu F_{x4} \overline{J} - F_{y4} \overline{J} = 0 \tag{A-35}$$

A-23

Moment equilibrium is given by the following expression, in which the pivot friction moment is expressed according to Equation (A-3b):

$$-M_{o4} - \rho_4 \mu ( \tilde{F}_{x4} + \tilde{F}_{y4} ) + r_{b4}F_{34} - \mu a_3(d_3 - a_3)F_{34} = 0 \qquad (A-36)$$

where $\rho_4$ represents the pivot radius. $d_3$ is the length of the line-of-action of the mesh from points of tangency to the base circles. $a_3$ is the distance on the line of action from the gear point of tangency to the contact point $C_3$.

Equation (A-35) gives the following component equations:

$$F_{34}\sin(\beta_3 + \theta_3) + \mu a_3 F_{34}\cos(\beta_3 + \theta_3) + T_4\cos\gamma_4 + F_{x4} + \mu F_{y4} = 0 \qquad (A-37)$$

$$-F_{34}\cos(\beta_3 + \theta_3) + \mu a_3 F_{34}\sin(\beta_3 + \theta_3) + T_4\sin\gamma_4 - F_{y4} + \mu F_{x4} = 0 \qquad (A-38)$$

A-24

Simultaneous solution of the above for $F_{x4}$ and $F_{y4}$ results in

$$F_{x4} = \frac{1}{1 + \mu^2} \left\{ -T_4 \left[ \cos\gamma_4 + \mu\sin\gamma_4 \right] - F_{34} \left[ (1 + \mu^2 s_3)\sin(\beta_3 + \theta_3) \right. \right.$$
$$\left. \left. - \mu(1 - s_3)\cos(\beta_3 + \theta_3) \right] \right\} \qquad \text{(A-39)}$$

and

$$F_{y4} = \frac{1}{1 + \mu^2} \left\{ T_4 \left[ \sin\gamma_4 - \mu\cos\gamma_4 \right] - F_{34} \left[ (1 + \mu^2 s_3)\cos(\beta_3 + \theta_3) \right. \right.$$
$$\left. \left. + \mu(1 - s_3)\sin(\beta_3 + \theta_3) \right] \right\} \qquad \text{(A-40)}$$

To obtain conservative values for the pivot friction moment in equation (A-36) according to equation (A-3b), one substitutes the largest possible values for $F_{x4}$ and $F_{y4}$. This is accomplished by making the signs of $T_4$ and $F_{34}$ positive and by using the absolute values of their respective coefficients in Equations (A-39) and (A-40). Equation (A-36) then becomes

$$-M_{o4} - \mu\rho_4(T_4 A_1 + F_{34} A_2 + T_4 A_3 + F_{34} A_4) + r_{b4} F_{34}$$
$$- \mu s_3(d_3 - a_3)F_{34} = 0 \qquad \text{(A-41)}$$

where

$$A_1 = \left| \frac{\sin\gamma_4 - \mu\cos\gamma_4}{1 + \mu^2} \right| \qquad \text{(A-42)}$$

A-25

$$A_2 = \left| \frac{(1 + \mu^2 s_3)\cos(\beta_3 + \theta_3) + \mu(1 - s_3)\sin(\beta_3 + \theta_3)}{1 + \mu^2} \right| \qquad \text{(A-43)}$$

$$A_3 = \left| \frac{\cos\gamma_4 + \mu\sin\gamma_4}{1 + \mu^2} \right| \qquad \text{(A-44)}$$

$$A_4 = \left| \frac{(1 + \mu^2 s_3)\sin(\beta_3 + \theta_3) - \mu(1 - s_3)\cos(\beta_3 + \theta_3)}{1 + \mu^2} \right| \qquad \text{(A-45)}$$

Finally equation (A-41) is solved for $F_{34}$

$$F_{34} = \frac{M_{o4}}{D_1} + \frac{T_4 C_1}{D_1} \qquad \text{(A-46)}$$

where

$$C_1 = \mu \rho_4 (A_1 + A_3) \qquad \text{(A-47)}$$

$$D_1 = r_{b4} - \mu \left[ s_3(d_3 - a_3) + \rho_4(A_2 + A_4) \right] \qquad \text{(A-48)}$$

b. EQUILIBRIUM OF GEAR AND PINION SET NO. 3

Figure A-7 shows the free body diagram of gear and pinion
set no. 3.    Contact point $C_3$, between pinion 4 and gear 3,
is, as shown previously in Figure A-6, before pitch point $P_3$.
The normal force, along the line-of-action, is given [see
equation (A-29)] by

$$\overline{F}_{43} = -F_{34}\overline{n}_{34} \qquad\qquad\text{(A-49)}$$

and the associated friction force $\overline{F}_{f43}$ is given [see
equation (A-30)] by

$$\overline{F}_{f43} = -s_3\mu F_{34}\overline{n}_{N34} \qquad\qquad\text{(A-50)}$$

The unit vectors along (and perpendicular to) the line-
of-action of gear 2 and pinion 3 are given by

$$\overline{n}_{23} = -\sin(\beta_2 - \theta_2)\,\overline{I} + \cos(\beta_2 - \theta_2)\,\overline{J} \qquad\qquad\text{(A-51)}$$

and

$$\overline{n}_{N23} = -\cos(\beta_2 - \theta_2)\,\overline{I} - \sin(\beta_2 - \theta_2)\,\overline{J} \qquad\qquad\text{(A-52)}$$
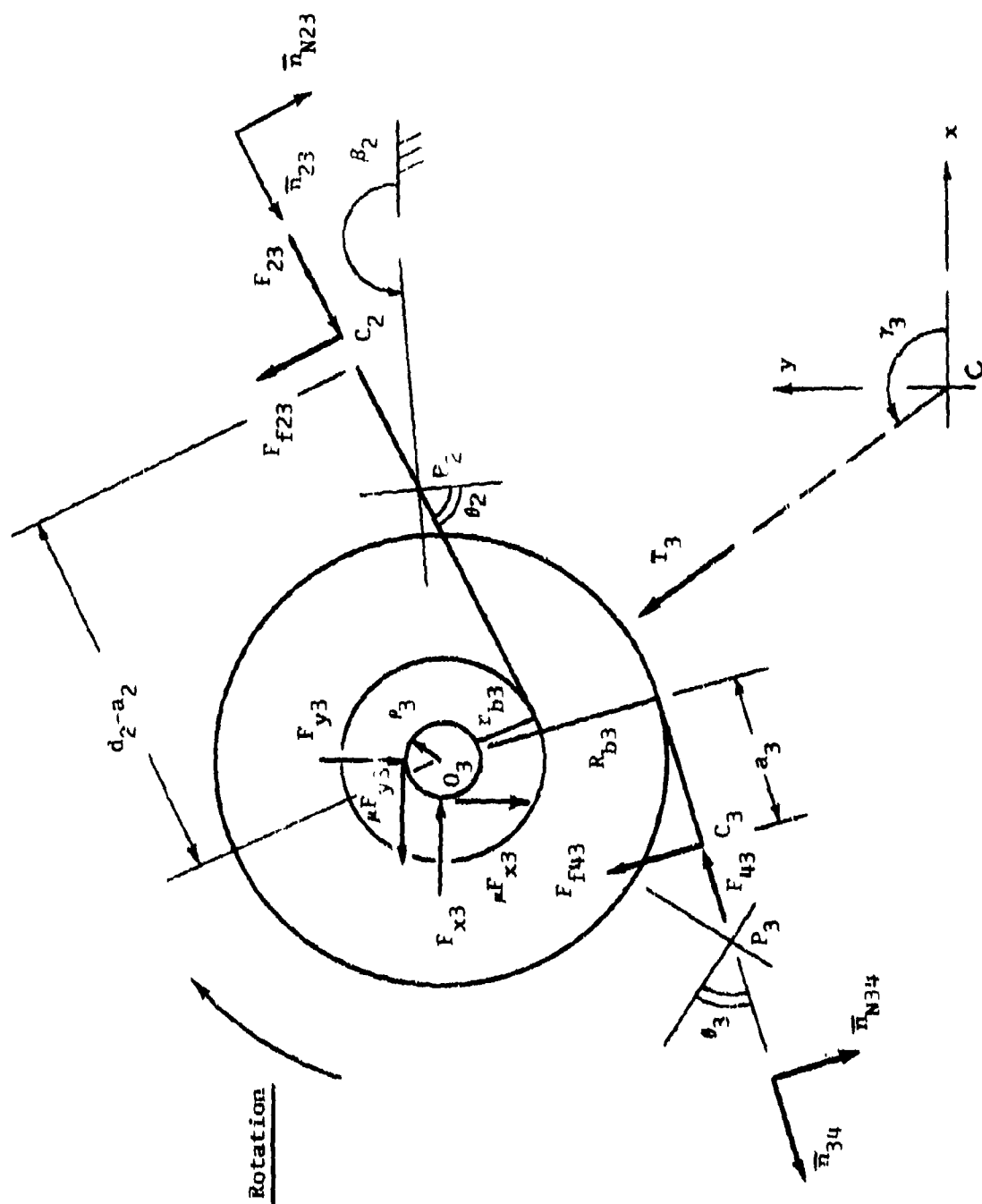
FIGURE A-7. EQUILIBRIUM OF GEAR AND PINION SET NO. 3

A-28

The contact point $C_2$ between gear 2 and pinion 3 is also shown before pitch point $P_2$ is passed.

The normal contact force between these teeth then becomes

$$\overline{F}_{23} = F_{23}\overline{n}_{23} \tag{A-53}$$

The associated friction force is given by

$$\overline{F}_{f23} = -\mu s_2 F_{23}\overline{n}_{N23} \tag{A-54}$$

where $s_2 = +1$ for contact before pitch point $P_2$

$s_2 = 0$ for contact at pitch point $P_2$

$s_2' = -1$ for contact after pitch point $P_2$

The normal forces on the pivot shaft are given by

$$\overline{F}_{x3} = F_{x3}\overline{I} \tag{A-55}$$

and

$$\overline{F}_{y3} = -F_{y3}\overline{J} \tag{A-56}$$

The associated pivot friction forces are represented by $(-)\mu F_{y3}\overline{I}$

and $(-) \mu F_{x3} \bar{J}$ for the indicated direction of gear rotation.

The centrifugal force, $\bar{T}_3$, on the assembly is given by

$$\bar{T}_3 = T_3 (\cos \gamma_3 \bar{I} + \sin \gamma_3 \bar{J}) \tag{A-57}$$

where

$$T_3 = R_3 \omega^2 m_3 \tag{A-58a}$$

with

$$m_3 = \text{mass of pinion and gear 3} \tag{A58b}$$

Force equilibrium is given by

$$F_{23} \bar{n}_{23} - \mu s_2 F_{23} \bar{n}_{N23} - F_{34} n_{34} - \mu s_3 F_{34} \bar{n}_{N34} + T_3 (\cos \gamma_3 \bar{I} + \sin \gamma_3 \bar{J})$$

$$+ F_{x3} \bar{I} - \mu F_{y3} \bar{I} - F_{y3} \bar{J} - \mu F_{x3} \bar{J} = 0 \tag{A-59}$$

Moment equilibrium requires

$$R_{b3} F_{34} - \mu s_3 a_3 F_{34} - r_{b3} F_{23} + \mu s_2 (d_2 - a_2) F_{23}$$

$$+ \mu \rho_3 (\tilde{F}_{x3} + \tilde{F}_{y3}) = 0 \tag{A-60}$$

Note the use of equation (A-3b) for the pivot friction moment. $\rho_3$ represents the pivot radius and length $d_2$ is the length of the

line-of-action between the points of tangency to the base circles. $a_2$ is the distance along the line-of-action from the gear point of tangency to the contact point $C_2$.

The component form of equation (A-59) is represented by the following two expressions:

$$-F_{23}\sin(\beta_2 - \theta_2) + \mu a_2 F_{23}\cos(\beta_2 - \theta_2) - F_{34}\sin(\beta_3 + \theta_3)$$
$$- \mu a_3 F_{34}\cos(\beta_3 + \theta_3) + T_3\cos\gamma_3 + F_{x3} - \mu F_{y3} = 0 \qquad (A-61)$$

and

$$F_{23}\cos(\beta_2 - \theta_2) + \mu a_2 F_{23}\sin(\beta_2 - \theta_2) + F_{34}\cos(\beta_3 + \theta_3)$$
$$- \mu a_3 F_{34}\sin(\beta_3 + \theta_3) + T_3\sin\gamma_3 - F_{y3} - \mu F_{x3}$$
$$= 0 \qquad (A-62)$$

Simultaneous solution of the above for $F_{x3}$ and $F_{y3}$ leads to

$$F_{x3} = \frac{1}{1 + \mu^2}\left\{F_{23}\left[(1 + \mu^2 a_2)\sin(\beta_2 - \theta_2) + \mu(1 - a_2)\cos(\beta_2 - \theta_2)\right]\right.$$
$$+ T_3\left[\mu\sin\gamma_3 - \cos\gamma_3\right] + F_{34}\left[(1 - \mu^2 a_3)\sin(\beta_3 + \theta_3)\right.$$
$$\left.\left. + \mu(1 + a_3)\cos(\beta_3 + \theta_3)\right]\right\} \qquad (A-63)$$

and

$$F_{y3} = \frac{1}{1 + \mu^2} \left\{ F_{23}\left[(1 + \mu^2 s_2)\cos(\beta_2 - \theta_2) + \mu(s_2 - 1)\sin(\beta_2 - \theta_2)\right] \right.$$
$$\left. + T_3\left[\sin\gamma_3 + \cos\gamma_3\right] + F_{34}\left[(1 - \mu^2 s_3)\cos(\beta_3 + \theta_3)\right.\right.$$
$$\left.\left. - \mu(s_3 + 1)\sin(\beta_3 + \theta_3)\right]\right\} \tag{A-64}$$

Now, equations (A-63) and (A-64) are substituted into the moment equation (A-60) with consideration of the pivot friction moment according to equation (A-3b). This gives

$$R_{b3}F_{34} - \mu s_3 a_3 F_{34} - r_{b3}F_{23} + \mu s_2(d_2 - a_2)F_{23}$$
$$+ \mu \rho_3\left[F_{23}(A_5 + A_8) + T_3(A_6 + A_9) + F_{34}(A_7 + A_{10})\right]$$
$$= 0 \tag{A-65}$$

where

$$A_5 = \left| \frac{(1 + \mu^2 s_2)\cos(\beta_2 - \theta_2) + \mu(s_2 - 1)\sin(\beta_2 - \theta_2)}{1 + \mu^2} \right| \tag{A-66}$$

$$A_6 = \left| \frac{\sin\gamma_3 + \mu\cos\gamma_3}{1 + \mu^2} \right| \tag{A-67}$$

A-32

$$A_7 = \left| \frac{(1 - \mu^2 s_3)\cos(\beta_3 + \theta_3) - \mu(1 + s_3)\sin(\beta_3 + \theta_3)}{1 + \mu^2} \right| \quad \text{(A-68)}$$

$$A_8 = \left| \frac{(1 + \mu^2 s_2)\sin(\beta_2 - \theta_2) + \mu(1 - s_2)\cos(\beta_2 - \theta_2)}{1 + \mu^2} \right| \quad \text{(A-69)}$$

$$A_9 = \left| \frac{\mu \sin \gamma_3 - \cos \gamma_3}{1 + \mu^2} \right| \quad \text{(A-70)}$$

$$A_{10} = \left| \frac{(1 - \mu^2 s_3)\sin(\beta_3 + \theta_3) + \mu(1 + s_3)\cos(\beta_3 + \theta_3)}{1 + \mu^2} \right| \quad \text{(A-71)}$$

Finally equation (A-65) is solved for $F_{23}$

$$F_{23} = \frac{F_{34}C_2 + T_3 C_3}{D_2} \quad \text{(A-72)}$$

where

$$C_2 = R_{b3} - \mu\left[s_3 a_3 - \rho_3(A_7 + A_{10})\right] \quad \text{(A-73)}$$

$$C_3 = \mu \rho_3(A_6 + A_9) \quad \text{(A-74)}$$

$$D_2 = r_{b3} - \mu\left[s_2(d_2 - a_2) + \rho_3(A_5 + A_8)\right] \quad \text{(A-75)}$$

A-33

## c.  EQUILIBRIUM OF GEAR AND PINION SET NO. 2

Figure A-8 shows the free body diagram of gear and pinion set no. 2.  The contact point $C_2$, between gear 2 and pinion 3, is again shown before the pitch point $P_2$ is passed.  (See also Figure A-7.) The normal force, along the line-of-action, becomes with equation (A-72)

$$\bar{F}_{32} = -F_{23}\bar{n}_{23} \qquad \text{(A-76)}$$

The associated friction force, $\bar{F}_{f32}$, is given by

$$\bar{F}_{f32} = \mu s_2 F_{23}\bar{n}_{N23} \qquad \text{(A-77)}$$

The unit vectors along(and perpendicular to)the line-of-action of gear 1 and pinion 2 are given by

$$\bar{n}_{12} = \sin(\beta_1 + \theta_1)\bar{I} - \cos(\beta_1 + \theta_1)\bar{J} \qquad \text{(A-78)}$$

and

$$\bar{n}_{N12} = \cos(\beta_1 + \theta_1)\bar{I} + \sin(\beta_1 + \theta_1)\bar{J} \qquad \text{(A-79)}$$

The contact point $C_1$ between gear 1 and pinion 2 is also shown before pitch point $P_1$ is passed.

FIGURE A8. FREE BODY DIAGRAM OF GEAR AND PINION SET NO. 2

A-35
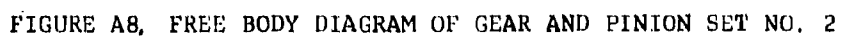
The normal contact force between the teeth of this mesh becomes

$$\overline{F}_{12} = F_{12}\overline{n}_{12} , \tag{A-80}$$

while the associated friction force is given by
(See Section 1 of this Appendix.)

$$\overline{F}_{f12} = \mu s_1 F_{12}\overline{n}_{N12} \tag{A-81}$$

where   $s_1 = +1$   for contact before pitch point $P_1$

   $s_1 = 0$   for contact at pitch point $P_1$

   $s_1 = -1$   for contact after pitch point $P_1$

The normal forces on the pivot shaft are

$$\overline{F}_{x2} = -F_{x2}\overline{I} \tag{A-82}$$

and

$$\overline{F}_{y2} = -F_{y2}\overline{J} \tag{A-83}$$

The associated pivot friction forces are again chosen such that

A-36

their friction moments oppose the indicated rotation.

The centrifugal force, $\overline{T}_2$, on this gear and pinion assembly is given by

$$\overline{T}_2 = T_2(\cos\gamma_2\overline{I} + \sin\gamma_2\overline{J}) \qquad\qquad (A-84)$$

where $T_2 = \mathcal{R}_2\omega^2 m_2$ $\qquad\qquad (A-85a)$

with

$\qquad m_2 = $ mass of gear and pinion set no. 2 $\qquad (A-85b)$

Force equilibrium is given by

$$-F_{23}\overline{n}_{23} + \mu s_2 F_{23}\overline{n}_{N23} + F_{12}\overline{n}_{12} + \mu s_1 F_{12}\overline{n}_{N12}$$

$$\div T_2(\cos\gamma_2\overline{I} + \sin\gamma_2\overline{J}) - F_{x2}\overline{I} - F_{y2}\overline{J} + \mu F_{y2}\overline{I} - \mu F_{x2}\overline{J}$$

$$= 0 \qquad\qquad (A-86)$$

Moment equilibrium is given by

$$-R_{b2}F_{23} + \mu s_2 a_2 F_{23} + r_{b2}F_{12} - \mu s_1(d_1 - a_1)F_{12}$$

$$- \mu \rho_2(\widetilde{F}_{x2} + \widetilde{F}_{y2}) = 0 \qquad (A-87)$$

Again, equation (A-3b) is used to account for the pivot friction moment. $\rho_2$ represents the pivot radius. $d_1$ is the length of the line-of-action between the points of tangency to the base circles, and $a_1$ is the distance from the point of tangency of the gear to the contact point $C_1$.

The component form of equation (A-86) is given by

$$F_{23}\sin(\beta_2 - \theta_2) - \mu s_2 F_{23}\cos(\beta_2 - \theta_2) + F_{12}\sin(\beta_1 + \theta_1)$$
$$+ \mu s_1 F_{12}\cos(\beta_1 + \theta_1) + T_2\cos\gamma_2 - F_{x2} + \mu F_{y2}$$
$$= 0 \qquad (A-88)$$

$$-F_{23}\cos(\beta_2 - \theta_2) - \mu s_2 F_{23}\sin(\beta_2 - \theta_2) - F_{12}\cos(\beta_1 + \theta_1)$$
$$+ \mu s_1 F_{12}\sin(\beta_1 + \theta_1) + T_2\sin\gamma_2 - F_{y2} - \mu F_{x2}$$
$$= 0 \qquad (A-89)$$

Simultaneous solution of the above furnishes

$$F_{x2} = \frac{1}{1 + \mu^2}\left\{-F_{12}\left[\mu(1 - a_1)\cos(\beta_1 + \theta_1) - (1 + \mu^2 s_1)\sin(\beta_1 + \theta_1)\right]\right.$$
$$+ T_2\left[\mu\sin\gamma_2 + \cos\gamma_2\right]$$
$$\left.+ F_{23}\left[(1 - \mu^2 s_2)\sin(\beta_2 - \theta_2) - \mu(1 + s_2)\cos(\beta_2 - \theta_2)\right]\right\}$$
$$(A-90)$$

A-38

and

$$F_{y2} = \frac{1}{1 + \mu^2} \left\{ -F_{12}\left[ \mu(1 - s_1)\sin(\beta_1 + \theta_1) + (1 + \mu^2 s_1)\cos(\beta_1 + \theta_1) \right] \right.$$
$$+ T_2\left[ \sin\gamma_2 - \mu\cos\gamma_2 \right]$$
$$\left. + F_{23}\left[ -\mu(1 + s_2)\sin(\beta_2 - \theta_2) - (1 - \mu^2 s_2)\cos(\beta_2 - \theta_2) \right] \right\}$$

(A-91)

Now, equations (A-90) and (A-91) are substituted into the moment equation (A-87). With consideration of the pivot friction moment according to equation (A-3b), this gives

$$-R_{b2}F_{23} + \mu s_2 a_2 F_{23} + r_{b2}F_{12} - \mu s_1(d_1 - a_1)F_{12}$$
$$- \mu P_2\left[ F_{12}(A_{11} + A_{14}) + T_2(A_{12} + A_{15}) + F_{23}(A_{13} + A_{16}) \right]$$
$$= 0 \qquad (A-92)$$

In the above

$$A_{11} = \left| \frac{\mu(1 - s_1)\sin(\beta_1 + \theta_1) + (1 + \mu^2 s_1)\cos(\beta_1 + \theta_1)}{1 + \mu^2} \right| \qquad (A-93)$$

$$A_{12} = \left| \frac{\sin\gamma_2 - \mu\cos\gamma_2}{1 + \mu^2} \right| \qquad (A-94)$$

$$A_{13} = \left| \frac{\mu(1 + s_2)\sin(\beta_2 - \theta_2) + (1 - \mu^2 s_2)\cos(\beta_2 - \theta_2)}{1 + \mu^2} \right| \quad (A-95)$$

$$A_{14} = \left| \frac{\mu(1 - s_1)\cos(\beta_1 + \theta_1) - (1 + \mu^2 s_1)\sin(\beta_1 + \theta_1)}{1 + \mu^2} \right| \quad (A-96)$$

$$A_{15} = \left| \frac{\mu\sin\gamma_2 + \cos\gamma_2}{1 + \mu^2} \right| \quad (A-97)$$

$$A_{16} = \left| \frac{(1 - \mu^2 s_2)\sin(\beta_2 - \theta_2) - \mu(1 + s_2)\cos(\beta_2 - \theta_2)}{1 + \mu^2} \right| \quad (A-98)$$

Finally, equation (A-92) is solved for $F_{12}$

$$F_{12} = \frac{F_{23}C_4 + T_2 C_5}{D_3} \quad (A-99)$$

where

$$C_4 = R_{b2} - \mu\left[s_2 a_2 - \rho_2(A_{13} + A_{16})\right] \quad (A-100)$$

$$C_5 = \mu\rho_2(A_{12} + A_{15}) \quad (A-101)$$

$$D_3 = r_{b2} - \mu\left[s_1(d_1 - a_1) + \rho_2(A_{11} + A_{14})\right] \quad (A-102)$$

A-40

d. EQUILIBRIUM OF GEAR NO. 1

Figure A-9 shows the free body diagram of gear no. 1, the input gear.

The contact point $C_1$ is identical with that shown in Figure A-8.

The normal force on gear no. 1 is given by   (See equation (A-80).)

$$\overline{F}_{21} \;=\; -F_{12}\overline{n}_{12} \tag{A-103}$$

The associated friction force is

$$\overline{F}_{f21} \;=\; -\mu s_1 F_{12}\overline{n}_{N12} \tag{A-104}$$

The normal forces on the pivot shaft are given by

$$\overline{F}_{x1} \;=\; -F_{x1}\overline{I} \tag{A-105}$$

and

$$\overline{F}_{y1} \;=\; F_{y1}\overline{J} \tag{A-106}$$

The associated pivot friction forces are chosen in such a direction that their moments oppose the indicated rotation.

A-41
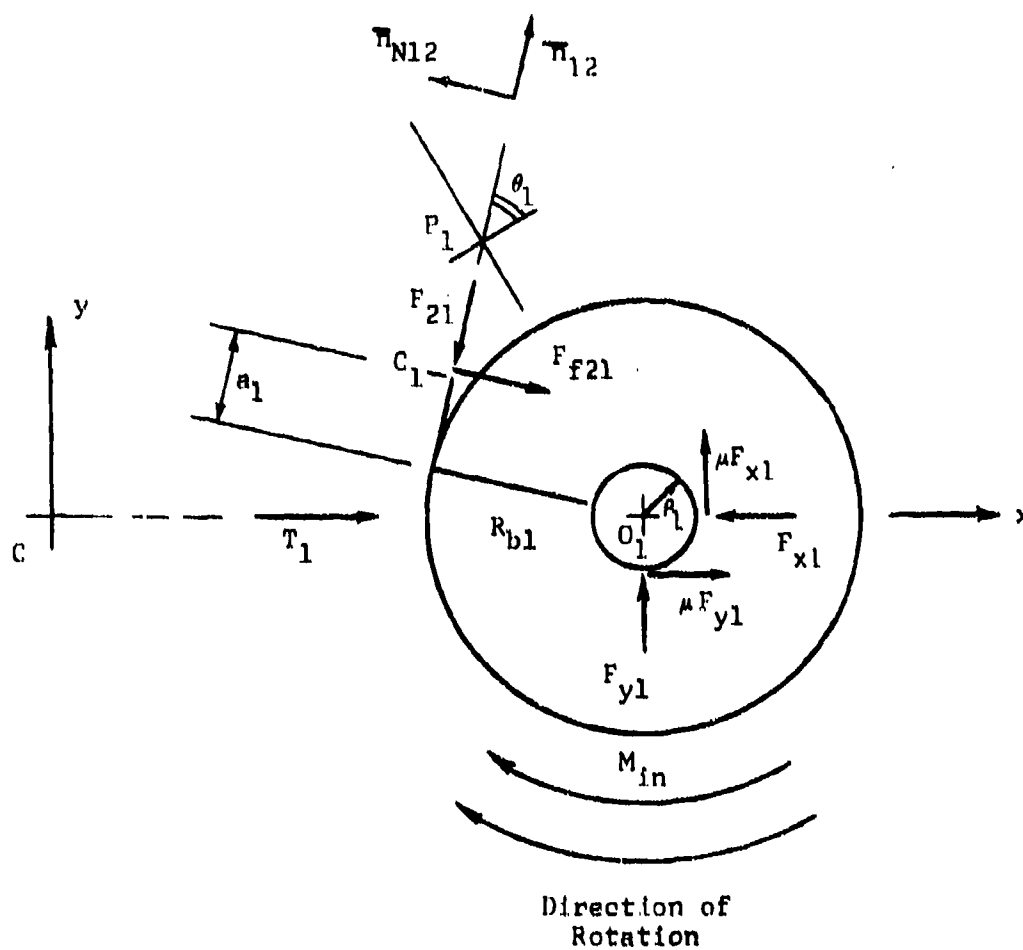
FIGURE A9. FREE BODY DIAGRAM OF GEAR 1

The centrifugal force $\overline{T}_1$ on gear 1 is given by

$$\overline{T}_1 = T_1 \overline{I} \tag{A-107}$$

where

$$T_1 = \mathcal{R}_1 \omega^2 m_1 \tag{A-108a}$$

$$m_1 = \text{mass of gear 1} \tag{A-108b}$$

Force equilibrium is given by

$$-F_{12}\overline{R}_{12} - \mu s_1 F_{12}\overline{R}_{N12} + T_1\overline{I} - F_{x1}\overline{I} + F_{y1}\overline{J} + \mu F_{y1}\overline{I}$$
$$+ \mu F_{x1}\overline{J} = 0 \tag{A-109}$$

Moment equilibrium is found from

$$R_{b1}F_{12} - \mu s_1 a_1 F_{12} - M_{1n} + \mu \rho_1(\widetilde{F}_{x1} + \widetilde{F}_{y1}) = 0 \tag{A-110}$$

The form of equation (A-3b) is again utilized to obtain the
pivot friction moment.  $\rho_1$ represents the pivot radius of
gear 1.

The component form of equation (A-109) becomes

$$-F_{12}\sin(\beta_1 + \theta_1) - \mu s_1 F_{12}\cos(\beta_1 + \theta_1) - F_{x1} + \mu F_{y1} + T_1$$
$$= 0 \qquad (A-111)$$

and

$$F_{12}\cos(\beta_1 + \theta_1) - \mu s_1 F_{12}\sin(\beta_1 + \theta_1) + F_{y1} + \mu F_{x1}$$
$$= 0 \qquad (A-112)$$

Simultaneous solution of equations (A-111) and (A-112) furnishes the forces on the pivot, i.e.,

$$F_{x1} = \frac{-F_{12}\left[(1 - \mu^2 s_1)\sin(\beta_1 + \theta_1) + \mu(1 + s_1)\cos(\beta_1 + \theta_1)\right] + T_1}{1 + \mu^2}$$
$$\qquad (A-113)$$

$$F_{y1} = \frac{F_{12}\left[\mu(1 + s_1)\sin(\beta_1 + \theta_1) - (1 - \mu^2 s_1)\cos(\beta_1 + \theta_1)\right] - \mu T_1}{1 + \mu^2}$$
$$\qquad (A-114)$$

Equations (A-113) and (A-114) are now substituted into the moment equation (A-110) in the following manner: (Again, the method of equation (A-3b) is applied.)

$$R_{b1}F_{12} - \mu s_1 a_1 F_{12} - M_{1n} + \mu \rho_1 \left[ F_{12}(A_{17} + A_{19}) + T_1(A_{18} + A_{20}) \right]$$
$$= 0 \qquad (A-115)$$

where

$$A_{17} = \left| \frac{(1 - \mu^2 s_1)\sin(\beta_1 + \theta_1) + \mu(1 + s_1)\cos(\beta_1 + \theta_1)}{1 + \mu^2} \right| \qquad (A-116)$$

$$A_{18} = \left| \frac{1}{1 + \mu^2} \right| \qquad (A-117)$$

$$A_{19} = \left| \frac{\mu(1 + s_1)\sin(\beta_1 + \theta_1) - (1 - \mu^2 s_1)\cos(\beta_1 + \theta_1)}{1 + \mu^2} \right| \qquad (A-118)$$

$$A_{20} = \left| \frac{\mu}{1 + \mu^2} \right| \qquad (A-119)$$

Finally, equation (A-115) is solved for $F_{12}$

$$F_{12} = \frac{M_{1n}}{D_4} - \frac{T_1 C_6}{D_4} \qquad (A-120)$$

where

$$C_6 = \mu \rho_1 (A_{18} + A_{20}) \qquad (A-121)$$

$$D_4 = R_{b1} - \mu \left[ s_1 a_1 - \rho_1 (A_{17} + A_{19}) \right] \qquad (A-122)$$

A-45

e. <u>INPUT-OUTPUT RELATIONSHIP</u>

To obtain the input-output relationship for the complete gear train, equation (A-120) is now equated to equation (A-99). This furnishes

$$F_{23} = \frac{D_3}{C_4 D_4} (M_{1n} - T_1 C_6) - T_2 \frac{C_5}{C_4} \qquad (A-123)$$

Further, the above is equated to equation (A-72). This results in the following expression for $F_{34}$

$$F_{34} = \frac{D_2 D_3}{C_7 C_4 D_4} (M_{1n} - T_1 C_6) - T_2 \frac{C_5 D_2}{C_2 C_4} - T_3 \frac{C_3}{C_2} \qquad (A-124)$$

Finally, equation (A-124) is equated to equation (A-46). This establishes the input-output relationship

$$M_{o4} = \frac{D_1 D_2 D_7}{C_2 C_4 D_4} (M_{1n} - T_1 C_6) - T_2 \frac{C_5 D_1 D_2}{C_2 C_4} - T_3 \frac{C_3 D_1}{C_2} - T_4 C_1$$

$$(A-125)$$

A-46

## 5. MOMENT INPUT-OUTPUT RELATIONSHIP FOR INVOLUTE TWO STEP-UP GEAR TRAIN IN SPIN ENVIRONMENT

Figure A-10 shows the basic configuration of a two step-up gear train with involute teeth for which the relationship between the equilibrant output moment, $M_{o3}$, acting on pinion 3, and the input moment, $M_{in}$, acting on gear 1 is to be found. All nomenclature is identical with that used in Section 4 in connection with the three step-up gear train.

Again, the general relationship between input and output is found by assembling the input-output relationships of the individual component gears.

## a. EQUILIBRIUM OF PINION 3

Figure A-11 shows the free body diagram of pinion 3. The contact point $C_2$ between gear 2 and pinion 3 is shown before the pitch point $P_2$ is passed.
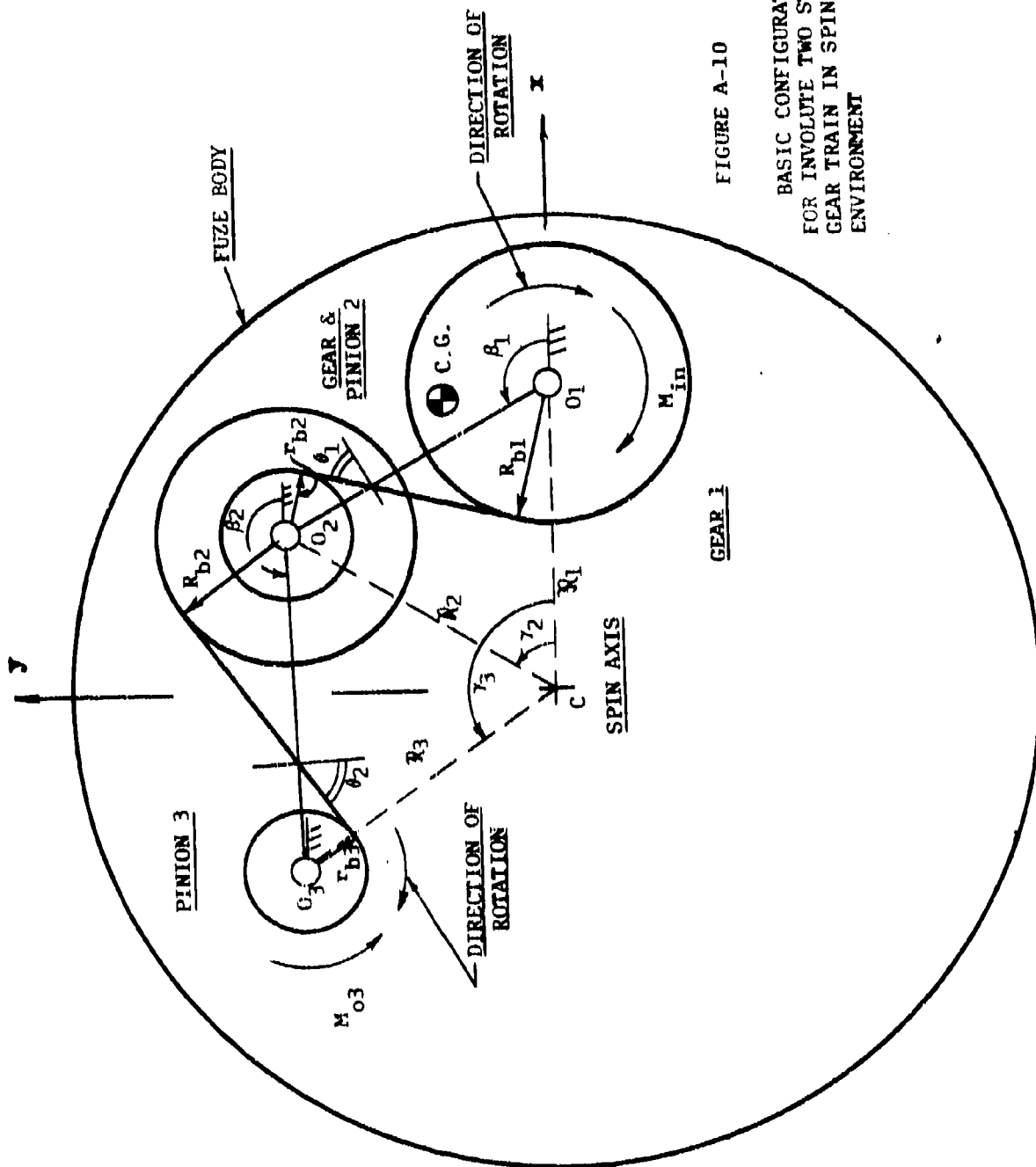
FIGURE A-10

BASIC CONFIGURATION
FOR INVOLUTE TWO STEP-UP
GEAR TRAIN IN SPIN
ENVIRONMENT
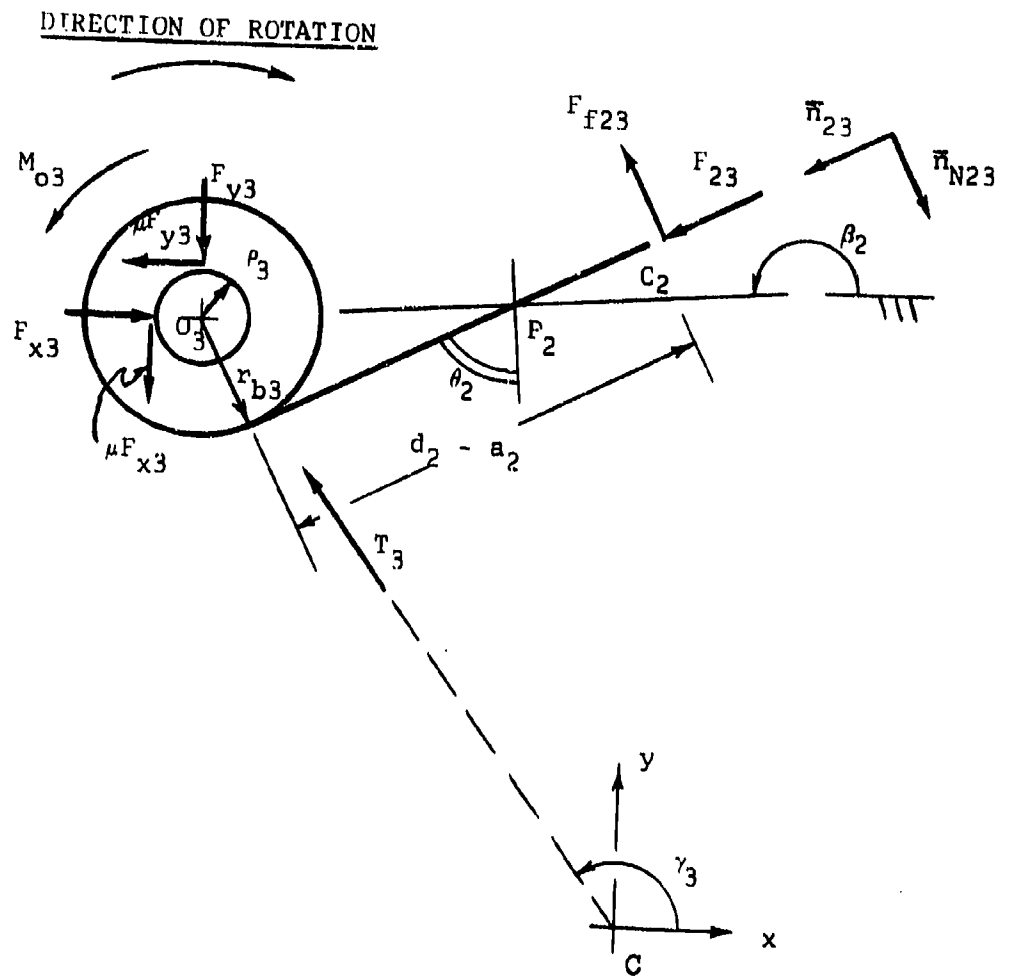
FIGURE A-11    FREE BODY DIAGRAM OF PINION 3

As in equation (A-53), the normal force between the teeth of gear 2 and pinion 3 is given by

$$\overline{F}_{23} = F_{23}\overline{n}_{23} \qquad (A-126)$$

The associated friction force is given by equation (A-54), i.e.,

$$\overline{F}_{f23} = -\mu s_2 F_{23}\overline{n}_{N23} \qquad (A-127)$$

where  $s_2 = +1$ ,  for contact before pitch point $P_2$

$s_2 = 0$ ,  for contact at  pitch point $P_2$

$s_2 = -1$ ,  for contact after  pitch point $P_2$

The unit vectors in equations (A-126) and (A-127) were defined by equations (A-51) and (A-52), respectively. The normal forces on the pivot shaft are given by

$$\overline{F}_{x3} = F_{x3}\overline{I} \qquad (A-128)$$

and

$$\overline{F}_{y3} = -F_{y3}\overline{J} \qquad (A-129)$$

The pivot friction forces become $(-)\mu F_{y3}\bar{I}$ and $(-)\mu F_{x3}\bar{J}$ for the indicated direction of rotation.

As in equation (A-57), the centrifugal force on the pinion is given by

$$\bar{T}_3 = T_3(\cos\gamma_3\bar{I} + \sin\gamma_3\bar{J}) \qquad (A-130)$$

where

$$T_3 = R_3\omega^2 m_3 \qquad (A-131)$$

with

$$m_3 = \text{mass of pinion 3} \qquad (A-132)$$

Force equilibrium is given by

$$F_{23}\bar{n}_{23} - \mu s_2 F_{23}\bar{n}_{N23} + T_3(\cos\gamma_3\bar{I} + \sin\gamma_3\bar{J}) + F_{x3}\bar{I}$$
$$- \mu F_{y3}\bar{I} - F_{y3}\bar{J} - \mu F_{x3}\bar{J} = 0 \qquad (A-133)$$

Moment equilibrium is obtained from

$$M_{o3} - r_{b3}F_{23} + \mu s_2(d_2 - a_2)F_{23} + \mu\rho_3(\tilde{F}_{x3} + \tilde{F}_{y3}) = 0$$
$$(A-134)$$

A-51

Note the use of equation (A-3b) for the pivot friction moment. $r_3$ represents the pivot radius. $d_2$ is the length of the line of action between the points of tangency to the base circles of pinion 3 and gear 2. $a_2$ is the distance along the line-of-action from the gear point of tangency to contact point $C_2$.

The x and y components of equation (A-133) are given by

$$-F_{23}\sin(\beta_2 - \theta_2) + \mu s_3 F_{23}\cos(\beta_2 - \theta_2) + T_3\cos\gamma_3 + F_{x3}$$
$$- \mu F_{y3} = 0 , \qquad \text{(A-135)}$$

and

$$F_{23}\cos(\beta_2 - \theta_2) + \mu s_3 F_{23}\sin(\beta_2 - \theta_2) + T_3\sin\gamma_3 - F_{y3}$$
$$- \mu F_{x3} = 0 \qquad \text{(A-136)}$$

Simultaneous solution of these expressions for $F_{x3}$ and $F_{y3}$ yields

$$F_{x3} = \frac{1}{1 + \mu^2}\left\{ F_{23}\left[(1 + \mu^2 s_2)\sin(\beta_2 - \theta_2) + \mu(1 - s_2)\cos(\beta_2 - \theta_2)\right]\right.$$
$$\left. + T_3\left[\mu\sin\gamma_3 - \cos\gamma_3\right] \right\} \qquad \text{(A-137)}$$

and

$$F_{y3} = \frac{1}{1 + \mu^2} \left\{ F_{23} \left[ (1 + \mu^2 s_2)\cos(\beta_2 - \theta_2) + \mu(s_2 - 1)\sin(\beta_2 - \theta_2) \right] \right.$$
$$\left. + T_3 \left[ \sin\gamma_3 + \mu\cos\gamma_3 \right] \right\} \qquad (A\text{-}138)$$

Now, equations (A-137) and (A-138) are substituted into the moment equation (A-134), with the pivot friction moment given according to the formulation of equation (A-3b):

$$M_{o3} - r_{b3}F_{23} + \mu s_2(d_2 - a_2 F_{23} + \mu\rho_3 \left[ F_{23}(A_1 + A_3) + T_3(A_2 + A_4) \right]$$
$$= 0 \qquad (A\text{-}139)$$

where

$$A_1 = \left| \frac{(1 + \mu^2 s_2)\cos(\beta_2 - \theta_2) + \mu(s_2 - 1)\sin(\beta_2 - \theta_2)}{1 + \mu^2} \right| \qquad (A\text{-}140)$$

$$A_2 = \left| \frac{\sin\gamma_3 + \mu\cos\gamma_3}{1 + \mu^2} \right| \qquad (A\text{-}141)$$

$$A_3 = \left| \frac{(1 + \mu^2 s_2)\sin(\beta_2 - \theta_2) + \mu(1 - s_2)\cos(\beta_2 - \theta_2)}{1 + \mu^2} \right| \qquad (A\text{-}142)$$

A-53

$$A_4 = \left| \frac{\mu \sin\gamma_3 - \cos\gamma_3}{-1 + \mu^2} \right| \tag{A-143}$$

Finally, equation (A-139) is solved for $F_{23}$

$$F_{23} = \frac{M_{o3}}{D_1} + T_3 \frac{C_1}{D_1} \tag{A-144}$$

where

$$C_1 = \mu \rho_3 (A_2 + A_4) \tag{A-145}$$

$$D_1 = r_{b3} - \mu \left[ s_2(d_2 - a_2) + \rho_3(A_1 + A_3) \right] \tag{A-146}$$

## b.   EQUILIBRIUM OF GEAR AND PINION SET NO. 2

Figure A-12 shows the free body diagram of gear and pinion set no. 2.  The contact point, $C_2$, between gear 2 and pinion 3 is again shown before the pitch point, $P_2$.  With equation (A-126), the normal force between the teeth of this mesh becomes

$$\bar{F}_{32} = -F_{23}\bar{n}_{23} \tag{A-147}$$

The associated friction force is the negative of equation (A-127), i.e.,

$$\bar{F}_{f32} = \mu s_2 F_{23}\bar{n}_{N23} \tag{A-148}$$

Similar to equation (A-80), the normal force between gear 1 and pinion 2 is given by

$$\bar{F}_{12} = F_{12}\bar{n}_{12} \tag{A-149}$$

(See equation (A-78) for the definition of unit vector $\bar{n}_{12}$.)
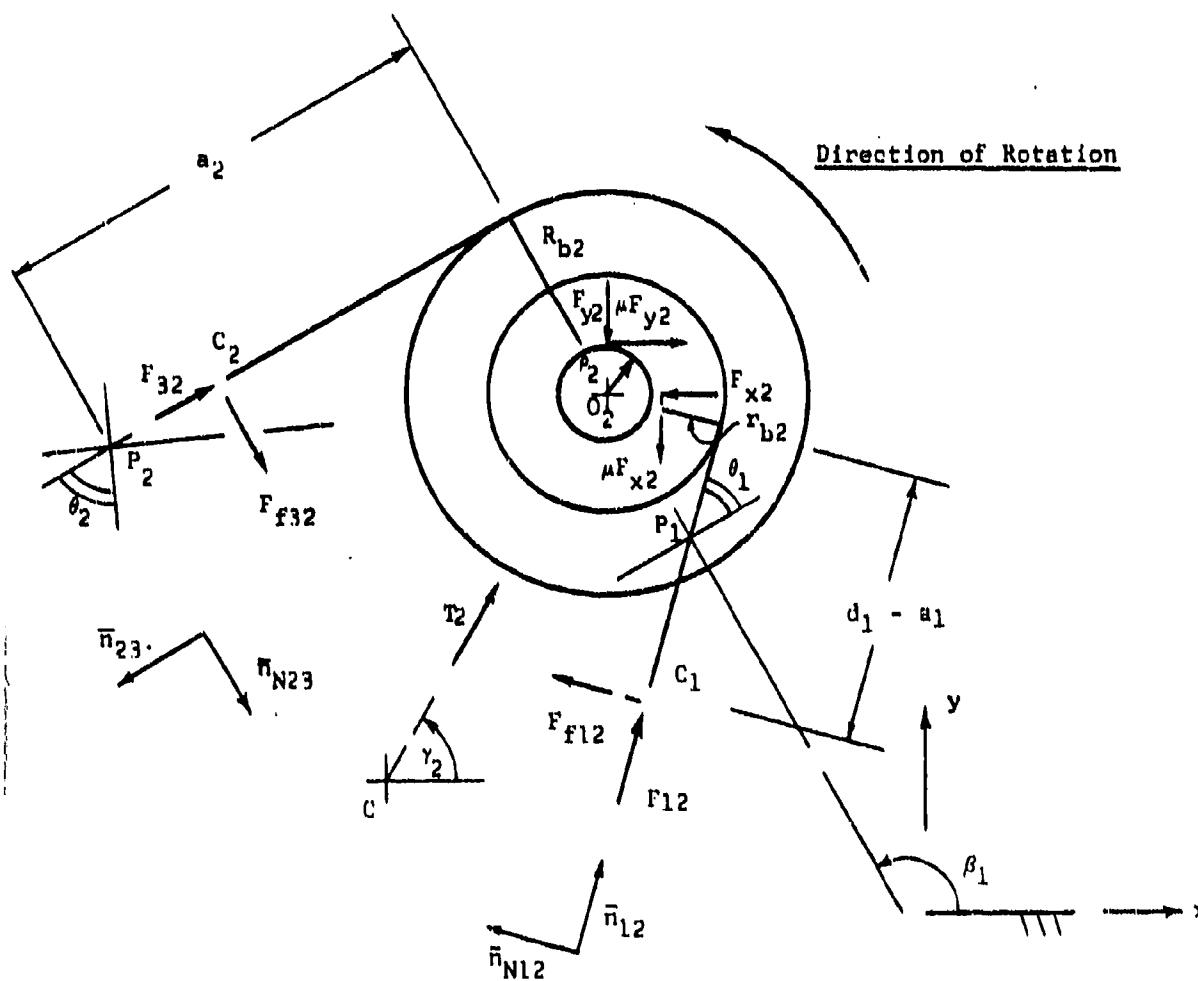
FIGURE A-12  FREE BODY DIAGRAM OF GEAR AND PINION SET NO. 2

Figure A-12 shows the contact point $C_1$ before the pitch point $P_1$ is passed.

The associated friction force is given by

$$\bar{F}_{f12} = \mu s_1 F_{12} \bar{n}_{N12} \tag{A-150}$$

where $s_1 = +1$ , for contact before pitch point $P_1$

$\qquad s_1 = 0$ , for contact at pitch point $P_1$

$\qquad s_1 = -1$ , for contact after pitch point $P_1$

The unit vector $\bar{n}_{N12}$ is given by equation (A-79).

The normal forces on the pivot shaft are given by

$$\bar{F}_{x2} = -F_{x2}\bar{I} \tag{A-151}$$

and

$$\bar{F}_{y2} = -F_{y2}\bar{J} \tag{A-152}$$

The associated friction forces $\mu F_{y2}\bar{I}$ and $(-)\mu F_{x2}\bar{J}$ were chosen so that their moments oppose the indicated rotation.

The centrifugal force on this gear and pinion assembly is expressed by

$$\bar{T}_2 = T_2(\cos\gamma_2 \bar{I} + \sin\gamma_2 \bar{J}) \tag{A-153}$$

$$\text{where } T_2 = \mathcal{R}_2 \omega^2 m_2 \tag{A-154}$$

$$m_2 = \text{mass of gear and pinion set no. 2} \tag{A-155}$$

Force equilibrium is given by

$$-F_{23}\bar{n}_{23} + \mu s_2 F_{23}\bar{n}_{N23} + F_{12}\bar{n}_{12} + \mu s_1 F_{12}\bar{n}_{N12}$$
$$+ T_2(\cos\gamma_2\bar{I} + \sin\gamma_2\bar{J}) - F_{x2}\bar{I} + \mu F_{y2}\bar{I} - F_{y2}\bar{J} - \mu F_{x2}\bar{J}$$
$$= 0 \tag{A-156}$$

Moment equilibrium is given by

$$-R_{b2}F_{23} + \mu s_2 a_2 F_{23} + r_{b2}F_{12} - \mu s_1(d_1 - a_1)F_{12}$$
$$- \mu \rho_2(\breve{F}_{x2} + \tilde{F}_{y2}) = 0 \tag{A-157}$$

Again, equation (A-3b) is used to obtain a conservative pivot friction moment. $\rho_2$ represents the pivot radius. $d_1$ and $a_1$ are similar to the previously used distances along the lines of action of the other meshes.

A-58

The component form of equation (A-156) becomes

$$F_{23}\sin(\beta_2 - \theta_2) - \mu s_2 F_{23}\cos(\beta_2 - \theta_2) + F_{12}\sin(\beta_1 + \theta_1)$$

$$+ \mu s_1 F_{12}\cos(\beta_1 + \theta_1) + T_2\cos\gamma_2 - F_{x2} + \mu F_{y2} = 0 \qquad (A-158)$$

and

$$-F_{23}\cos(\beta_2 - \theta_2) - \mu s_2 F_{23}\sin(\beta_2 - \theta_2) - F_{12}\cos(\beta_1 + \theta_1)$$

$$+ \mu s_1 F_{12}\sin(\beta_1 + \theta_1) + T_2\sin\gamma_2 - F_{y2} - \mu F_{x2} = 0 \qquad (A-159)$$

Simultaneous solution of the above expressions for $F_{x2}$ and $F_{y2}$ gives

$$F_{x2} = -\frac{1}{1 + \mu^2} \left\{ -F_{12}\left[\mu(1 - s_1)\cos(\beta_1 + \theta_1) - (1 + \mu^2 s_1)\sin(\beta_1 + \theta_1)\right] \right.$$

$$+ T_2\left[\mu\sin\gamma_2 + \cos\gamma_2\right]$$

$$\left. + F_{23}\left[(1 - \mu^2 s_2)\sin(\beta_2 - \theta_2) - \mu(1 + s_2)\cos(\beta_2 - \theta_2)\right] \right\}$$

$$(A-160)$$

A-59

$$F_{y2} = \frac{1}{1 + \mu^2}\left\{- F_{12}\left[\mu(1 - s_1)\sin(\beta_1 + \theta_1) + (1 + \mu^2 s_1)\cos(\beta_1 + \theta_1)\right]\right.$$

$$+ T_2\left[\sin\gamma_2 - \mu\cos\gamma_2\right]$$

$$\left.+ F_{23}\left[-\mu(1 + s_2)\sin(\beta_2 - \theta_2) - (1 - \mu^2 s_2)\cos(\beta_2 - \theta_2)\right]\right\}$$

$$(A\text{-}161)$$

Now equations (A-160) and (A-161) are substituted into the moment equation (A-157) with the pivot friction moment formulated again according to equation (A-3b):

$$-R_{b2}F_{23} + \mu s_2 a_2 F_{23} + r_{b2}F_{12} - \mu s_1(d_1 - s_1)F_{12}$$

$$- \mu P_2 \left[ F_{12}(A_5 + A_8) + T_2(A_6 + A_9) + F_{23}(A_7 + A_{10}) \right] = 0 \tag{A-162}$$

where

$$A_5 = \left| \frac{\mu(1 - s_1)\sin(\beta_1 + \theta_1) + (1 + \mu^2 s_1)\cos(\beta_1 + \theta_1)}{1 + \mu^2} \right| \tag{A-163}$$

$$A_6 = \left| \frac{\sin\gamma_2 - \mu\cos\gamma_2}{1 + \mu^2} \right| \tag{A-164}$$

$$A_7 = \left| \frac{\mu(1 + s_2)\sin(\beta_2 - \theta_2) + (1 - \mu^2 s_2)\cos(\beta_2 - \theta_2)}{1 + \mu^2} \right| \tag{A-165}$$

$$A_8 = \left| \frac{\mu(1 - s_1)\cos(\beta_1 + \theta_1) - (1 + \mu^2 s_1)\sin(\beta_1 + \theta_1)}{1 + \mu^2} \right| \tag{A-166}$$

$$A_9 = \left| \frac{\mu\sin\gamma_2 + \cos\gamma_2}{1 + \mu^2} \right| \tag{A-167}$$

A-61

$$A_{10} = \left| \frac{(1 - \mu^2 s_2)\sin(\beta_2 - \theta_2) - \mu(1 + s_2)\cos(\beta_2 - \theta_2)}{1 + \mu^2} \right| \qquad \text{(A-168)}$$

Finally equation (A-162) is solved for $F_{12}$

$$F_{12} = \frac{F_{23}C_2}{D_2} + \frac{T_2C_3}{D_2} \qquad \text{(A-169)}$$

where

$$C_2 = R_{b2} - \mu\left[s_2 a_2 - \rho_2(A_7 + A_{10})\right] \qquad \text{(A-170)}$$

$$C_3 = \mu\rho_2(A_6 + A_9) \qquad \text{(A-171)}$$

$$D_2 = r_{b2} - \mu\left[s_1(d_1 - a_1) + \rho_2(A_5 + A_8)\right] \qquad \text{(A172)}$$

c.   EQUILIBRIUM OF GEAR NO. 1

Figure A-13 shows the free body diagram of gear no. 1, the input gear.

The contact point, $C_1$, corresponds to that shown in Figure A-12.

According to equation (A-149), the normal contact force between pinion 2 and gear 1 becomes

$$\bar{F}_{21} \quad = \quad -F_{12}\bar{n}_{12} \tag{A-173}$$

The associated friction force is the negative of equation (A-150), i.e.,

$$\bar{F}_{f21} \quad = \quad - \mu s_1 F_{12}\bar{n}_{N12} \tag{A-174}$$

The normal forces on the pivot shaft are given by

$$\bar{F}_{x1} \quad = \quad -F_{x1}\bar{I} \tag{A-175}$$

and

$$\bar{F}_{y1} \quad = \quad F_{y1}\bar{J} \tag{A-176}$$
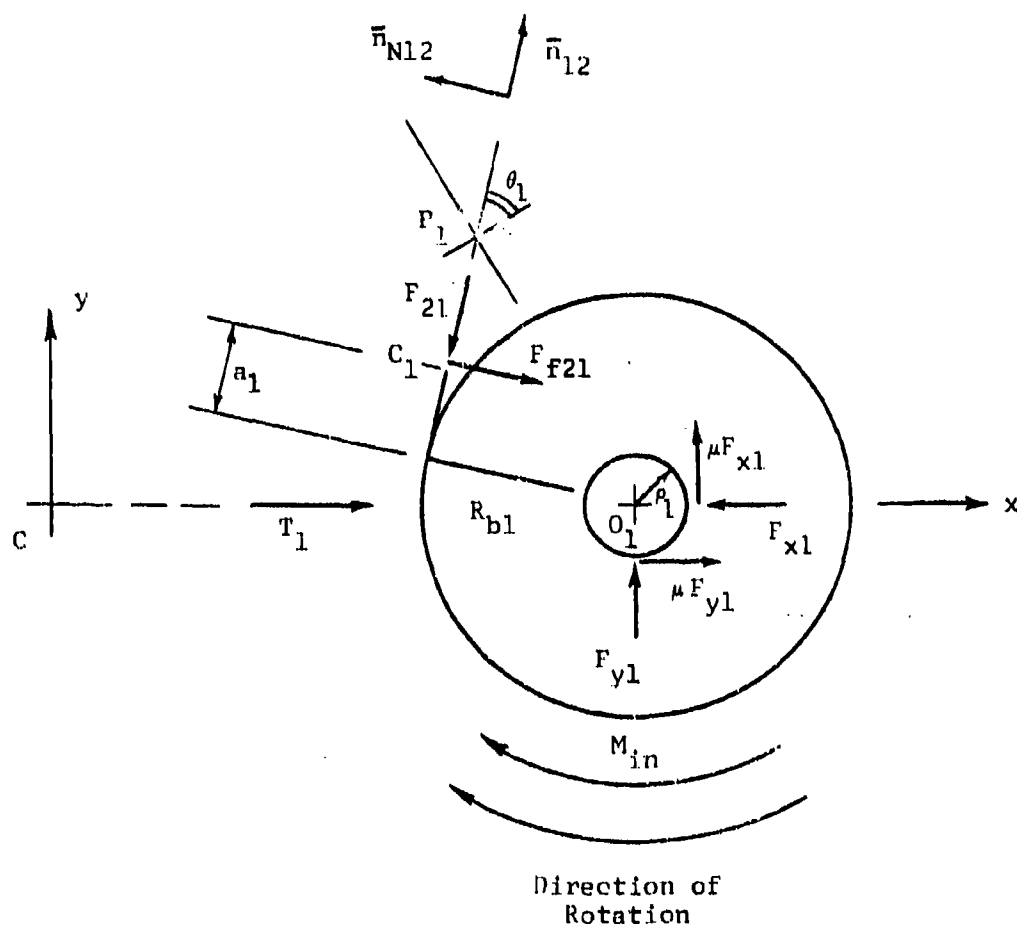
FIGURE A-13. FREE BODY DIAGRAM OF GEAR 1

A-64

The associated friction forces $\mu F_{y1}\bar{I}$ and $\mu F_{x1}\bar{J}$ were chosen with such directions that their moments oppose rotation due to input moment $M_{in}$.

Centrifugal force $\bar{T}_1$ on gear 1 is given by

$$\bar{T}_1 = T_1\bar{I} \tag{A-177}$$

where, as with equation (A-107)

$$T_1 = \mathcal{R}_1 \omega^2 m_1 \tag{A-178}$$

and

$$m_1 = \text{mass of gear 1} \tag{A-179}$$

Force equilibrium of gear 1 is given by

$$-F_{12}\bar{n}_{12} - \mu s_1 F_{12}\bar{n}_{N12} + T_1\bar{I} - F_{x1}\bar{I} + F_{y1}\bar{J} + \mu F_{y1}\bar{I}$$
$$+ \mu F_{x1}\bar{J} = 0 \tag{A-180}$$

Moment equilibrium is given by

$$R_{b1}F_{12} - \mu s_1 a_1 F_{12} - M_{in} + \mu \rho_1 (\hat{F}_{x1} + \hat{F}_{y1}) = 0 \tag{A-181}$$

Note that equations (A-180) and (A-181) have the same forms as equations (A-109) and (A-110).

The force component expressions are the same as given by equations (A-111) and (A-112), and their simultaneous solution for the pivot forces is identical to that given by equations (A-113) and (A-114), i.e.,

$$F_{x1} = \frac{-F_{12}\left[(1 - \mu^2 s_1)\sin(\beta_1 + \theta_1) + \mu(1 + s_1)\cos(\beta_1 + \theta_1)\right] + T_1}{1 + \mu^2}$$

(A-182)

and

$$F_{y1} = \frac{F_{12}\left[\mu(1 + s_1)\sin(\beta_1 + \theta_1) - (1 - \mu^2 s_1)\cos(\beta_1 + \theta_1)\right] - \mu T_1}{1 + \mu^2}$$

(A-183)

Equations (A-182) and (A-183) are now substituted into equation (A-181) according to the method of equation (A-3b)

$$R_{b1}F_{12} - \mu s_1 a_1 F_{12} - M_{1n} + \mu \rho_1 \left[F_{12}(A_{11} + A_{13}) + T_1(A_{12} + A_{14})\right]$$

$$= 0$$

(A-184)

where

$$A_{11} = \left| \frac{(1 - \mu^2 s_1)\sin(\beta_1 + \theta_1) + \mu(1 + s_1)\cos(\beta_1 + \theta_1)}{1 + \mu^2} \right| \quad \text{(A-185)}$$

$$A_{12} = \left| \frac{1}{1 + \mu^2} \right| \quad \text{(A-186)}$$

$$A_{13} = \left| \frac{\mu(1 + s_1)\sin(\beta_1 + \theta_1) - (1 - \mu^2 s_1)\cos(\beta_1 + \theta_1)}{1 + \mu^2} \right| \quad \text{(A-187)}$$

$$A_{14} = \left| \frac{\mu}{1 + \mu^2} \right| \quad \text{(A-188)}$$

Finally, equation (A-184) is solved for $F_{12}$

$$F_{12} = \frac{M_{in}}{D_3} - \frac{T_1 C_4}{D_3} \quad \text{(A-189)}$$

where

$$C_4 = \mu \rho_1 (A_{12} + A_{14}) \quad \text{(A-190)}$$

$$D_3 = R_{b1} - \mu \left[ s_1 a_1 - \rho_1 (A_{11} + A_{13}) \right] \quad \text{(A-191)}$$

A-67

## d. INPUT-OUTPUT RELATIONSHIP

To obtain the input-output relationship for the complete gear train, equation (A-189) is now set equal to equation (A-169). This furnishes

$$F_{23} = \frac{D_2}{C_2 D_3}\left[M_{in} - T_1 C_4\right] - T_2 \frac{C_3}{C_2} \qquad (A-192)$$

The above is then set equal to equation (A-144). This results in the input-output moment relationship

$$M_{o3} = \frac{D_1 D_2}{C_2 D_3}\left[M_{in} - T_1 C_4\right] - T_2 \frac{C_3 D_1}{C_2} - T_3 C_1 \qquad (A-193)$$

## 6. AUXILIARY GEOMETRIC AND KINEMATIC EXPRESSIONS FOR TWO AND THREE STEP-UP GEAR TRAINS WITH INVOLUTE TEETH

### a. NOMENCLATURE FOR INVOLUTE GEAR TEETH

$R_{pi}$, $r_{pi}$ = pitch radii of gear and pinion of $i^{th}$ gear and pinion set

$R_{bi}$, $r_{bi}$ = base radii of gear and pinion of $i^{th}$ gear and pinion set

$R_{oi}$, $r_{oi}$ = outside radii of gear and pinion of $i^{th}$ gear and pinion set

$\theta_j$ = effective pressure angle of $j^{th}$ mesh

$\mathcal{R}_i$ = distance from spin axis to pivot of $i^{th}$ gear and pinion set

### b. ANGULAR RELATIONSHIPS BETWEEN PIVOT HOLES

Figure A-14 shows the angular relationships between the lines connecting the pivot holes as well as the spin center. (See also Figures A-5 and A-10.) The following serves to determine the angles $\gamma_i$ and $\beta_i$ for certain combinations of gears and pinions as well as spin radii $\mathcal{R}_i$.
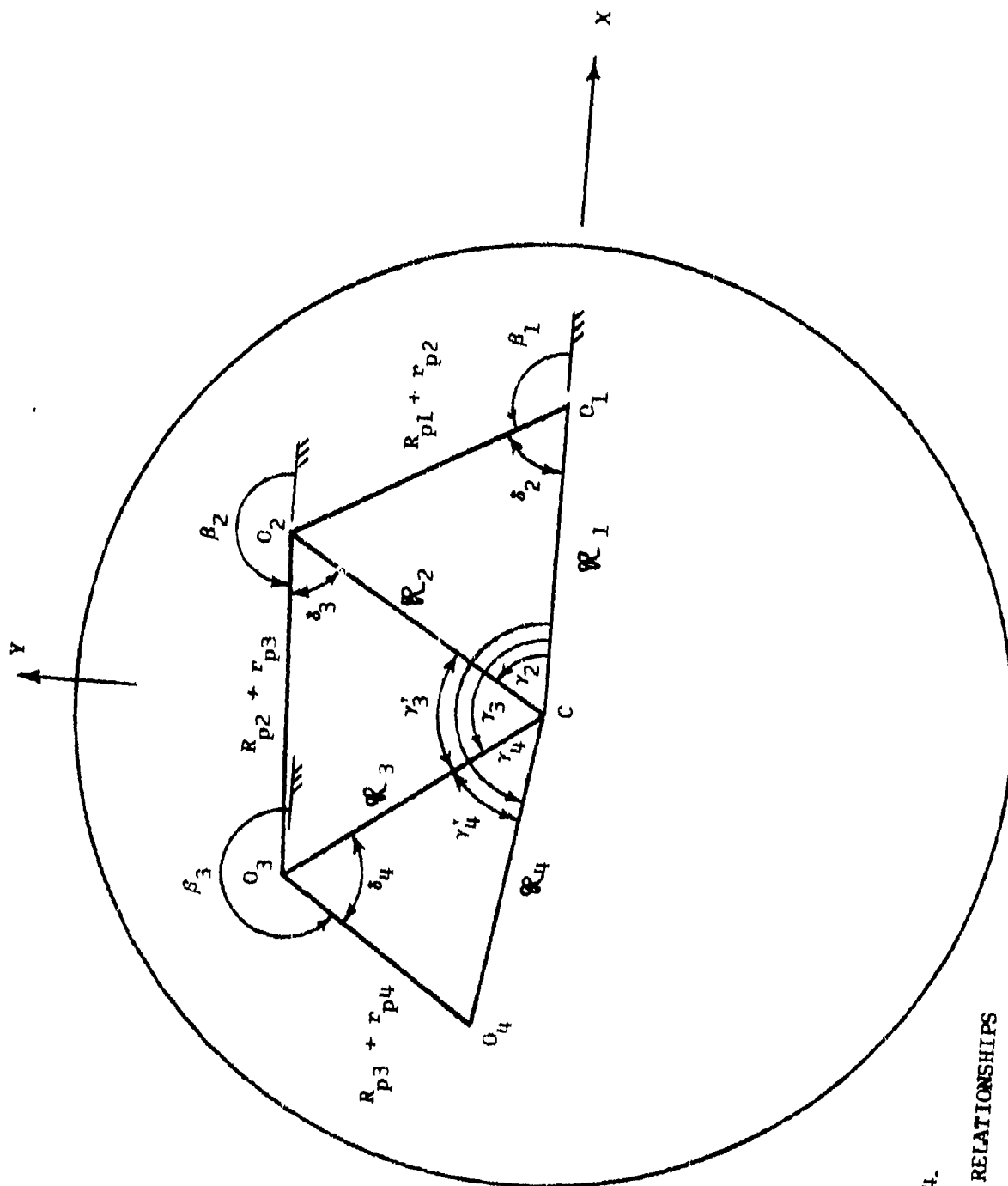
FIGURE A-14.

PIVOT HOLE RELATIONSHIPS

ANGLE $\gamma_1$

From

$$(R_{p1} + r_{p2})^2 = \mathcal{R}_1^2 + \mathcal{R}_2^2 - 2\mathcal{R}_1\mathcal{R}_2\cos\gamma_2$$

one obtains

$$\gamma_2 = \cos^{-1}\left[\frac{\mathcal{R}_1^2 + \mathcal{R}_2^2 - (R_{p1} + r_{p2})^2}{2\mathcal{R}_1\mathcal{R}_2}\right] \qquad (A-194)$$

Similarly, from

$$\gamma_3' = \cos^{-1}\left[\frac{\mathcal{R}_2^2 + \mathcal{R}_3^2 - (R_{p2} + r_{p3})^2}{2\mathcal{R}_2\mathcal{R}_3}\right]$$

one obtains

$$\gamma_3 = \gamma_2 + \gamma_3' \qquad (A-195)$$

A-71

Also with

$$\gamma_4' = \cos^{-1}\left[\frac{\mathcal{R}_3^2 + \mathcal{R}_4^2 - (R_{p3} + r_{p4})^2}{2\mathcal{R}_3\mathcal{R}_4}\right]$$

one obtains

$$\gamma_4 = \gamma_3 + \gamma_4' \tag{A-196}$$

### ANGLES $\delta_1$

Since

$$\mathcal{R}_2^2 = (R_{p1} + r_{p2})^2 + \mathcal{R}_1^2 - 2(R_{pi} + r_{p2})\mathcal{R}_1\cos\delta_2$$

$$\delta_2 = \cos^{-1}\left[\frac{(R_{p1} + r_{p2})^2 + \mathcal{R}_1^2 - \mathcal{R}_2^2}{2\mathcal{R}_1(R_{p1} + r_{p2})}\right] \tag{A-197}$$

Similarly,

$$\delta_3 = \cos^{-1}\left[\frac{(R_{p2} + r_{p3})^2 + \mathcal{R}_2^2 - \mathcal{R}_3^2}{2\mathcal{R}_2(R_{p2} + r_{p3})}\right] \tag{A-198}$$

A-72

and

$$\delta_4 = \cos^{-1}\left[\frac{(R_{p3} + r_{p4})^2 + \mathcal{R}_3^2 - \mathcal{R}_4^2}{2\mathcal{R}_3(R_{p3} + r_{p4})}\right]$$

(A-199)

## ANGLES $\beta_1$

With Equation (A-197)

$$\beta_1 = \pi - \delta_2$$

(A-200)

Further, with Equations (A-194) and (A-198)

$$\beta_2 = \gamma_2 + (\pi - \delta_3)$$

(A-201)

Finally, with Equations (A-195) and (A-199)

$$\beta_3 = \gamma_3 + (\pi - \delta_4)$$

(A-202)

A-73

## c. DETERMINATION OF CONTACT POINT C FOR VARIOUS MESHES

Figure A-15 shows the points of interest along the line of action of an involute gear which drives an involute pinion.

Points L and L' are the points of tangency to the base circles of radius $R_b$ and $r_b$, respectively, and the distance d = LL'. Initial contact is made at point M, where the line of action intersects the pinion addendum circle of radius $r_o$. Final contact corresponds to point N. Here the line of action intersects the gear addendum circle of radius $R_o$.

The position of the instantaneous contact point C with respect to point L, i.e., the length, a, is expressed with the help of instantaneous angle $\alpha$ which has its origin at the line $O_1L$. Then,

$$a = LC = R_b \alpha \tag{A-203}$$

A computer procedure for the determination of instantaneous angle $\alpha$ of any mesh must first find the associated initial and final angles of contact $\alpha_{in}$ and $\alpha_{fin}$. In addition, it must contain a method for incrementing angle $\alpha$. The following shows such a procedure for each of the meshes of a two pass and a three pass step-up gear train, together with a means of obtaining the signs of the signum terms.
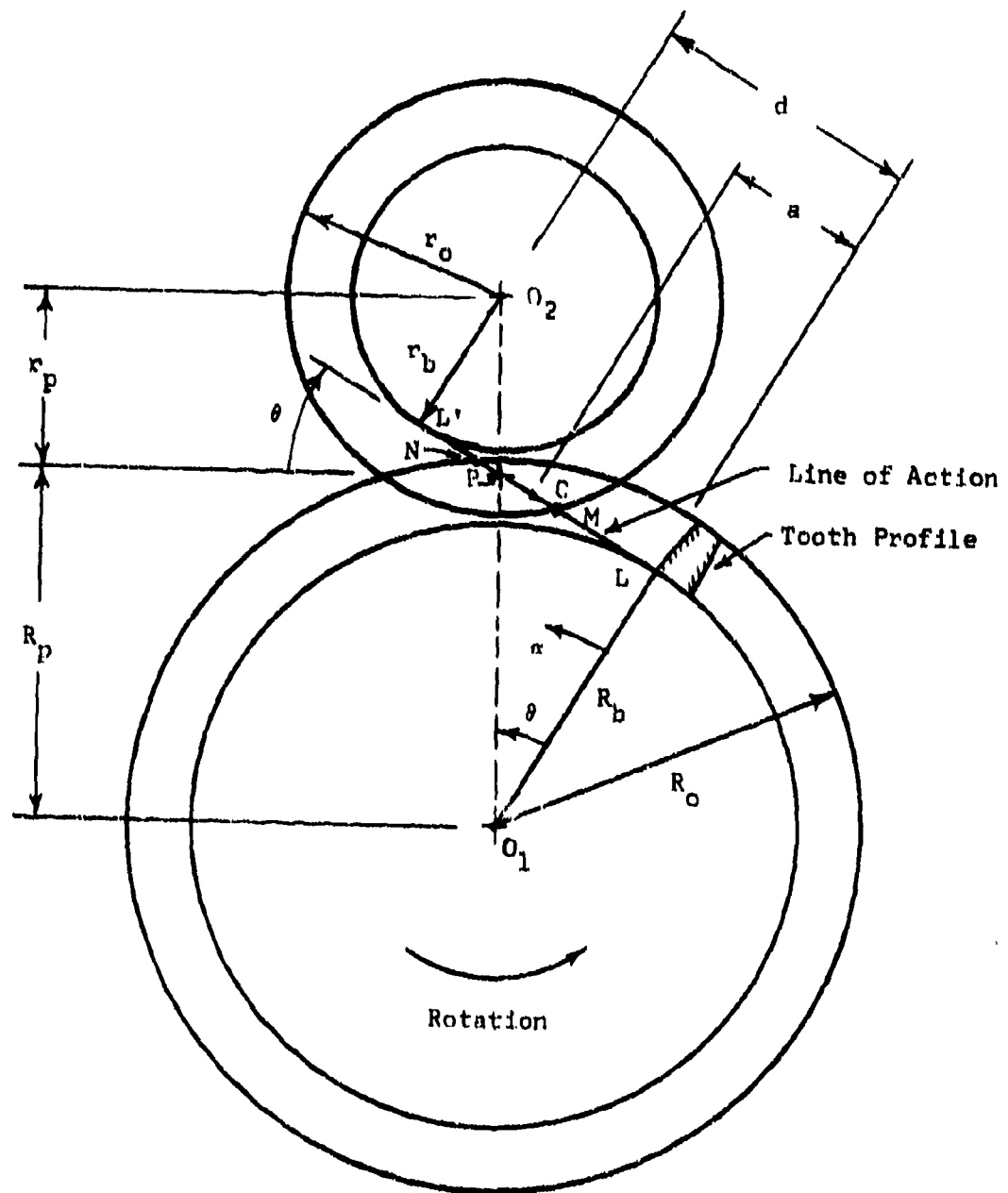
FIGURE A-15.

INVOLUTE MESH GEOMETRY

A-75

## 1.) MESH OF GEAR 1 AND PINION 2

The total length, $d_1 = LL'$, is given by

$$d_1 = (R_{b1} + r_{b1})\tan\theta_1 \qquad\qquad (A\text{-}204)$$

The initial angle of contact, $\alpha_{1IN}$, is obtained from

$$\alpha_{1in} = \frac{ML}{R_{b1}} = \frac{(R_{b1} + r_{b2})\tan\theta_1 - \sqrt{r_{o2}^2 - r_{b2}^2}}{R_{b1}} \qquad\qquad (A\text{-}205)$$

Similarly, the final angle of contact, $\alpha_{1FIN}$, is given by

$$\alpha_{1fin} = \frac{\sqrt{R_{o1}^2 - R_{b1}^2}}{R_{b1}} \qquad\qquad (A\text{-}206)$$

The magnitude of the increment $\Delta\alpha_1$ depends on whether one deals with a two step-up or a three step-up gear train.

Assuming that a two step-up train is involved and that one wishes to compute the length $a_2$ of mesh 2 $K_2$ times after the first contact, the angular increment, $\Delta\alpha_{22}$, has the magnitude

$$\Delta \alpha_{22} = \frac{\alpha_{2fin} - \alpha_{2in}}{K_2} \qquad \text{(A-207)}$$

(The second subscript refers to a two step-up configuration.)
Because of the transmission ratio between gear sets 1 and 2, the
associated angular increment of gear 1 will be smaller than $\Delta \alpha_{22}$, i.e.,

$$\Delta \alpha_{12} = \Delta \alpha_{22} \frac{r_{b2}}{R_{b1}} \qquad \text{(A-208)}$$

The instantaneous angle, $\alpha_1$, will then be given by

$$\alpha_1 = \alpha_{1in} + J_{12} \Delta \alpha_{12} \qquad \text{(A-209)}$$

and, the instantaneous distance, $a_1$, becomes

$$a_1 = R_{b1}( \alpha_{1in} + J_{12} \Delta \alpha_{12}) \qquad \text{(A-210)}$$

In the above, $J_{12}$ represents the number of times the angle $\alpha_1$
has been incremented. While the total number of increments depends
on the length of contact, the incrementing of $\alpha_1$ comes to an end
when $\alpha_1 \geq \alpha_{1fin}$. This also corresponds to a complete mechanism

cycle.  Since mesh 2 goes through $R_{b1}/r_{b2}$ times as many cycles as mesh no. 1, the angle $\alpha_2$ has to be re-initialized to $\alpha_{2in}$ when $\alpha_2 \geq \alpha_{2fin}$, i.e., after $K_2$ increments.  (For simplicity it is assumed that the motion starts when all meshes are at their initial contact angles, $\alpha_{in}$.)

When a three step-up gear train is involved, one must make sure that enough computations are made for mesh 3.  Assuming that $K_3$ increments are to be made, one obtains

$$\Delta\alpha_{33} = \frac{\alpha_{3fin} - \alpha_{3in}}{K_3} \tag{A-211}$$

The associated angular increments for meshes 2 and 1 then become

$$\Delta\alpha_{23} = \Delta\alpha_{33} \frac{r_{b3}}{R_{b2}} \tag{A-212}$$

and

$$\Delta\alpha_{13} = \Delta\alpha_{33} \frac{r_{b3}}{R_{b2}} \times \frac{r_{b2}}{R_{b1}} \tag{A-213}$$

For this case the instantaneous distance $a_1$ becomes

$$a_1 = R_{b1}(\alpha_{11n} + j_{13}\Delta\alpha_{13}) \tag{A-214}$$

A-78

$j_{13}$ stands for the number of times $\alpha_1$ has been incremented at any given instant. This incrementing again ends when $\alpha_1 \geq \alpha_{1fin}$. Meshes 2 and 3 are re-initialized to $\alpha_{2in}$ and $\alpha_{3in}$ as often as is necessary to complete one cycle for mesh no. 1.

The sign of $s_1$ is best obtained from the fact that at pitch point $P_1$

$$\alpha_{1p} = \frac{LP}{R_{b1}} = \frac{R_{b1} \tan \theta_1}{R_{b1}} = \tan \theta_1 \qquad \text{(A-215)}$$

then for

$$\begin{aligned}
\alpha_1 &< \tan \theta_1 : & s_1 &= +1 \\
\alpha_1 &= \tan \theta_1 : & s_1 &= 0 \\
\alpha_1 &> \tan \theta_1 : & s_1 &= -1
\end{aligned} \qquad \text{(A-216)}$$

## 2.) MESH OF GEAR 2 AND PINION 3

Similar to the previous section.

$$d_2 = (R_{b2} + r_{b3})\tan\theta_2 \qquad\qquad\text{(A-217)}$$

$$\alpha_{2in} = \frac{(R_{b2} + r_{b3})\tan\theta_2 - \sqrt{r_{o3}^2 - r_{b3}^2}}{R_{b2}} \qquad\qquad\text{(A-218)}$$

$$\alpha_{2fin} = \frac{\sqrt{R_{o2}^2 - R_{b2}^2}}{R_{b2}} \qquad\qquad\text{(A-219)}$$

For a two step-up gear train, the instantaneous length, $a_2$, becomes

$$a_2 = R_{b2}(\alpha_{2in} + J_{22}\Delta\alpha_{22}) \qquad\qquad\text{(A-220)}$$

where $\Delta\alpha_{22}$ is given by Equation (A-207) and

$$J_{22} = 1, 2, \ldots, K_2 .$$

When $\alpha_2 \geq \alpha_{2fin}$ it must be re-initialized to $\alpha_{2in}$ until mesh no. 1 has completed its full cycle.

For a three step-up gear train, the length, $a_2$, becomes

$$a_2 = R_{b2}(\alpha_{2in} + j_{23}\Delta\alpha_{23}) \qquad \text{(A-221)}$$

with $\Delta\alpha_{23}$ given by Equation (A-212). $j_{23}$ stands for the number
of times $\alpha_2$ has been incremented and again depends on the length
of contact. Re-initialization follows the same rule as given above.

The sign of $a_2$ is obtained as follows: (See Equation (A-215).)

For

$$\alpha_2 < \tan\theta_2 : \quad s_2 = +1$$
$$\alpha_2 = \tan\theta_2 : \quad s_2 = 0 \qquad \text{(A-222)}$$
$$\alpha_2 > \tan\theta_2 : \quad s_2 = -1$$

3.) MESH OF GEAR 3 AND PINION 4

Again, in the same vein as in Section 1,

$$d_3 = (R_{b3} + r_{b4}) \tan \theta_3 \qquad (A\text{-}223)$$

$$\alpha_{3in} = \frac{(R_{b3} + r_{b4}) \tan \theta_3 - \sqrt{r_{o4}^2 - r_{b4}^2}}{R_{b3}} \qquad (A\text{-}224)$$

$$\alpha_{3fin} = \frac{\sqrt{R_{o3}^2 - R_{b3}^2}}{R_{b3}} \qquad (A\text{-}225)$$

The instantaneous length, $a_3$, is given by

$$a_3 = R_{b3}(\alpha_{3in} + j_{33}\Delta\alpha_{33}) \qquad (A\text{-}226)$$

where $\Delta\alpha_{33}$ is given by Equation (A-211) and $j_{33} = 1, 2, \ldots, K_3$. whenever $\alpha_3 \geq \alpha_{3fin}$ it must be re-initialized until mesh no. 1 completed its full cycle.

The sign of $s_3$ is again obtained with the help of an expression Equation (A-215):

A-82

$$\alpha_3 < \tan\theta_3 : \quad s_3 = +1$$

$$\alpha_3 = \tan\theta_3 : \quad s_3 = 0 \tag{A-227}$$

$$\alpha_3 > \tan\theta_3 : \quad s_3 = -1$$

APPENDIX B

DESIGN OF UNEQUAL ADDENDUM INVOLUTE GEAR SETS WITH STANDARD

CENTER DISTANCES

One of the ways of preventing undercutting in pinions with
small numbers of teeth, which must mesh with gears of unequal
and larger numbers of teeth, is to decrease the pinion dedendum
together with the gear addendum by the necessary amount. To
maintain standard working depth for such a mesh, the addendum of
the pinion as well as the dedendum of the gear are increased by
the same amount. (This, of course, presupposes that the gear
is not undercut by this modification.) This can be accomplished
without any change in the standard base and pitch radii or the
associated standard center distance by "withdrawing" the hob
during the cutting of the pinion and feeding it "deeper" when the
gear is cut. In this way the pinion, which has its ouside radius
increased by the hob withdrawal distanco, will have a larger than
standard circular tooth thickness at its standard pitch circle.
The outside radius of the gear is decreased by the same amount.
Because the cutter is fed to full depth, the gear tooth thickness
at the standard pitch circle will be less than standard.

The following gives the design steps for this type of gearing
and illustrates them by way of an example.

# 1. STANDARD GEAR NOMENCLATURE

$N$ = number of teeth of gear

$n$ = number of teeth of pinion

$\theta$ = pressure angle

$P_d$ = $\dfrac{N}{2R_p}$ = $\dfrac{n}{2r_p}$ = diametral pitch

$P_{cs}$ = standard circular pitch at pitch circle, where also

$$P_{cs} = \dfrac{\pi}{P_d}$$

$R_p, r_p$ = pitch radii of gear and pinion, respectively

$R_b$ = $R_p \cos\theta$ , base circle radius of gear

$r_b$ = $r_p \cos\theta$ , base circle radius of pinion

$T_{cs}, t_{cs}$ = $\dfrac{P_{cs}}{2}$ , circular tooth thickness of gear and pinion,

respectively, at standard pitch circles

$\dfrac{1}{P_d}$ = standard gear addendum

$\dfrac{1.157}{P_d}$ = standard gear dedendum

## 2. DETERMINATION OF HOB WITHDRAWAL DISTANCE C

Figure B-1 indicates the relationship between the pinion pitch radius, $r_p$, its root radius, $r_r$, and the rack cutter addendum A. The root radius is formed by the addendum line of the cutter tooth, so that

$$r_r = r_p - A \qquad\qquad (B-1)$$

In order to avoid undercutting, the addendum line of a sharp cornered cutter must not pass below point L, the tangent point of the line of action and the base circle (see Figure B-2). The minimum root radius, $r_{rm}$, becomes for this case

$$r_{rm} = r_b \cos\theta = r_p \cos^2\theta \qquad\qquad (B-2)$$

When the rack tooth corner is rounded off with a radius $r_c$, as shown in Figure B-3, the effective addendum line of the cutter tooth moves up the distance $r_c(1 - \sin\theta)$. To avoid undercutting with this type of cutter, the effective addendum line must not pass the base circle below point L in Figure B-2. This allows a reduction of the minimum allowable root radius to

B-3
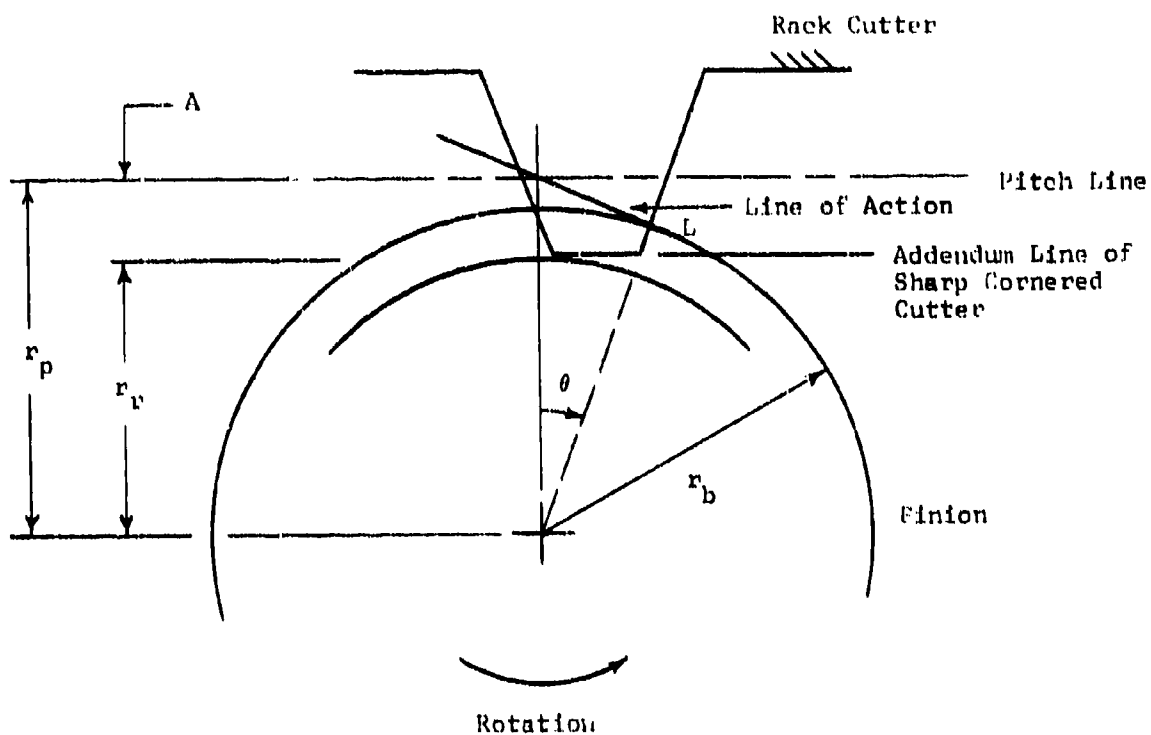
FIGURE B-1

RELATIONSHIP BETWEEN PINION PITCH RADIUS, $r_p$, RACK CUTTER ADDENDUM A
AND RESULTING PINION ROOT RADIUS, $r_r$
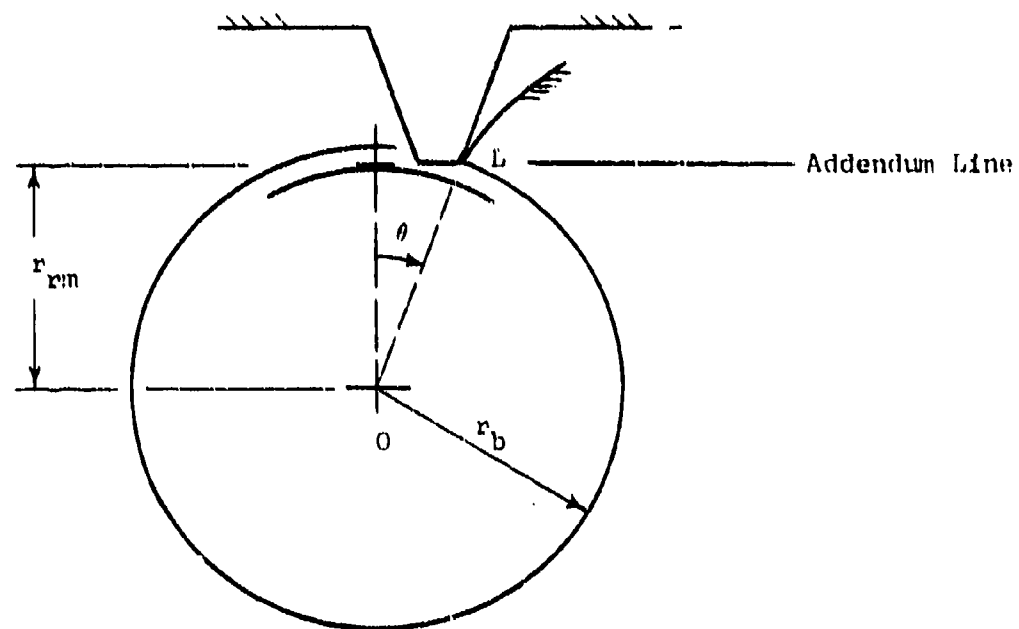
FIGURE B-2

MINIMUM ROOT RADIUS, $r_{rm}$, FOR RACK CUTTER WITH SHARP CORNER
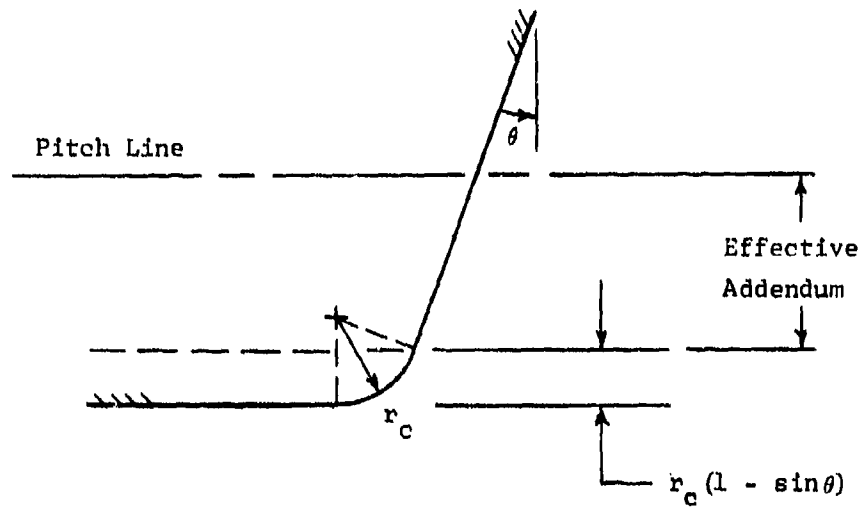
FIGURE B-3

RACK CUTTER WITH CORNER RADIUS, $r_c$ , (EFFECTIVE ADDENDUM OF CUTTER IS DECREASED)

$$r_{rmc} = r_p \cos^2\theta - r_c(1 - \sin\theta) \qquad\qquad (B-3)$$

By common usage the corner radius may either be

$$r_c = .1\, t_s = \frac{.05\,\pi}{P_d} , \qquad\qquad (B-4)$$

or it is chosen such that the second term in Equation (B-3) becomes

$$r_c(1 - \sin\theta) \approx \frac{.157}{P_d} \qquad\qquad (B-5)$$

(This makes for an effective cutter addendum of $1/P_d$.)

Hob withdrawal becomes necessary if the root radius, obtained by setting the cutter to standard depth, is smaller than the minimum given by Equation (B-3). Thus, the hob withdrawal C is obtained from Equations (B-1) and (B-3), i.e.,

$$C = r_{rmc} - r_r \qquad\qquad (B-6)$$

With the cutter addendum $A = 1.157/P_d$, and using the expression of Equation (B-4) for the rack corner radius, Equation (B-6) becomes

B-7

$$C = \frac{1}{P_d}(1 + .157\sin\theta) - r_p\sin^2\theta \qquad (B-7)$$

If Equation (B-5) is used, with the same definition of the cutter addendum, one obtains for C

$$C = \frac{1}{P_d} - r_p\sin^2\theta \qquad (B-7)$$

## 3. OUTSIDE RADII OF PINION AND GEAR BLANKS

The outside radius of the pinion blank becomes

$$r_o = r_p + \frac{1}{P_d} + C \qquad (B-8)$$

The outside radius of the gear blank becomes

$$R_o = R_p + \frac{1}{P_d} - C \qquad (B-9)$$

B-8

## 4. TOOTH THICKNESS AT PITCH CIRCLES OF PINION AND GEAR

Since the thickness of the hobtooth at the pinion pitch circle will be reduced by the amount $2C\tan\theta$, due to the withdrawal C, the circular thickness of the pinion tooth at this location will be increased by this amount, i.e.,

$$t_c = t_{cs} + 2C\tan\theta \qquad (B-10)$$

The gear tooth thickness will be decreased to

$$T_c = T_{cs} - 2C\tan\theta \qquad (B-11)$$

at its pitch circle.

# 5. TOOTH THICKNESS AT OUTSIDE AND BASE RADII BY INVOLUTOMETRY

The circular tooth thickness at an arbitrary radius of an involute tooth may be obtained if the tooth thickness, radius, and pressure angle of any other location, such as the pitch circle, are known together with the pressure angle at the arbitrary radius. [See Equation (5-5), pg. 80, E. Buckingham: Analytical Mechanics of Gears, McGraw-Hill Book Co., Inc. New York, 1949.]

Accordingly, the circular tooth thickness, $t_o$, at the outside radius of the pinion, may be obtained from

$$t_o = t_c \frac{r_o}{r_p} - 2r_o (INV\theta_{OP} - INV\theta) \qquad \text{(B-12)}$$

where

$INV\theta = \tan\theta - \theta$ , the involute function corresponding to the pressure angle $\theta$

$\theta = $ pressure angle of mesh. This is also the pressure angle at the pitch circle.

$\theta_{OP} = \cos^{-1}\frac{r_b}{r_o}$ , the pressure angle associated with the outside radius of the pinion

Similarly, the circular tooth thickness, $T_o$, at the outside radius of the gear is obtained from:

$$T_o = T_c \frac{R_o}{R_p} - 2R_o(INV\theta_{OG} - INV\theta) \qquad (B-13)$$

where

$\theta_{OG} = cos^{-1} \frac{R_b}{R_o}$ , the pressure angle associated with the outside radius of the gear

The tooth thickness at the base circle radius of the modified pinion is given by

$$t_b = t_c cos\theta + 2r_b INV\theta \qquad (B-14)$$

This becomes for the gear

$$T_b = T_c cos\theta + 2R_b INV\theta \qquad (B-15)$$

(The above is only of theoretical interest in case the inter-tooth space is not cut below the base circle radius.)

## 6. EXPRESSIONS FOR CONTACT RATIO AND PINION OUSIDE RADIUS FOR

## UNITY CONTACT RATIO

The expression for contact ratio [Equation 4-19, pg. 72,
E. Buckingham: Analytical Mechanics of Gears] is given by

$$m_p = \frac{\sqrt{R_o^2 - R_b^2} + \sqrt{r_o^2 - r_b^2} - (R_b + r_b)\tan\theta}{p_{cs}\cos\theta} \qquad (B\text{-}16)$$

If the contact ratio of a certain mesh is larger than unity,
and one wishes to reduce it to unity by reducing the outside
radius of the pinion, one may find this new pinion outside radius
with the help of

$$r_o' = \sqrt{r_b^2 + \left[ p_{cs}\cos\theta + (R_b + r_b)\tan\theta - \sqrt{R_o^2 - R_b^2} \right]^2}$$

$$(B\text{-}17)$$

## 7. EXAMPLE

Design a pinion and a gear with the following specifications:

$$P_d = 44, \quad N = 42, \quad n = 8, \quad \theta = 20°$$

This gives

$R_p$ = .47727 in. (1.212 cm)     $r_p$ = .09091 in. (.231 cm)

$R_b$ = $R_p \cos 20°$ = .44848 in. (1.139 cm)  $r_b$ = .08542 in. (.217 cm)

$P_{cs}$ = $\dfrac{\pi}{44}$ = .07140 in. (.181 cm)  $T_{cs}$ = $t_{cs}$ = .03570 in. (.091 cm)

According to Equation (B-7), the hob withdrawal, C, is computed as

$$C = \frac{1}{44} - .09091 \sin^2 20° = .012093 \text{ in. } (.031 \text{ cm})$$

Then, according to Equations (B-8) and (B-9)

$r_o$ = .09091 + .022727 + .012093 = .12573 in. (.319 cm)

$R_o$ = .47727 + .022727 - .012093 = .48790 in. (1.239 cm)

The new tooth thickness at the pitch radii is found with the
help of Equations (B-10) and (B-11)

$$t_c = .03570 + 2(.012093)\tan20^{\circ} = .04450 \text{ in. } (.113 \text{ cm})$$

$$T_c = .03570 - 2(.012093)\tan20^{\circ} = .02689 \text{ in. } (.068 \text{ cm})$$

For the purposes of the present analysis, the pinion outside
radius is now reduced to obtain a contact ratio of unity. With
Equation (B-17)

$$r_o' = .110 \text{ in. } (.279 \text{ cm}) \quad \text{This is essentially the unmodified pinion radius of}$$
$$.113 \text{ in. } (.287 \text{ cm}).$$

Now compute the circular tooth thickness at the outside
radii $r_o'$ and $R_o$ using the following data in Equations (B-12)
and (B-13):

$$\beta = 20^{\circ} \qquad\qquad\qquad \text{INV } 20^{\circ} = .01490$$

$$\theta_{OP} = \cos^{-1}\frac{.08542}{.110} = 39.0509^{\circ} \qquad \text{INV } 39.0509^{\circ} = .12969$$

$$\theta_{OG} = \cos^{-1}\frac{.44848}{.48790} = 23.1889^{\circ} \qquad \text{INV } 23.1889^{\circ} = .02365$$

B-14

Then

$$t_o = .04450 \left( \frac{.110}{.09091} \right) - 2(.110)(.12969 - .0149) = .02860 \text{ in. } (.073 \text{ cm})$$

and

$$T_o = .02689 \left( \frac{.4879}{.47727} \right) - 2(.4879)(.02365 - .0149) = .01896 \text{ in. } (.048 \text{ cm})$$

These are sufficient to allow for rounding off the teeth.

Finally, check that the gear is not undercut. The actual root radius of the gear is

$$R_o = \frac{2.157}{P_d} = .4879 - .04902 = .4389 \text{ in. } (1.115 \text{ cm})$$

The minimum permissible root radius without undercutting, according to Equations (B-3) and (B-4), is computed from

$$R_{rm} = R_p \cos^2 \theta - \frac{.05\pi}{P_d}(1 - \sin \theta) = .47727(.883) - .0023$$

$$= .419 \text{ in. } (1.064 \text{ cm})$$

Thus, the gear is not undercut.

APPENDIX C

COMPUTER MODELS FOR STEP-UP GEAR TRAINS WITH INVOLUTE TEETH

The present appendix contains descriptions, listings, and sample outputs of the following involute gear-train-related computer programs:

1. Program INVOL 1 : Design of unequal addendum involute gear and pinion set with unity contact ratio.

2. Program INVOL 2 : Point and cycle efficiencies for single pass involute step-up gear mesh with unity contact ratio.

3. Program INVOL 3 : Point and cycle efficiencies for three pass involute step-up gear train in spin environment.(All meshes have unity contact ratio).

4. Program INVOL 4 : Point and cycle efficiencies for two pass involute step-up gear train in spin environment.(All meshes have unity contact ratio).

The relevant background, the input parameters, the manner of the computations and the form of the output of each program are discussed in detail. The program proper forms the last part of each section.

# 1. Program INVOL 1: Design of Unequal Addendum Involute Gear and Pinion Set with Unity Contact Ratio.

The program INVOL 1 is based on Appendix B which shows the design equations for unequal addendum involute gear sets with standard center distances.

The nomenclature of the program is chosen to coincide as much as possible with that of Appendix B.

## a. Input Parameters (See also Program INVOL 1, below)

The following parameters represent the input data of the program:

PSUBD      =      $P_d$ , the diametral pitch

NG      =      $N_G$ , the number of teeth of the gear

NP      =      $N_P$ , the number of teeth of the pinion

THETAD      =      $\Theta$ , the pressure angle of the mesh as well as of the hob (in degrees)

ISTOP      =      arbitrary single-digit integer for multiple data sets. Must be zero for last set of data.

## b. Computations

The program computes the following quantities (where not otherwise indicated, consult Section 1 of Appendix B for nomenclature):

CAPRP      =      $R_p$

RP      =      $r_p$

CAPRB      =      $R_b$

RB      =      $r_b$

| | | |
|---|---|---|
| PSUBC | = | $p_c$ |
| TSTAND | = | $t_{cs}$ |
| C | = | hob withdrawal distance. See eq (B-7) |
| RO | = | $r_o$ , See eq (B-8). This is the original pinion blank radius. |
| CAPRO | = | $R_o$ , see eq (B-9) |
| TC | = | $t_c$ , see eq (B-10) |
| CAPTC | = | $T_c$ , see eq (B-11) |
| TB | = | $t_b$ , see eq (B-14). Note also the computation of the involute function at the end of the program. |
| CAPTB | = | $T_b$, see eq (B-15) |
| ROFIN | = | $r_o'$ , see eq (B-17). This is the pinion outside radius for unity contact ratio. |
| THETOG | = | $\theta_{OG}$, according to expression associated with eq (B-13) |
| THETOP | = | $\theta_{OP}$, according to expression associated with eq (B-12) |
| CAPTO | = | $T_o$ , see eq (B-13) |
| TO | = | $t_o$ , see eq (B-12). Computed with ROFIN. |
| CRATIO | = | $m_p$ , see eq (B-16). This is the original contact ratio ,which uses the unmodified pinion blank radius RO. |
| CRFIN | = | represents the contact ratio when computed with the final pinion blank radius ROFIN. |
| ROOT | = | actual root radius of pinion |
| CAPROOT | = | actual root radius of gear |
| CAPRMIN | = | $R_{rm}$ , see equations (B-3) and (B-4) |

c. Output of Program

The output of the program is best explained by means of
the first sample computations which are shown at the end of
the program. This example is identical with that given in
section 7 of Appendix B. The output lists the following:

I. The input parameters, PSUBD, NG, NP, and pressure
angle, THETAD, are printed out.

II. The above is followed by computational results for

CAPRP

RP

CAPRB

TSTAND

C

CAPRO

RO, this is the original pinion blank radius before the
unity contact ratio modification.

CRATIO, note that for the given case, this original contact
ratio equals 1.3

ROFIN, the pinion outside radius which corresponds to unity
contact ratio

CRFIN, this, of course, has to be unity because of the use of
ROFIN. This computation serves as a check.

CAPTC, this tooth thickness, as well as the following one,
is useful for strength computations

TC

THEOGD, this pressure angle, as well as the following one, corresponds to the final outside radii of the gear and pinion, respectively, and is needed for the computations of the tooth thicknesses at the outside radii

THEOPD

CAPTO, this tooth thickness at the outside radius of the gear, as well as the following one for the pinion, must be sufficiently large to allow for the presence of a tip radius

TO

CAPTB

TB

CAPROOT

ROOT

CAPRMIN

The values of the last five parameters are to some extent interconnected. First, it is important to see whether the gear is undercut. This does not occur as long as CAPROOT is larger than CAPRMIN, i.e., the actual root radius is larger than the minimum allowable one. The gear tooth thickness at the base circle is of interest for strength purposes. Since, for the given case, CAPRB = .44849 in. (1.139 cm) and CAPROOT = .4388 in. (1.115 cm) the base circle lies above the root circle, and the greatest cross-section of the tooth is approximately equal to CAPTB. TB is called the theoretical

pinion tooth thickness at the base circle since it is possible
that the base circle lies below the minimum allowable root circle
of the pinion. In such a case, the actual tooth may be weaker
than indicated by this dimension. For the present case, RB =
.08543 in. (.217 cm) and ROOT = .07671 in. (.195 cm), and therefore,
the base circle, which lies above the root circle, gives a good
indication of the actual tooth cross-section at the root.

Program INVOL 1

This program contains five different sets of gears. This
will be of use in programs INVOL 2, INVOL 3 and INVOL 4.

```
        PROGRAM INVOL1 (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C   DESIGN OF UNEQUAL ADDENDUM INVOLUTE GEAR AND PINION SET
C   WITH UNITY CONTACT RATIO.
C
        REAL INV
1       READ(5,2)PSUBD,NG,NP,THETAD,ISTOP
2       FORMAT(F10.4,2I10,F10.4,9X,I1)
        PI = 3.14159
        Z = PI/180.
        THETA = THETAD*Z
        CAPRP = NG/(2.*PSUBD)
        RP = NP/(2.*PSUBD)
        CAPRB = CAPRP*COS(THETA)
        RB = RP*COS(THETA)
        PSUBC = PI/PSUBD
        TSTAND = PSUBC/2.
        C = 1./PSUBD
        CAPRO = CAPRP + 1./PSUBD - C
        RO = RP + 1./PSUBD
        CAPTC = TSTAND - 2.*C*TAN(THETA)
        TC = TSTAND + 2.*C*TAN(THETA)
        CAPTB = CAPTC*COS(THETA) + 2.*CAPRB*INV(THETA)
        TB = TC*COS(THETA) + 2.*RB*INV(THETA)
        ROFIN = SQRT(RB*RB + (PSUBC*COS(THETA) + (CAPRB + RB)*TAN(THETA)
     1   - SQRT(CAPRO*CAPRO - CAPRB*CAPRB))**2)
        THETOG = ACOS(CAPRB/CAPRO)
        THEOGD = THETOG/Z
        THETOP = ACOS(RB/ROFIN)
        THEOPD = THETOP/L
        CAPTO = CAPTC*CAPRO/CAPRP - 2.0*CAPRO*(INV(THETOG) - INV(THETA))
        TO = TC*RO*IN/RP - 2.*ROFIN*(INV(THETOP) - INV(THETA))
        CRATIO = (SQRT(CAPRO*CAPRO - CAPRB*CAPRB) + SQRT(RO*RO - RB*RB)
     1   - (CAPRB + RB)*TAN(THETA))/(PSUBC*COS(THETA))
        CRFIN = (SQRT(CAPRO*CAPRO - CAPRB*CAPRB) + SQRT(ROFIN*ROFIN
     1   - RB*RB) - (CAPRB + RB)*TAN(THETA))/(PSUBC*COS(THETA))
        ROOT = RO - 2.157/PSUBD
        CAPROOT = CAPRO - 2.157/PSUBD
        CAPRMIN = CAPRP*COS(THETA)*COS(THETA) - .05*PI*(1. - SIN(THETA))/
     1   PSUBD
        WRITE(6,3)PSUBD,NG,NP,THETAD
3       FORMAT(*1DIAMETRAL PITCH (PSUBD) =*,F6.2/*0GEAR NUMBER OF TEETH (N
     1G) =*,I13/*0PINION NUMBER OF TEETH (NP) =*,I13/*0PRESSURE ANGLE (THE
     2TA) =*,F6.2)
        WRITE(6,4)CAPRP,RP
4       FORMAT(*0GEAR PITCH RADIUS (CAPRP) =*,F9.5,3X,*PINION PITCH RADIUS
     1 (RP) =*,F9.5)
        WRITE(6,5)CAPRB,RB
5       FORMAT(*0GEAR BASE RADIUS (CAPRB) =*,F9.5,3X,*PINION BASE RADIUS (
     1RB) =*,F9.5)
        WRITE(6,6)TSTAND
6       FORMAT(*0STANDARD TOOTH THICKNESS AT PITCH RADII (TSTAND) =*,F9.5)
        WRITE(*,7)C
```

C-8

```
1        REAL FUNCTION INV(THETA)
         INV = TAN(THETA) - THETA
         RETURN
         END
```

```
         7 FORMAT(*0HOB WITHDRAWAL DISTANCE (C) =*,F8.5)
           WRITE(6,8)CAPRO,HO
         8 FORMAT(*0GEAR BLANK RADIUS (CAPRO) =*,F9.5,3X,*ORIGINAL PINION BLA
          1NK RADIUS (RO) =*,F9.5)
           WRITE(6,9)CRATIO
         9 FORMAT(*0ORIGINAL CONTACT RATIO (CRATIO) =*,F6.3)
           WRITE(6,10)ROFIN
        10 FORMAT(*0PINION OUTSIDE RADIUS FOR UNITY CONTACT RATIO (ROFIN) =*,
          1F9.5)
           WRITE(6,11)CRFIN
        11 FORMAT(*0FINAL CONTACT RATIO (CRFIN) =*,F6.3)
           WRITE(6,12)CAPTC,TC
        12 FORMAT(*0GEAR TOOTH THICKNESS AT PITCH CIRCLE (CAPTC) =*,F9.5,3X,
          1*PINION TOOTH THICKNESS AT PITCH CIRCLE (TC) =*,F8.5)
           WRITE(6,13)THEOGD,THEOPD
        13 FORMAT(*0GEAR PRESSURE ANGLE AT OUTSIDE RADIUS (THEOGD) =*,F9.5,
          13X,*PINION PRESSURE ANGLE AT FINAL OUTSIDE RADIUS (THEOPD) =*,
          1F9.5)
           WRITE(6,14)CAPTO,TO
        14 FORMAT(*0GEAR TOOTH THICKNESS AT OUTSIDE RADIUS (CAPTO) =*,F8.5,
          13X,*PINION TOOTH THICKNESS AT FINAL OUTSIDE RADIUS (TO) =*,F8.5)
           WRITE(6,15)CAPTB,TB
        15 FORMAT(*0GEAR TOOTH THICKNESS AT BASE CIRCLE (CAPTB) =*,F8.5,3X,
          1*THEORETICAL PINION TOOTH THICKNESS AT BASE CIRCLE (TB) =*,F8.5)
           WRITE(6,16)CAPROOT,CAPRMIN
        16 FORMAT(*0RADIUS OF ROOT CIRCLE OF GEAR (CAPROOT) =*,F8.5,3X,
          1*MINIMUM ALLOWABLE RADIUS OF ROOT CIRCLE OF GEAR (CAPRMIN) =*,F8.5
          2)
           IF(CAPROOT .GE. CAPRMIN)WRITE(6,17)
           IF(CAPROOT .LT. CAPRMIN)WRITE(6,18)
        17 FORMAT(*0THE GEAR IS NOT UNDERCUT*)
        18 FORMAT(*0THE GEAR IS UNDERCUT*)
           WRITE(6,19)ROOT
        19 FORMAT(*0RADIUS OF ROOT CIRCLE OF PINION (ROOT) =*,F8.5)
           IF(ISTOP .EQ. 0)GO TO 9999
           GO TO 1
      9999 STOP
           END
```

C-10

DIAMETRAL PITCH (PSUBD) = 44.00

GEAR NUMBER OF TEETH (NG) = 42

PINION NUMBER OF TEETH (NP) = 8

PRESSURE ANGLE (T=PIA) = 20.00

GEAR PITCH RADIUS (CAPRP) = .47727   PINION PITCH RADIUS (RP) = .09091

GEAR BASE RADIUS (CAPRG) = .44849   PINION BASE RADIUS (RB) = .08543

STANDARD TOOTH THICKNESS AT PITCH RADII (TSTAND) = .03570

MORE WITHDRAWAL DISTANCE (C) = .01207

GEAR BLANK RADIUS (CAPRO) = .48741   ORIGINAL PINION BLANK RADIUS (HO) = .12573

ORIGINAL CONTACT RATIO (CRATIO) = 1.342

PINION OUTSIDE RADIUS FOR UNITY CONTACT RATIO (ROFIN) = .11000

FINAL CONTACT RATIO (CRFIN) = 1.090

GEAR TOOTH THICKNESS AT PITCH CIRCLE (CAPHC) = .02690   PINION TOOTH THICKNESS AT PITCH CIRCLE (TC) = .04450

GEAR PRESSURE ANGLE AT OUTSIDE RADIUS (THEOG) = 23.18696   PINION PRESSURE ANGLE AT FINAL OUTSIDE RADIUS (THEOP) = 32.05693

GEAR TOOTH THICKNESS AT OUTSIDE RADIUS (CAPTO) = .01596   PINION TOOTH THICKNESS AT FINAL OUTSIDE RADIUS (TO) = .02860

GEAR TOOTH THICKNESS AT BASE CIRCLE (CAPTB) = .03564   THEORETICAL PINION TOOTH THICKNESS AT BASE CIRCLE (TB) = .04637

RADIUS OF ROOT CIRCLE OF GEAR (CAPRROT) = .43055   MINIMUM ALLOWABLE RADIUS OF ROOT CIRCLE OF GEAR (CAPRMIN) = .41909

THE GEAR IS NOT UNDERCUT

RADIUS OF ROOT CIRCLE OF PINION (RROT) = .07571

DIAMETRAL PITCH (P-SUBD) = 65.00

GEAR NUMBER OF TEETH (NG) = 27

PINION NUMBER OF TEETH (NP) = 9

PRESSURE ANGLE (T-FIA) = 28.00

GEAR PITCH RADIUS (CAPRP) = .20769   PINION PITCH RADIUS (RP) = .06923

GEAR BASE RADIUS (CAPRBG) = .18717   PINION BASE RADIUS (RB) = .06106

STANDARD TOOTH THICKNESS AT PITCH RADII (TSTAND) = .02417

POG WITHDRAWAL OFFTRACE (C) = .00729

GEAR DEDAR RADIUS (CAPRRG) = .21579   ORIGINAL PINION DEDAR RADIUS (RRO) = .09199

OUTSIDE CONTACT RATIO (CRATIO) = 1.371

PINION OUTSIDE RADIUS FOR UNITY CONTACT RATIO (ROFIN) = .04009

FINAL CONTACT RATIO (CRFIN) = 1.000

GEAR TOOTH THICKNESS AT PITCH CIRCLE (CAPTG) = .01606   PINION TOOTH THICKNESS AT PITCH CIRCLE (TC) = .02947

GEAR PRESSURE ANGLE AT OUTSIDE RADIUS (THEOGO) = 25.2527   PINION PRESSURE ANGLE AT FINAL OUTSIDE RADIUS (THEOPO) = 36.8597

GEAR TOOTH THICKNESS AT OUTSIDE RADIUS (CAPTGO) = .01267   PINION TOOTH THICKNESS AT FINAL OUTSIDE RADIUS (TO) = .02024

GEAR TOOTH THICKNESS AT BASE CIRCLE (CAPTBG) = .02254   THEORETICAL PINION TOOTH THICKNESS AT BASE CIRCLE (TB) = .02263

RADIUS OF ROOT CIRCLE OF GEAR (CAPRROOT) = .18261   MINIMUM ALLOWABLE RADIUS OF ROOT CIRCLE OF GEAR (CAPRMIN) = .16181

THE GEAR IS NOT UNDERCUT

RADIUS OF ROOT CIRCLE OF PINION (RROOT) = .05472

DIAMETRAL PITCH (PSUBD) = 11.80

GEAR NUMBER OF TEETH (NGT) = 27

PINION NUMBER OF TEETH (NPT) = 9

PRESSURE ANGLE (THETA) = 20.00

GEAR PITCH RADIUS (CAPRP) = .1532   PINION PITCH RADIUS (RP) = .05844

GEAR BASE RADIUS (CAPRB) = .16175   PINION BASE RADIUS (RB) = .05492

STANDARD TOOTH THICKNESS AT PITCH RADII (TSTAND) = .02540

HOB WITHDRAWAL DISTANCE (L) = .00815

GEAR BLANK RADIUS (CAPRC) = .18218   ORIGINAL PINION BLANK RADIUS (RB) = .01758

ORIGINAL CONTACT RATIO (CRATIO) = 1.371

PINION OUTSIDE RADIUS FOR UNITY CONTACT RATIO (ROP1IN) = .04828

FINAL CONTACT RATIO (CRFIN) = 1.500

GEAR TOOTH THICKNESS AT PITCH CIRCLE (CAP1C) = .01592   PINION TOOTH THICKNESS AT PITCH CIRCLE (1C) = .02488

GEAR PRESSURE ANGLE AT OUTSIDE RADIUS (THEOGD) = 25.25382   PINION PRESSURE ANGLE AT FINAL CUT (THEOPO) = 36.45992

GEAR TOOTH THICKNESS AT OUTSIDE RADIUS (CAP1C) = .01976   PINION TOOTH THICKNESS AT FINAL CUT (TT8) = .01210

GEAR TOOTH THICKNESS AT BASE CIRCLE (CAP1B) = .01987   THEORETICAL PINION TOOTH THICKNESS AT BASE CIRCLE (TTB) = .02501

RADIUS OF ROOT CIRCLE OF GEAR (CAPROOT) = .15415   MINIMUM ALLOWABLE RADIUS OF ROOT CIRCLE OF GEAR (CAPRMIN) = .15347

THE GEAR IS NOT UNDERCUT

RADIUS OF ROOT CIRCLE OF PINION (RROOT) = .04957

C-13

DIAMETRAL PITCH (PCUBD) = 44.00

GEAR NUMBER OF TEETH (NG) = 56

PINION NUMBER OF TEETH (NP) = 8

PRESSURE ANGLE (THETA) = 20.00

GEAR PITCH RADIUS (CAPRWP) = .63636   PINION PITCH RADIUS (RP) = .09091

GEAR BASE RADIUS (CAPWB) = .59799   PINION BASE RADIUS (RB) = .08543

STANDARD TOOTH THICKNESS AT PITCH RADII (TSTAND) = .03570

HOB WITHDRAWAL DISTANCE (C) = .91209

GEAR BLANK RADIUS (CAPWD) = .64700   ORIGINAL PINION BLANK RADIUS (RO) = .12573

ORIGINAL CONTACT RATIO (CRATIO) = 1.349

PINION OUTSIDE RADIUS FOR UNITY CONTACT RATIO (ROFIN) = .10970

FINAL CONTACT RATIO (CRFIN) = 1.800

GEAR TOOTH THICKNESS AT PITCH CIRCLE (CAPTC) = .02698   PINION TOOTH THICKNESS AT PITCH CIRCLE (TC) = .04450

GEAR PRESSURE ANGLE AT OUTSIDE RADIUS (THEOGO) = 22.44425   PINION PRESSURE ANGLE AT FINAL OUTSIDE RADIUS (THEOPO) = 38.85337

GEAR TOOTH THICKNESS AT OUTSIDE RADIUS (CAPTD) = .01991   PINION TOOTH THICKNESS AT FINAL OUTSIDE RADIUS (TD) = .02401

GEAR TOOTH THICKNESS AT BASE CIRCLE (CAPTB) = .04310   THEORETICAL PINION TOOTH THICKNESS AT BASE CIRCLE (TB) = .04437

RADIUS OF ROOT CIRCLE OF GEAR (CAPROOT) = .59798   MINIMUM ALLOWABLE RADIUS OF ROOT CIRCLE OF GEAR (CAPRMIN) = .55957

THE GEAR IS NOT UNDERCUT

RADIUS OF ROOT CIRCLE OF PINION (ROOT) = .07671

DIAMETRAL PITCH (PSUBD) = 65.00

GEAR NUMBER OF TEETH (NG) = 56

PINION NUMBER OF TEETH (NP) = 8

PRESSURE ANGLE (THETA) = 20.00

GEAR PITCH RADIUS (CAPRP) = .43877 - PINION PITCH RADIUS (RP) = .06154

GEAR BASE RADIUS (CAPRB) = .60479 - PINION BASE RADIUS (RB) = .05783

STANDARD TOOTH THICKNESS AT PITCH RADI (TSTAND) = .02417

HOB WITHDRAWAL DISTANCE (C) = -.00619

GEAR BLANK RADIUS (CAPRO) = .43797 - ORIGINAL PINION BLANK RADIUS (RO) = .06511

ORIGINAL CONTACT RATIO (CRATIO) = 1.349

PINION OUTSIDE RADIUS FOR UNITY CONTACT RATIO (ROFIN) = .07426

FINAL CONTACT RATIO (CRFIN) = 1.000

GEAR TOOTH THICKNESS AT PITCH CIRCLE (CAPIC) = .01821 - PINION TOOTH THICKNESS AT PITCH CIRCLE (TC) = .03012

GEAR PRESSURE ANGLE AT OUTSIDE RADIUS (THEOGR) = 22.64487 - PINION PRESSURE ANGLE AT FINAL OUTSIDE RADIUS (THEOPD) = 30.65337

GEAR TOOTH THICKNESS AT OUTSIDE RADIUS (CAPIO) = .21287 - PINION TOOTH THICKNESS AT FINAL OUTSIDE RADIUS (TO) = .01464

GEAR TOOTH THICKNESS AT BASE CIRCLE (CAPTB) = .02918 - THEORETICAL PINION TOOTH THICKNESS AT BASE CIRCLE (TB) = .03003

RADIUS OF ROOT CIRCLE OF GEAR (CAPROOT) = .49478 - MINIMUM ALLOWABLE RADIUS OF ROOT CIRCLE OF GEAR (CAPRMIN) = .37879

THE GEAR IS NOT UNDERCUT

RADIUS OF ROOT CIRCLE OF PINION (ROOT) = .05192

2. Program INVOL 2: Point and Cycle Efficiencies for Single
Pass Involute Step-Up Gear Mesh With
Unity Contact Ratio

The program INVOL 2 is based on section 3 of Appendix A,
which gives the moment input-output relationship for a single
step-up gear mesh with involute teeth. The mesh has unity contact
ratio. Again, the nomenclature of the program is chosen to
coincide as much as possible with that of the original deriva-
tion. The contact geometry is adapted from section 6c of
Appendix A.

a. Input Parameters (See also program in section d below)

The following parameters represent the input data of the
program. Most of these are taken from the results of INVOL 1
since the moment expressions are for unity contact ratio only.

CAPRP = $R_p$, the pitch radius of the gear

RP = $r_p$, the pitch radius of the pinion

CAPRO = $R_o$, the outside radius of the gear

ROFIN = $R_o'$, see eq. (B-17). This is the pinion outside radius
for unity contact ratio.

RHOCAPN = $\rho_N$, the pivot radius of the gear

RHON = $\rho_N$, the pivot radius of the pinion

MU = $\mu$, the coefficient of friction at both pivots as well as
at the gear and pinion contact point

K, range divisor, i.e.,it represents the number of times the
output moment and the efficiency are computed between
initial and final contact of the gear and the pinion

C-16

ISTOP, arbitrary single digit for multiple data sets.  It must
be zero for the last set of data.


b.  Computations

Both point and cycle efficiencies are based on eq. (A-25).
With the help of eq. (3), the point
efficiency becomes

$$\epsilon_p = \frac{\{r_b - \mu[\rho_n + s(d-a)]\}}{\{R_b + \mu[\rho_N - sa]\}} \frac{\dot{\psi}}{\dot{\phi}} \qquad (C-1)$$

Since the angular velocity is constant for involute gears,
and may be expressed in terms of the base circle radii $R_b$ and $r_b$,

$$K_{ratio} = \frac{\dot{\psi}}{\dot{\phi}} = \frac{R_b}{r_b} \qquad (C-2)$$

Therefore,

$$\epsilon_p = E_2 \qquad (POINTEF) \qquad (C-3)$$

as given by eq. (A-26).

The cycle efficiency expression is based on eq. (4).  If
one replaces both integrals by summations, one obtains

$$\epsilon_o = \frac{\Sigma\ M_o\ \Delta\psi}{\Sigma\ M_{1n}\ \Delta\phi} \qquad (C-4)$$

where $\Delta\phi$ and $\Delta\psi$ now represent incremental changes in the input
and output angles, respectively.  The input moment is constant

C-17

over the total interval.  Also

$$\Sigma\Delta\varphi = \alpha_{FIN} - \alpha_{IN} \tag{C-5}$$

(See eqs. (A205) and (A206).)

The increment $\Delta\psi$ is also constant and may therefore be taken outside the summation sign.  It is expressed with the help of $\Delta\varphi$

$$\Delta\varphi = \Delta\alpha = \frac{\alpha_{FIN} - \alpha_{IN}}{K} = DELALPH \tag{C-6}$$

(This is an adaptation of eq. (A-207) to the single step-up mesh.)

With the above

$$\Delta\psi = K_{ratio}\,\Delta\alpha \tag{C-7}$$

Substitution into eq. (C-4) gives

$$\iota_0 = \frac{K_{ratio}\,\Delta\alpha\,\Sigma M_o}{M_{in}\,(\alpha_{FIN} - \alpha_{IN})} \tag{C-8}$$

Since

$$\iota_p = K_{ratio}\,\frac{M_o}{M_{in}} \tag{C-9}$$

one obtains for the cycle efficiency

$$\iota_0 = \frac{\Delta\alpha\,\Sigma\iota_p}{(\alpha_{FIN} - \alpha_{IN})} \quad (CYCLEFF) \tag{C-10}$$

C-18

To arrive at expressions for both POINTEF and CYCLEFF in the program, the following other important computations are necessary:

ALPHIN $= \alpha_{IN}$, see eq. (A-205)
ALPHFIN $= \alpha_{FIN}$, see eq. (A-206)
D $=$ d, see eq. (A-204)
A $=$ a, see eq. (A-203)

The signum value s is obtained according to eqs. (A-215) and (A-216), which are alternate ways of expressing eqs. (A-6) to (A-8).

c. Output of Program (see Program INVOL 2, below)

The output of the program is best explained by means of the single sample computation which is shown at the end of the program. This example uses the data of the first sample output of INVOL 1. The output lists the following:

I. Input Parameters

The input parameters CAPRP, RP, CAPRO, RO and THETAD are reproduced. In addition, the following dimensions, which were selected from a practical viewpoint, are shown:

RHOCAPN $= \rho_N =$ .060 in. (.152 cm)
RHON $= \rho_n =$ .030 in. (.076 cm)
MU $= \mu =$ .2
K $=$ 25 (no substantial changes in cycle efficiency were
        encountered for larger values of K)

C-19

## II.  Computed Values

The point efficiency is listed as a function of the angle $\alpha$, while the cycle efficiency is computed for the interval from $\alpha_{IN}$ to $\alpha_{FIN}$. The signum parameter, s, is listed for checking purposes.

Program INVOL 2

C - 21

```
      PROGRAM INVOL? (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C     POINT AND CYCLE EFFICIENCIES FOR SINGLE PASS INVOLUTE STEP-UP
C     GEAR MESH WITH UNITY CONTACT RATIO
C
      REAL MU,MTOT
    1 READ(5,2)CAPRP,RP,CAPRG,ROFIN,THETAG,RHOCAPN,PHKN,MU,K,ISTOP
    2 FORMAT(5F16.4/3F10.4/F7,I3,9X,I1)
      WRITE(6,3)CAPRP,RP,CAPRG,ROFIN,THETAG,RHOCAPK,RHON,MU,K
    3 FORMAT(*1GEAR PITCH RADIUS (CAPRP) =*,F7,5,3X,*PINION PITCH RADIUS
     1 (RP) =*,F7,5/*0GEAR OUTSIDE RADIUS (CAPRG) =*,F7,5,3X,*PINION OUT
     2SIDE RADIUS FOR UNITY CONTACT RATIO (ROFIN) =*,F7,5/*0PRESSURE ANG
     3LE IN DEGREES (THETAG) =*,F6,2/*0GEAR PIVOT RADIUS (RHOCAPN) =*,
     4F5,3,3X,*PINION PIVOT RADIUS (RHON) =*,F5,3/*0COEFFICIENT OF FRICT
     5ION (MU) =*,F4,2/*0RANGE DIVISOR (K) =*,I4///)
      PI = 3.14159
      Z = PI/180.
      THETA = THETAG*Z
      CAPRB = CAPRP*COS(THETA)
      RB = RP*COS(THETA)
      ALPHIN = ((CAPRB+RB)*TAN(THETA) - SQRT(ROFIN*ROFIN - RB*RB))/CAPRB
      ALPHFIN = SQRT(CAPRO*CAPRO - CAPRB*CAPRB)/CAPRB
      DELALPH = (ALPHFIN-ALPHIN)/K
      MTOT = 0.
      DO 5 I = 1,K
      ALPHA = ALPHIN + (I-1)*DELALPH
      IF(ALPHA .LT. TAN(THETA))S = 1.
      IF(ALPHA .EQ. TAN(THETA))S = 0.
      IF(ALPHA .GT. TAN(THETA))S = -1.
      A = CAPRB*ALPHA
      AA = 1. - MU*(RHON + S*(D-A))/RB
      BB = 1. - MU*(RHOCAPH - S*A)/CAPRB
      POINTEF = AA/BB
      MTOT = MTOT + POINTEF
      ALPHAD = ALPHA/Z
      WRITE(6,4)ALPHAD,S,POINTEF
    4 FORMAT(* ALPHAD =*,F6,2,3X,*S =*,F4,1,3X,*POINTEF =*,F6,4)
    5 CONTINUE
      CYCLEFF = DELALPH*MTOT/(ALPHFIN-ALPHIN)
      WRITE(6,6)CYCLEFF
    6 FORMAT(///*0 CYCLE EFFICIENCY =*,F6,4)
      IF(ISTOP .EQ. 0)GO TO 9999
      GO TO 1
 9999 STOP
      END
```

GEAR PITCH RADIUS (CAPRP) = .47727     PINION PITCH RADIUS (RP) = .09091

GEAR OUTSIDE RADIUS (CAPRO) = .49791    PINION OUTSIDE RADIUS FOR UNITY CONTACT RATIO (ROFIN) = .11000

PRESSURE ANGLE IN DEGREES (THETAD) = 20.00

GEAR PIVOT RADIUS (RHOCADN) = .060     PINION PIVOT RADIUS (RHOIN) = .030

COEFFICIENT OF FRICTION (MU) = .20

RANGE DIVISOR (K) = 75

| | | |
|---|---|---|
| ALPHAD = 15.07 | U = 1.0 | POINTEF = .7905 |
| ALPHAD = 16.32 | U = 1.0 | POINTEF = .7979 |
| ALPHAD = 16.66 | U = 1.0 | POINTEF = .8054 |
| ALPHAD = 17.38 | U = 1.0 | POINTEF = .8129 |
| ALPHAD = 17.34 | U = 1.0 | POINTEF = .8204 |
| ALPHAD = 17.69 | U = 1.0 | POINTEF = .8279 |
| ALPHAD = 18.63 | U = 1.0 | POINTEF = .8355 |
| ALPHAD = 18.37 | U = 1.0 | POINTEF = .8430 |
| ALPHAD = 18.72 | U = 1.0 | POINTEF = .8506 |
| ALPHAD = 19.06 | U = 1.0 | POINTEF = .8582 |
| ALPHAD = 19.46 | U = 1.0 | POINTEF = .8658 |
| ALPHAD = 19.75 | U = 1.0 | POINTEF = .8735 |
| ALPHAD = 20.09 | U = 1.0 | POINTEF = .8811 |
| ALPHAD = 20.43 | U = -1.0 | POINTEF = .8888 |
| ALPHAD = 20.77 | U = -1.0 | POINTEF = .8965 |
| ALPHAD = 21.12 | U = -1.0 | POINTEF = .9067 |
| ALPHAD = 21.46 | U = -1.0 | POINTEF = .9000 |
| ALPHAD = 21.80 | U = -1.0 | POINTEF = .8933 |
| ALPHAD = 22.15 | U = -1.0 | POINTEF = .8866 |
| ALPHAD = 22.49 | U = -1.0 | POINTEF = .8800 |
| ALPHAD = 22.83 | U = -1.0 | POINTEF = .8734 |
| ALPHAD = 23.17 | U = -1.0 | POINTEF = .8667 |
| ALPHAD = 23.52 | U = -1.0 | POINTEF = .8601 |
| ALPHAD = 23.86 | U = -1.0 | POINTEF = .8535 |
| ALPHAD = 24.20 | U = -1.0 | POINTEF = .8470 |

CYCLE EFFICIENCY = .8566

3. **Program INVOL 3: Point and Cycle Efficiencies for Three Pass Involute Step-Up Gear Train in Spin Environment (All Meshes Have Unity Contact Ratio)**

The program INVOL 3 is based on section 4 of Appendix A, which derives the moment input-output relationship for a three pass step-up gear train operating in a spin environment. Again, all meshes have unity contact ratio. As previously, the nomenclature of the program is chosen to coincide as closely as possible with that of the original derivations. The expressions for the contact geometry and other auxiliary geometric terms may be found in section 6 of Appendix A.


a. **Input Parameters (see Program INVOL 3, below)**

The following parameters represent the input data for the program. Those which involve gear dimensions only must be obtained from the results of INVOL 1 since the moment expressions are derived for unity contact ratio only.

MU $= \mu$, the coefficient of friction at all pivots and at all tooth contact points

RPM, revolutions per minute of the fuze body

CAPRP1 $= R_{p1}$

CAPRP2 $= R_{p2}$

CAPRP3 $= R_{p3}$

RP2 $= r_{p2}$

RP3 $= r_{p3}$

RP4 $= r_{p4}$

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THETA1 = $\theta_1$

THETA2 = $\theta_2$

THETA3 = $\theta_3$

ISTOP, arbitrary single digit integer for multiple data set.

It must be zero for last set of data.

R1 = $\mathcal{R}_1$

R2 = $\mathcal{R}_2$

R3 = $\mathcal{R}_3$

R4 = $\mathcal{R}_4$

RHO1 = $\rho_1$

RHO2 = $\rho_2$

RHO3 = $\rho_3$

RHO4 = $\rho_4$

CAPRB1 = $R_{b1}$

CAPRB2 = $R_{b2}$

CAPRB3 = $R_{b3}$

RB2 = $r_{b2}$

RB3 = $r_{b3}$

RB4 = $r_{b4}$

CAPRO1 = $R_{o1}$

CAPRO2 = $R_{o2}$

CAPRO3 = $R_{o3}$

RO2 = $r'_{o2}$

RO3 = $r'_{o3}$

RO4 = $r'_{o4}$

M1 = $m_1$, mass of input gear 1

M2 = $m_2$, mass of gear and pinion 2

M3 $=$ $m_3$, mass of gear and pinion 3

M4 $=$ $m_4$, mass of pinion 4

MD $=$ $md^2$, the "mass-distance" product contained in the expression for the input moment $M_{in}$

K $=$ $K_3$, the range divisor which is associated with gear 3, the driving gear of the last mesh (see eq. (A-211))

b. Computations (see COMMENT cards in program)

I. Computation of MIN, GAMMAS and BETAS

To start with, the program computes the input moment

$$MIN = M_{in} = md^2\omega^2 \qquad (C-11)$$

Subsequently, the angles $\gamma_2$, $\gamma_3$, $\gamma_4$ and $\beta_1$, $\beta_2$, $\beta_3$ are established according to the expressions given in section 6b of Appendix A.

II. Determination of Gear Train Constants

The determination of the gear train constants consists of the following:

RATIO $=$ $K_{ratio}$. (see eq. (2)). Since the angular velocity is constant, this parameter may be expressed in terms of the applicable base radii, i.e.,

$$\frac{R_{b1} \times R_{b2} \times R_{b3}}{r_{b2} \times r_{b3} \times r_{b4}}$$

TEST1, TEST2 and TEST3 represent the tangent functions of the mesh pressure angles, which are used in conjunction with the values of the signum functions, s.

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D1, D2 and D3 are given by eqs. (A-204), (A-217) and (A-223), respectively, and represent the distances between the points of tangency to the base circles along the lines-of-action of the three meshes.

MTOT = 0 represents the initialization of the sum of the output moments. This is used for the determination of the cycle efficiency.

### III. Determination of Initial and Final Values of ALPHAS, Initialization of ALPHAS and Centrifugal Forces

The determination of the initial and final angles of rotation is accomplished with the help of subroutine ALPHA, at the end of the program, which makes use of eqs. (A-205), (A-206), (A-218), (A-219), (A-224) and (A-225). Thus, the initial values of the individual angles of rotation, ALPHA1, ALPHA2 and ALPHA3 are represented by AL1IN, AL2IN and AL3IN, while the final angles are given by AL1FIN, AL2FIN and AL3FIN.

The angular increments of gears 3, 2 and 1, i.e., DELAL3, DELAL2 and DELAL1, are determined with the help of eqs. (A-211) - (A-213), respectively.

The centrifugal forces, which act on the pivots of the various gear and/or pinion assemblies, are obtained by way of eqs. (A-33), (A-57), (A-84) and (A-107).

### IV. Point and Cycle Efficiencies (See "output moment" in program)

Both point and cycle efficiencies are based on eq. (A-125) for the output moment $MO4 = M_{o4}$.

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The point efficiency is computed directly in the manner of eq. (3), i.e.,

$$\eta_p = K_{ratio} \frac{M_o4}{M_{in}} = POINTEF \tag{C-12}$$

The cycle efficiency is treated in the manner of eq. (C-8), i.e.,

$$\eta_p = \frac{K_{ratio} \Delta\alpha_1 \Sigma M_o4}{M_{in} (\alpha_{1FIN} - \alpha_{1IN})} = CYCLEFF \tag{C-13}$$

The program gives the summation as

$$MTOT = \Sigma M_o4 \tag{C-14}$$

### V. Gear Train Motion Model

The simulation of the gear train motion, which is necessary for the computation of both POINTEF and CYCLEFF, is found in a loop which starts with statement label no. 14 (card no. 116) and ends with card no. 199. As discussed earlier, the motions of the individual driving gears are initialized at their respective angles, AL1IN, AL2IN and AL3IN. (This starting of the total train is arbitrary and is done only for convenience. There is an infinite number of other starting combinations each of which produces a different starting point efficiency.) The position of each mesh is subsequently incremented by the appropriate DELAL1, DELAL2 or DELAL3. When the angle ALPHA1 reaches the magnitude AL1FIN, CYCLEFF is determined, and the computation is ended. Since meshes 2 and 3 go through numerous cycles while mesh 1 goes through one cycle, they have

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to be reset to their initial angles of rotation once their respective final angles have been reached. This is accomplished by the conditional statements on cards 116 and 117.

The values of the signum functions $s_1$, $s_2$ and $s_3$ are determined continuously according to eqs. (A-216), (A-222) and (A-227).

The instantaneous positions of the contact $A1 = a_1$, $A2 = a_2$, and $A3 = a_3$ are determined for each of the meshes by an appropriate adaptation of eq. (A-203). (See also eqs. (A-214), (A-220) and (A-226).)

The determination of the instantaneous output moment, $MO4 = M_o4$, requires the continuous computation of the variable quantities $A_1$ to $A_{20}$, $C_1$ to $C_6$ and $D_1$ to $D_4$, which are given originally in conjunction with the various equilibrium conditions in section 4 of Appendix A. The program uses the following nomenclature for these variables:

| AA1 | to | AA20 |
| CC1 | to | CC6 |
| DD1 | to | DD4 |

o. Output (see Program INVOL 3, below)

Again, the output of the program is best explained by means of the sample computation which is shown at the end of the program. This example uses the gear data of the first three sample computations of program INVOL 1. The output lists the following:

## I. Input Parameters

### Mesh No. 1

CAPRP1 $= R_{p1} = .47727$ in. (1.212 cm)

CAPRB1 $= R_{b1} = .44849$ in. (1.139 cm)

CAPRO1 $= R_{o1} = .48791$ in. (1.239 cm)

RP2 $= r_{p2} = .09091$ in. (.231 cm)

RB2 $= r_{b2} = .08543$ in. (.217 cm)

RO2 $= r'_{o2} = .11000$ in. (.279 cm) (This is an ROFIN as given by INVOL 1.)

Also,

THETA1 $= \theta_1 = 20^o$

### Mesh No. 2

CAPRP2 $= R_{p2} = .20769$ in. (.527 cm)

CAPRB2 $= R_{b2} = .19517$ in. (.496 cm)

CAPRO2 $= R_{o2} = .21579$ in. (.548 cm)

RP3 $= r_{p3} = .06923$ in. (.176 cm)

RB3 $= r_{b3} = .06506$ in. (.165 cm)

RO3 $= r'_{o3} = .08089$ in. (.205 cm)

Also,

THETA2 $= \theta_2 = 20^o$

### Mesh No. 3

CAPRP3 $= R_{p3} = .17532$ in. (.445 cm)

CAPRB3 $= R_{b3} = .16475$ in. (.418 cm)

CAPRO3 $= R_{o3} = .18216$ in. (.463 cm)

RP4 $= r_{p4} = .05844$ in. (.148 cm)

RB4 $= r_{b4} = .05492$ in. (.139 cm)

RO4 $= r'_{o4} = .06828$ in. (.173 cm)

Also,

$$\text{THETA3} = \theta_3 = 20°$$

In addition,

$$\text{MU} = \mu = .2$$

$$\text{RPM} = 1000$$

$$\text{M1} = m_1 = .51079 \times 10^{-4} \text{ lb-sec}^2/\text{in.} \quad (8.943 \text{ g})$$

$$\text{M2} = m_2 = .17413 \times 10^{-4} \text{ lb-sec}^2/\text{in.} \quad (3.049 \text{ g})$$

$$\text{M3} = m_3 = .69788 \times 10^{-5} \text{ lb-sec}^2/\text{in.} \quad (1.222 \text{ g})$$

$$\text{M4} = m_4 = .7745 \times 10^{-6} \text{ lb-sec}^2/\text{in.} \quad (0.136 \text{ g})$$

$$\text{R1} = \mathcal{R}_1 = .75 \text{ in.} \quad (1.905 \text{ cm})$$

$$\text{R2} = \mathcal{R}_2 = .75 \text{ in.} \quad (1.905 \text{ cm})$$

$$\text{R3} = \mathcal{R}_3 = .75 \text{ in.} \quad (1.905 \text{ cm})$$

$$\text{R4} = \mathcal{R}_4 = .75 \text{ in.} \quad (1.905 \text{ cm})$$

$$\text{RHO1} = \rho_1 = .060 \text{ in.} \quad (.152 \text{ cm})$$

$$\text{RHO2} = \rho_2 = .030 \text{ in.} \quad (.076 \text{ cm})$$

$$\text{RHO3} = \rho_3 = .025 \text{ in.} \quad (.064 \text{ cm})$$

$$\text{RHO4} = \rho_4 = .020 \text{ in.} \quad (.051 \text{ cm})$$

$$\text{MD} = md^2 = .15 \times 10^{-4} \text{ lb-sec}^2 \text{ in.} \quad (16.944 \text{ g-cm}^2)$$

$$\text{K} = 25$$

## II. Computed Values

The point efficiency is given as a function of the angle $\alpha_1$, together with the signum parameters $s_1$, $s_2$ and $s_3$ (given for checking purposes). The cycle efficiency is shown at the end of the output. In addition, the input moment, MIN, is printed out.

Program INVOL 3

```
        PROGRAM INVOL3(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C       POINT AND CYCLE EFFICIENCIES FOR THREE PASS INVOLUTE STEP-UP
C       IN SPIN ENVIRONMENT (ALL MESHES HAVE UNITY CONTACT RATIO)
C
        REAL MIN,MU,M1,M2,M3,M4,MO3,MO4,MTOT,MD
C
C       READ AND WRITE INPUT DATA
C
100     READ(5,1)MU,RPM,CAPRP1,CAPRP2,CAPRP3,RP7,RP4,RP4,THETA1,
       1THETA2,THETA3,ISTOP
        READ(5,2)R1,R2,R3,R4
        READ(5,3)RHO1,RHO2,RHO3,RH04
        READ(5,4)CAPRB1,CAPRB2,CAPRB3,RB2,RB3,RB4
        READ(5,5)CAPRO1,CAPRO2,CAPRO3,RO2,RO3,RO4
        READ(5,6)M1,M2,M3,M4
        READ(5,7)MD,K
        PI = 3.14159
        OMEGA = RPM*2.*PI/60.
        OM2 = OMEGA*OMEGA
1       FORMAT(F16.3,F10.076F(6.5/3F10.4/1)
2       FORMAT(4F10.4)
3       FORMAT(4F10.4)
4       FORMAT(6F10.5)
5       FORMAT(6F10.5)
6       FORMAT(4F10.2)
7       FORMAT(E10.2/I3)
C
C       COMPUTATION OF MIN, GAMMAS AND BETAS
C
        MIN = MD*OM2
        GAMMA2 = ACOS((R1*R1 + R2*R2 - (CAPRP1+RP2)*(CAPRP1+RP2))/
       1(2.*R1*R2))
        GAMA3P = ACOS((R2*R2 + R3*R3 - (CAPRP2+RP3)*(CAPPP2+RP3))/
       1(2.*R2*R3))
        GAMMA3 = GAMMA2 - GAMA3P
        GAMA4P = ACOS((R3*R3 + R4*R4 - (CAPRP3+RP4)*(CAPRP3+RP4))/
       1(2.*R3*R4))
        GAMMA4 = GAMMA3 + GAMA4P
        DELTA2 = ACOS(((CAPRP1+RP2)*(CAPRP1+RP2) + R1*R1 - R2*R2)/
       1(2.*R1*(CAPRP1 + RP2)))
        DELTA3 = ACOS(((CAPRP2+RP3)*(CAPRP2+RP3) + R2*R2 - R3*R3)/
       1(2.*R2*(CAPRP2 + RP3)))
        DELTA4 = ACOS(((CAPRP3+RP4)*(CAPRP3+RP4) + R3*R3 - R4*R4)/
       1(2.*R3*(CAPRP3 + RP4)))
        BETA1 = PI - DELTA2
        BETA2 = GAMMA2 + PI - DELTA3
        BETA3 = GAMMA3 + PI - DELTA4
        WRITE(6,8)MIN,RPM,MU,CAPRP1,CAPRP2,CAPRP3,RP2,RP3,RP4,THETA1,
       1THETA2,THETA3
        WRITE(6,9)R1,R2,R3,R4,M1,M2,M3,M4
        WRITE(6,10)RHO1,RHO2,RHO3,RH04
        WRITE(6,11)CAPRB1,CAPRP2,CAPRB3,RB2,RB3,RB4
```

```
55          WRITE(6,12)CAPRC1,CAPRO2,CAPRO3,CAPRO3,RO2,RO3,RO4
            WRITE(6,13)MD,K
          8 FORMAT(1P1*,5X,*MK =*,F12.5,3X,*MU =*,F6.3,3X,*PPM =*,F6.0//
           16X,*CAPRP1 =*,F8.5,3X,*CAPRP* =*,F5.5,3X,*CAPRP3 =*,F8.5//6X,
           2*RP2 =*,F8.5,3X,*RP3 =*,F* ,E  X,*RP4 =*,F8.5//6X,
           3*THETA1 =*,F6.3,3X,*THETA2  =*,F8.3,3X,*THETA3 =*,F8.3/)
          9 FORMAT(6X,*R1 =*,F7.5,3X,*R2 =*,F7.5,3X,*R3 =*,F7.5,3X,*R4 =*,F7.5
60         1//6X,*M1 =*,E15.5,3X,*M2 =*,F7.5,3X,*M3 =*,F7.5,3X,*M3 =*,E15.5/
           2*M4 =*,E15.5/)
         10 FORMAT(6X,*RHO1 =*,F7.5,3X,*RHO2 =*,F7.5,3X,*RHO3 =*,F7.5,3X,
           1*RHO4 =*,F7.5/)
         11 FORMAT(6X,*CAPRR1 =*,F7.5,3X,*CAPRR2 =*,F7.5,3X,*CAPRR2 =*,F7.5,
65         13X,*RR2 =*,F7.5,3X,*RR3 =*,F7.5,3X,*R84 =*,F7.5/)
         12 FORMAT(6X,*CAPRO1 =*,F7.5,3X,*CAPRO2 =*,F7.5,3X,*CAPRO3 =*,F7.5,3X
           1,*RO2 =*,F7.5,3X,*RO3 =*,F7.5,3X,*RO4 =*,F7.5/)
         13 FORMAT(6X,*MD =*,E10.3//6X,*RANGE DIVISOR =*,I4//)
70    C
      C   CONVERSION TO RADIANS
      C
            Z = PI/180.
            THETA1 = THETA1*Z
75          THETA2 = THETA2*Z
            THETA3 = THETA3*Z
      C
      C   DETERMINATION OF GEAR TRAIN CONSTANTS
      C
80          RATIO = CAPRR3*CAPRR2*CAPRR1/(PB2*RB3*RR4)
            TEST1 = TAN(THETA1)
            TEST2 = TAN(THETA2)
            TEST3 = TAN(THETA3)
            D1 = (CAPRR91 + RB2)*TAN(THETA1)
85          D2 = (CAPRRP2 + RR3)*TAN(THETA2)
            D3 = (CAPRR3 + RR4)*TAN(THETA3)
            WTGT = 0.
      C
      C   DETERMINATION OF INITIAL AND FINAL VALUES OF ALPHAS
      C
90          CALL ALPHA(CAPRR1,RB2,THETA1,CAPRO1,RO2,AL1IN,AL1FIN)
            CALL ALPHA(CAPRR2,RR3,THETA2,CAPRO2,RO3,AL2IN,AL2FIN)
            CALL ALPHA(CAPRR3,RR4,THETA3,CAPRO3,RO4,AL3IN,AL3FIN)
      C
            DELAL3 = (AL3FIN - AL3IN)/K
95          DELAL2 = DELAL3*RB3/CAPRB2
            DELAL1 = DELAL2*RB2/CAPRB1
      C
      C   INITIALIZATION OF ALPHAS
      C
100         ALPHA1 = AL1IN
            ALPHA2 = AL2IN
            ALPHA3 = AL3IN
      C
      C   CENTRIFUGAL FORCES
105   C
```

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```
          T1 = M1+T1-OM2
          T2 = M2+T2+OM2
          T3 = M3+T3+OM2
          T4 = M4+M4-TOM2

   110  C
        C   DENOM = 1. + MU+MU
        C
   115  C   UPDATE VALUES OF ALPHAS
        C
          14 IF(ALPHA2 .LT.  0.)F(1)+LPHA2 = AL21N
          IF(ALPHA3 .GT. 0.)F(IN)ALPHA3 = AL31N
        C
   120  C   TEST TO DETERMINE IF CONTACT POINT IS IN APPROACH OF RECESS
        C       1. APPROACH, S = 1.
        C       IF RECESS, S = -1.
        C       IF PITCH POINT, S = 0.
        C
   125     IF(ALPHA1 .LT. TEST1)S1 = 1.
          IF(ALPHA2 .LT. TEST2)S2 = 1.
          IF(ALPHA3 .LT. TEST3)S3 = 1.
          IF(ALPHA1 .GT. TEST1)S1 = -1.
          IF(ALPHA2 .GT. TEST2)S2 = -1.
          IF(ALPHA3 .GT. TEST3)S3 = -1.
   130     IF(ALPHA1 .EQ. TEST1)S1 = 0.
          IF(ALPHA2 .EQ. TEST2)S2 = 0.
          IF(ALPHA3 .EQ. TEST3)S3 = 0.
        C
        C   DETERMINATION OF INPUT FOR MOMENT EXPRESSIONS
   135  C
          A1 = ALPHA1+CAPR61
          A2 = ALPHA2+CAPR62
          A3 = ALPHA3+CAPR63
          2A1 = ABS(SIN(GAMMA1) - MU+COS(GAMMA3))/DENOM)
   140  AA2 = ABS((1.-S3+MU+M1)+COS(BETA3-THETA3) + MU+(1.-S3)
          (+SIN(BETA3-THETA3))/DENOM)
          AA3 = ABS((COS(GAMMA4) + MU+SIN(GAMMA4))/DENOM)
          AA4 = ABS((1.+S3+MU+M1)+SIN(BETA3-THETA3) - MU+(1.-S3)
          1+COS(BETA3-THETA3))/DENOM)
   145  AA5 = ABS((1.+MU+MU+S3)+COS(BETA2-THETA2) + MU+(S2-1.)
          1+SIN(BETA2-THETA2))/DENOM)
          AA6 = ABS(SIN(GAMMA3) + MU+COS(GAMMA3))/DENOM)
          AA7 = ABS((1.-MU+MU+S3)+COS(BETA3-THETA3) - MU+(1.+S3)
          1+SIN(BETA3-THETA3))/DENOM)
   150  AA8 = ABS((1.+MU+MU+S3)+SIN(BETA2-THETA2) + MU+(1.-S2)
          1+COS(BETA2-THETA2) - COS(GAMMA3))/DENOM)
          AA9 = ABS(MU+SIN(GAMMA3))/DENOM)
          AA10 = ABS((1.-MU+MU+S3)+SIN(BETA3-THETA3) + MU+(1.+S3)
          1+COS(BE+A3-THETA3))/DENOM)
          AA11 = ABS(MU+(1.-S1)+SIN(RC+A1+THETA1) + (1.-S1+MU+MU)
          1+COS(BETA1-THETA1))/DENOM)
   155  AA12 = ABS(SIN(GAMMA2) - MU+COS(GAMMA2))/DENOM)
          AA13 = ABS(MU+(1.+S2)-SIN(GE+A2-THETA2) + (1.-S2+MU+S2)
          1+COS(BETA2-THETA2))/DENOM)
```

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```
160      AA14 = ABS((MU)*(1.-S1)*COS(BETA1+THETA1) - (1.+MU+MU)*S1)
         1*SIN(BETA1+THETA1))/DE+DM)
         AA15 = ABS((MU*SIN(GAM*A2) - COS(GAM*A2))/DENOM)
         AA16 = ABS((1.-MU*MU+2)*SIN(BETA2-THETA2) - MU*(1.-S2)
         1*COS(BETA2-THETA2))/DE+DM)
165      AA17 = ABS(((1.-MU*MU*S1)*SIN(BETA1+THETA1) + MU*(1.-S1)
         1*COS(BETA1+THETA1))/DENOM)
         AA18 = ABS(1./DENOM)
         AA19 = ABS((MU*(1.+S1)*SIN(BETA1+THETA1) - (1.-MU+PI)*S1)
         1*COS(BETA1+THETA1))/DE+DM)
170      AA20 = ABS(MU/DENOM)
         DD1 = RR4 - MU*(S3*(D3.+A3) + RHO4*(AA2+AA4))
         DD2 = -MU*(RH+O3*(AA5+AA8) + S2*(D2-A2)) + PR3
         DD3 = PR2 - MU*(S1*(D1-A1) + RHO2*(AA1+AA14))
         DD4 = CAPRR1 - MU+(S3+A1 - RHO3*(AA17+AA19))
175      CC1 = MU*RHO2+(AA1+AA3)
         CC2 = CAPRR3 - MU*(S3+A3 - RHO3*(AA7+AA10))
         CC3 = MU*RH+O3*(AA6+AA9)
         CC4 = CAPRR2 - MU*(S2+A2 - RHO2*(AA13+AA16))
         CC5 = MU*RHO2+(AA12+AA15)
180      CC6 = MU*R+O1*(AA18+AA20)
C
C   OUTPUT MOMENT
C

         ALPHIG = ALPHA1+198./PI
185      R01=DD2+DD3/(CC2+CC6+DD4+(MIN-T1+CC6). - T2+CC5+DD1+DD2
         1/(CC2+CC4) - T3+CC3+DD1/CC2 - T4+CC1
         POINTEF = RATIO+R04/MI+
         WRITE(6.15)ALPH+1D-S1+S3,POINTEF
15       FORMAT(6X+ALPHA1 =+F4.2++ (DEG)+,3X,+S1 =+F5.1,3X,+S2 =+F5.1,
190      13X,+S3 =+F5.1,3X,+POINT EFFICIENCY =+F7.5;
         MTOT = MTOT + M0+
C
C   A VANCE GEAR TRAIN TO NEXT POSITION
C
         ALPHA1 = ALPHA1 + DELA1
195      IF(ALPHA1 .GT. ALIFIN)GO TO 16
         ALPHA2 = ALPHA2 + DELA1
         ALPHA3 = ALPHA3 + DELA1 3
         GO TO 14
16       CYCLEFF = RATIO+DELA1+(+MTOT/(MIN*(ALIFIN-ALIIN))
200      WRITE(6.17)CYCLEFF
17       FORMAT(+0+,5X,+CYCLE EFFICIENCY =+,F5.3)
         IF((STOP .NE. 0)GO TO 128
         STOP
205      END
```

```
      1          SUBROUTINE ALPHA(CAPRR,RR,THETA,CAPRO,RO,ALIN,ALFIN)
                 C
                 C   THIS SUBROUTINE COMPUTES THE INITIAL AND FINAL VALUES OF ALPHAS
                 C
      5          ALIN = ((CAPRR + RR)*TAN(THETA) - SQRT(PO*PO - RR*RR))/CAPRR
                 ALFIN = SQRT(CAPRO*CAPRO - CAPRR*CAPRB)/CAPRB
                 RETURN
                 END
```

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ALPHA1 = 24.07 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .45544
ALPHA1 = 24.10 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .46864
ALPHA1 = 24.13 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .46588
ALPHA1 = 24.17 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .47115
ALPHA1 = 24.19 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .47646
ALPHA1 = 24.27 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .48100
ALPHA1 = 24.30 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .48718
ALPHA1 = 24.34 (DEG) S1 = -1.0 S2 = 1.0 S3 = 1.0 POINT EFFICIENCY = .9268
ALPHA1 = 24.37 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .49968
ALPHA1 = 24.45 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .45709
ALPHA1 = 24.44 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .43459
ALPHA1 = 24.47 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .45149
ALPHA1 = 24.51 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .48928
ALPHA1 = 24.54 (DEG) S1 = -1.0 S2 = 1.0 S3 = -1.0 POINT EFFICIENCY = .48657

CYCLE EFFICIENCY = .601

4. Program INVOL 4: Point and Cycle Efficiencies for Two
Pass Involute Step-Up Gear Train in
Spin Environment (All Meshes Have
Unity Contact Ratio)

The program INVOL 4 is based on section 5 of Appendix A,
which derives the moment input-output relationship for a two
pass step-up gear train, operating in a spin environment.
Here again, all meshes have unity contact ratio. INVOL 4 is
very similar to INVOL 3 in its construction. Again, the
expressions for the contact geometry and other auxiliary
geometric terms may be found in section 6 of Appendix A.

a. Input Parameters (see Program INVOL 4, below)

The following parameters represent the input data for the
program. Those which involve gear dimensions only must be
obtained from the results of INVOL 1 since the moment expressions
are again derived for unity contact ratio only.

MU = $\mu$, coefficient of friction at all pivots and at all
tooth contact points

RPM, revolutions per minute of the fuze body

CAPRP1 = $R_{p1}$

CAPRP2 = $R_{p2}$

RP2 = $r_{p2}$

RP3 = $r_{p3}$

THETA1 = $\theta_1$

THETA2 = $\theta_2$

ISTOP,  arbitrary single-digit integer for multiple data sets.

It must be zero for last set of data.

R1 $= \mathcal{R}_1$

R2 $= \mathcal{R}_2$

R3 $= \mathcal{R}_3$

RHO1 $= \rho_1$

RHO2 $= \rho_2$

RHO3 $= \rho_3$

CAPRB1 $= R_{b1}$

CAPRB2 $= R_{b2}$

RB2 $= r_{b2}$

RB3 $= r_{b3}$

CAPRO1 $= R_{o1}$

CAPRO2 $= R_{o2}$

RO2 $= r'_{o2}$

RO3 $= r'_{o3}$

M1 $= m_1$,  mass of input gear 1

M2 $= m_2$,  mass of gear and pinion 2

M3 $= m_3$,  mass of pinion 3

MD $= md^2$,  "mass-distance" product contained in the following

expression  for the input moment $M_{in}$

K $= K_2$,  the range divisor which is associated with gear 2,

the driving gear of the last mesh for this case

(see eq. (A-207))

b. Computations (see COMMENT cards in program)

### I. Computation of MIN, Gammas and Betas

To start with, the program computes the input moment

$$MIN = M_{in} = md\omega^2 \qquad\qquad (C-15)$$

The program computes the angles $\gamma_2$, $\gamma_3$ and $\beta_1$, $\beta_2$ according to the expressions given in section 6b of Appendix A.

### II. Determination of the Gear Train Constants

The determination of the gear train constants consists of the following:

RATIO = $K_{ratio}$. (see eq. (2)). Since the angular velocity is constant, this parameter may be expressed in terms of the applicable base radii, i.e.,

$$\frac{R_{b1} \times R_{b2}}{r_{b2} \times r_{b3}}$$

TEST1 and TEST2 represent the tangent functions of the mesh pressure angles, which are used in conjunction with the values of the signum functions s.

D1, and D2 are given by eqs. (A-204) and (A-217), respectively, and represent the distances between the points of tangency to the base circles along the lines-of-action of the two meshes.

MTOT = 0 represents the initialization of the sum of the output moments. This is used for the determination of the cycle efficiency.

## III. Determination of Initial and Final Values of ALPHAS, Initialization of ALPHAS and Centrifugal Forces

The determination of the initial and final angles of rotation is accomplished with the help of subroutine ALPHA, at the end of the program, which makes use of eqs. (A-205), (A-206) as well as (A-218) and (A-219). Thus, the initial values of the individual angles of rotation, ALPHA1 and ALPHA2, are represented by AL1IN and AL2IN, respectively, while the final ones are given by AL1FIN and AL2FIN.

The angular increments of gears 2 and 1, i.e., DELAL2 and DELAL1, are det mined with the help of eqs. (A-207) and (A-208), respectively.

The centrifugal forces, which act on the pivots of the various gear and/or pinion assemblies, are obtained by way of eqs. (A-131), (A-154) and (A-178).

## IV. Point and Cycle Efficiencies (See "output moment" in program)

Both point and cycle efficiencies are based on eq. (A-193) for the output moment $MO3 = M_{o3}$.

The point efficiency is computed directly in the manner of eq. (3), i.e.,

$$\eta_p = K_{ratio} \frac{M_{o3}}{M_{in}} = POINTEF \tag{C-16}$$

The cycle efficiency is treated in the manner of eq. (C-13), i.e.,

$$r_o = \frac{K_{ratio} \, \Delta\alpha_1 \, \Sigma M_{o3}}{M_{in} \, (\alpha_{1FIN} - \alpha_{1IN})} = \text{CYCLEFF} \qquad (C-17)$$

The program gives the summation es

$$\text{MTOT} = \Sigma M_{o3} \qquad (C-18)$$

## V. Gear Train Motion Model

The simulation of the gear train motion, which is necessary for the computation of both POINTEF and CYCLEFF, is found in a loop which begins with statement label no. 14 (card no. 98) and ends with card no. 161.

As discussed earlier, the motions of the individual driving gears are initialized at their respective angles, AL1IN and AL2IN.                        (This starting of the total train is arbitrary and is done only for convenience. There is an infinite number of other starting combinations, each of which produces a different starting point efficiency.) The position of each mesh is subsequently incremented by the appropriate DELAL1 or DELAL2. When the angle ALPHA1 reaches the magnitude AL1FIN, CYCLEFF is determined, and the computation is ended. Since mesh 2 goes through a number of cycles while mesh 1 goes through one cycle, mesh 2 has to be reset to its starting position, AL2IN, once the angle AL2FIN has been reached. This is accomplished by the conditional statement on card no. 98.

The values of the signum functions $s_1$ and $s_2$ are determined continuously according to eqs. (A-216) and (A-222).

The instantaneous positions of the contact, A1 = $a_1$ and A2 = $a_2$, are determined for each of the meshes by appropriate

adaptations of eq. (A-203). (See also eqs. (A-214) and (A-220).)

The determination of the instantaneous output moment, $MO3 = M_{o3}$, requires the continuous computation of the variable quantities $A_1$ to $A_{14}$, $C_1$ to $C_4$ and $D_1$ to $D_3$, which are given originally in conjunction with the various equilibrium conditions in section 5 of Appendix A. The program uses the following nomenclature for these variables:

AA1 to AA14

CC1 to CC4

DD1 to DD3

c. Output (see Program INVOL 4, below)

The output of the program is again best explained with the help of the sample computation shown at the end of the program. This example uses the gear data of the fourth and fifth sample computations of program INVOL 1. The output lists the following:

I. Input Parameters

Mesh No. 1

CAPRP1 = $R_{p1}$ = .63636 in. (1.616 cm)

CAPRB1 = $R_{b1}$ = .59799 in. (1.519 cm)

CAPRO1 = $R_{o1}$ = .64700 in. (1.643 cm)

RP2 = $r_{p2}$ = .09091 in. (0.231 cm)

RB2 = $r_{b2}$ = .08543 in. (0.217 cm)

RO2 = $r'_{o2}$ = .1097 in. (0.279 cm) (This is a ROFIN as given by INVOL 1.)

Also

THETA1 = $\theta_1$ = 20°

## Mesh No. 2

$CAPRP2 = R_{p2} = .43077$ in. (1.094 cm)

$CAPRB2 = R_{b2} = .40479$ in. (1.028 cm)

$CAPRO2 = R_{o2} = .43977$ in. (1.117 cm)

$RP3 = r_{p3} = .06154$ in. (0.156 cm)

$RB3 = r_{b3} = .05783$ in. (0.147 cm)

$RO3 = r'_{o3} = .07426$ in. (0.189 cm)

Also

$THETA2 = \theta_2 = 20^{\circ}$

In addition,

$MU = \mu = .2$

$RPM = 1000$

$M1 = m_1 = .51079 \times 10^{-4}$ lb-sec$^2$/in. (8.943 g)

$M2 = m_2 = .17413 \times 10^{-4}$ lb-sec$^2$/in. (3.049 g)

$M3 = m_3 = .69788 \times 10^{-5}$ lb-sec$^2$/in. (1.222 g)

$R1 = \mathcal{R}_1 = .75$ in. (1.905 cm)

$R2 = \mathcal{R}_2 = .75$ in. (1.905 cm)

$R3 = \mathcal{R}_3 = .75$ in. (1.905 cm)

$RHO1 = \rho_1 = .060$ in. (0.152 cm)

$RHO2 = \rho_2 = .030$ in. (0.076 cm)

$RHO3 = \rho_3 = .025$ in. (0.064 cm)

$MD = md^2 = .15 \times 10^{-4}$ lb-sec$^2$ in. (16.944 g-cm$^2$)

$K = 25$

## II. Computed Values

The point efficiency is given as a function of the angle $\alpha_1$, together with the signum parameters $s_1$ and $s_2$ (given for checking purposes). The cycle efficiency is shown at the end of the output. In addition, the input moment, MIN, is printed out.

Program INVOL 4

```
              PROGRAM INVOL 6(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)

       C      POINT AND CYCLE EFFICIENCIES FOR TWO PASS INVOLUTE STEP-UP GEAR TRAIN
       C      IN SPIN ENVIRONMENT (ALL MESHES HAVE UNITY CONTACT RATIO)
       C
  5           REAL MIN,M1,M2,M3,M03,MTOT,MD
       C
       C      READ AND WRITE INPUT DATA
       C
 10    100    READ(5,1)MU,RPM,CAPRP1,CAPRP2,RP2,RP3,THETA1,THETA2,ISTOP
              READ(5,2)R1,R2,R3
              READ(5,3)RHO1,RHO2,RHO3
              READ(5,4)CAPRR1,CAPRR2,RR2,RR3
              READ(5,5)CAPRO1,CAPRO2,RO2,RO3
 15           READ(5,6)M1,M2,M3
              READ(5,7)MD,K
       1      FORMAT(F10.3,F10.0/4F10.5/2F10.4/I1)
       2      FORMAT(3F10.5)
       3      FORMAT(3F10.5)
       4      FORMAT(4F10.5)
 20    5      FORMAT(4F10.5)
       6      FORMAT(3E10.4)
       7      FORMAT(F10.4/I3)
              PI = 3.14159
              OMEGA = RPM*2.*PI/60.
 25           OM2 = OMEGA*OMEGA
       C
       C      COMPUTATION OF MIN, GAMMAS AND BETAS
       C
              MIN = MD*OM2
 30           GAMMA2 = ACOS((R1*R1 + R2*R2 - (CAPRP1*PP2)*(CAPRP1+RP2))/
             1(2.*R1*R2))
              GAMMA3P = ACOS((R2*R2 + R3*R3 - (CAPRP2+RP3)*(CAPRP2-RP3))/
             1(2.*R2*R3))
              GAMMA3 = GAMMA2 + GAMA3P
 35           DELTA2 = ACOS((((CAPRP1*PP2)*(CAPRP1+PP2) + P1*R1 - R2*R7)/
             :(2.*R1*(CAPRP1 - RP2)))
              DELTA3 = ACOS((((CAPRP2*RP3)*(CAPRP2+RP3) + R2*R2 - R3*R3)/
             1(2.*R2*(CAPRP2 + RP3)))
              BETA1 = PI - DELTA2
 40           BETA2 = GAMMA2 + PI - DELTA3
              WRITE(6,8)MIN,MU,RPM,CAPRP1,CAPRP2,RP2,RP3,THETA1,THETA2
              WRITE(6,9)R1,R2,R3,M1,M2,M3
              WRITE(6,10)RHO1,RHO2,R403
              WRITE(6,11)CAPRR1,CAPRR2,RR2,RR3
 45           WRITE(6,12)CAPRO1,CAPRO2,RO2,RO3
              WRITE(6,13)MD,K
       8      FORMAT(*1*,5X,*MIN =*,F12.5,3X,*MU =*,F6.3,3X,*RPM =*,F6.0//6X,
             1*CAPRP1 =*,F8.5,3X,*CAPRP2 =*,F8.5/6X,*PP2 =*,F8.5,3X,*RP3 =*,
             7F8.5//6X,*THETA1 =*,F8.3,3X,*THETA2 =*,F8.3/)
       9      FORMAT(6X,*R1 =*,F7.5,3X,*R2 =*,F7.5,3X,*R3 =*,F7.5//6X,*M1 =*,
             1F12.5,3X,*M2 =*,E12.5,3X,*M3 =*,E12.5/)
 50    10     FORMAT(5X,*RHO1 =*,F7.5,3X,*RHO2 =*,F7.5,3X,*RHO3 =*,F7.5/)
```

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```
       11 FORMAT(6X,*CAPRB1 =*,F7.5,3X,*CAPRB2 =*,F7.5,3X,*RB2 =*,F7.5,3X,
         1*RB3 =*,F7.5/)
       12 FORMAT(6X,*CAPRO1 =*,F7.5,3X,*CAPRO2 =*,F7.5,3X,*RO2 =*,F7.5,3X,
         1* RO3 =*,F7.5/)
       13 FORMAT(6X,*MU =*D =*,E10.3//6X,*RANGE DIVISOR =*,I6//)
C
C      CONVERSION TO RADIANS
C
         Z = PI/180.
         THETA1 = THETA1*Z
         THETA2 = THETA2*Z
C
C      DETERMINATION OF GEAR TRAIN CONSTANTS
C
         RATIC = (CAPRA2+CAPRB1)/(RB2+RB3)
         TEST1 = TAN(THETA1)
         TEST2 = TAN(THETA2)
         D1 = (CAPRB1 + RB2)*TA(THETA1)
         D2 = (CAPRA2 + RB3)*TAN(THETA2)
         PTOT = 0.
C
C      DETERMINATION OF INITIAL AND FINAL VALUES OF ALPHAS
C
         CALL ALPHA(CAPRB1,RB2,THETA1,CAPRO1,RO2,ALI1N,ALIFIN)
         CALL ALPHA(CAPRB2,RB3,THETA2,CAPRO2,RO3,AL2IN,AL2FIN)
C
         DELAL2 = (AL2FIN - AL2IN)/K
         DELAL1 = DELAL2*RR2/CAPRB1
C
C      INITIALIZATION OF ALPHAS
C
         ALPHA1 = ALI1N
         ALPHA2 = AL2IN
C
C      CENTRIFUGAL FORCES
C
         T1 = M1*R1*W2
         T2 = M2*R2*W2
         T3 = M3*R3*W2
C
         DENOM = 1. + MU*MU
C
C      UPDATE VALUES OF ALPHAS
C
       14 IF(ALPHA2 .GT. AL2FIN)ALPHA2 = AL2IN
C
C      TEST TO DETERMINE IF CONTACT POINT IS IN APPROACH OR RECESS
C      IF APPROACH, S = 1.
C      IF RECESS, S = -1.
C      AT PITCH POINT, S = 0.
C
         IF(ALPHA1 .LT. TEST1)S1 = 1.
         IF(ALPHA2 .LT. TEST2)S2 = 1.
```

```
           IF(ALPHA1 .GT. TES1)IS1 = -1.
           IF(ALPHA2 .GT. TES1)IS2 = -1.
           IF(ALPHA1 .EQ. TES1)IS1 = 0.
110        IF(ALPHA2 .EQ. TES1)IS2 = 0.
     C
     C     DETERMINATION OF INPUT FOR MOMENT EXPRESSIONS
     C
           A1 = ALPHA1*CAPPH1
           A2 = ALPHA2*CAPPH2
115        AA1 = ABS((1.+MU*MU*S2)*COS(BETA2-THETA2) + MU*(S2-1.)*
          1SIN(BETA2-THETA2))/DENOM)
           AA2 = ABS((SIN(GAMMA3) + MU*COS(GAMMA3))/DENOM)
           AA3 = ABS((1.-MU*MU*S2)*SIN(BETA2-THETA2) + MU*(1.-S2)*COS(BETA2
          1-THETA2))/DENOM)
120        AA4 = ABS((MU*SIN(GAMMA3) - COS(GAMMA3))/DENOM)
           AA5 = ABS((MU-(1.-S1)*SIN(BETA1+THETA1) + (1.-S1-MU*MU))*
          1COS(BETA1+THETA1))/DENOM)
           AA6 = ABS(SIN(GAMMA2) - MU*COS GAMMA2))/DENOM)
125        AA7 = ABS((MU*(1.-S2)*SIN(BETA2-THETA2) + (1.-MU-MU*S2)*
          1COS(BETA2-THETA2))/DENOM)
           AA8 = ABS((MU*(1.-S1)*COS(BETA1+THETA1) - (1.-MU+MU*S1))*
          1SIN(BETA1+THETA1))/DENOM)
           AA9 = ABS((MU*SIN(GAMMA2) + COS(GAMMA2))/DENOM)
130        AA10 = ABS((-MU*RHO3*S2)*SIN(BETA2-THETA2) - MU*(1.+S2)*
          1COS(BETA2-THETA2))/DENOM)
           AA11 = ABS((1.-MU*MU*S1))*SIN(BETA1+THETA1) + MU*(1.+S1)*
          1COS(BETA1-THETA1))/DENOM)
           AA12 = 1./DENOM
135        AA13 = ABS((MU*(1.-S1)*SIN(BETA1+THETA1) - (1.-MU*MU*S1)*
          1COS(BETA1+THETA1))/DENOM)
           AA14 = MU/RENOM
           CC1 = MU*RHO3*(AA2-AA4)
           CC2 = ALPHB2 - MU*(S2*A2-RHO2*(AA7+AA10))
140        CC3 = RHO*RHO2*(AA6-AA9)
           CC4 = MU*R-R01*(AA12+AA14)
           DO1 = RB3 - MU*(S2*(02-A2)+RHO3*(AA1+AA3))
           DO2 = RB2 - MU*(S1*-01-A11-RHO2*(AA5-AA8))
           DO3 = CAP+81 - MU*(S1*A1-RHO1*(AA11+AA13))
145        ALPH1D = ALPHA1//4
     C
     C     OUTPUT MOMENT
     C
           MO3 = (DO1+DO2)/(CC2+DO3)+(MIN-T1+CC4) - T2+CC3+DO1/CC2 - T3*CC1
150        POINTEF = ZATIO+RO3/MIN
        15 WRITE(6,15)ALPH1D-S1,S2,POINTEF
           FORMAT(6(1H ,ALPHA1 =,F6.2,3H,*S1 =*,F5.1,3X,*S2 =*,F5.1,3X,
          19*POINT EFFICIENCY =*,F7.5)
           MTOT = MTOT + MO3.
     C
     C     ADVANCE GEAR TRAIN TO NEXT POSITION
     C
155        ALPHA1 = ALPHA1 - DELBL1
           IF(ALPHA1 .GT. ALIFIN)GO TO 16
```

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```
160      ALPHA2 = ALPHA2 + DELAL1*CAPR81/R92
         GO TO 14
16       CYCLEFF = (RATIO*DELAL1*MTOT)/(MIN*(ALJFIN-ALJIN))
         WRITE(6.17)CYCLEFF
17       FORMAT(1H*,6X,*CYCLE EFFICIENCY =*,F5.3)
165      IF(J .NE. 0)GO TO 100
         STOP
         END
```

```
      1        SUBROUTINE ALPHA(CAPRB,RB,THETA,CAPRO,RO,ALIN,ALFIN)
               C
               C   THIS SUBROUTINE COMPUTES THE INITIAL AND FINAL VALUES OF ALPHAS

               ALIN = ((CAPRB + RB)*TAN(THETA) - SQRT(RO*RO - RB*RB))/CAPRB
      5        ALFIN = SQRT(CAPRO*CAPRO - CAPRB*CAPRB)/CAPRB
               RETURN
               END
```

MIN = .1544E+05   NU = .288   RPM = 1800.

CAPRP1 = 6.3636   CAPRP2 = .3977

RP2 = .90001   RP3 = .56154

THETA1 = 20.000   THETA2 = 20.000

R1 = .75000   R2 = .75000   R3 = .75000

M1 = .5187E-04   M2 = .17613E-04   M3 = .6978E-05

RHO1 = .86986   RHO2 = .8398   RHO3 = .6258

CAPRO1 = .59794   CAPRO2 = .68679   NBC = .88543   DE3 = .65703

CAPHO1 = .56700   CAPHO2 = .63797   HO2 = .10070   HO3 = .07626

HO = .150E-04

RANGE DIVIS = 25

| ALPHA1 | S1 | S2 | POINT EFFICIENCY |
|---|---|---|---|
| 17.26 | 1.0 | 1.0 | .61855 |
| 17.28 | 1.0 | 1.0 | .61634 |
| 17.31 | 1.0 | 1.0 | .62267 |
| 17.35 | 1.0 | 1.0 | .62704 |
| 17.39 | 1.0 | 1.0 | .63360 |
| 17.42 | 1.0 | 1.0 | .63952 |
| 17.46 | 1.0 | 1.0 | .64539 |
| 17.50 | 1.0 | 1.0 | .65128 |
| 17.53 | 1.0 | 1.0 | .65720 |
| 17.57 | 1.0 | 1.0 | .66314 |
| 17.61 | 1.0 | 1.0 | .66910 |
| 17.64 | 1.0 | 1.0 | .67509 |
| 17.68 | 1.0 | 1.0 | .68110 |
| 17.72 | 1.0 | -1.0 | .68714 |
| 17.75 | 1.0 | -1.0 | .69321 |
| 17.79 | 1.0 | -1.0 | .51625 |
| 17.83 | 1.0 | -1.0 | .51258 |
| 17.86 | 1.0 | -1.0 | .50898 |
| 17.90 | 1.0 | -1.0 | .50522 |
| 17.94 | 1.0 | -1.0 | .50153 |
| 17.97 | 1.0 | -1.0 | .49786 |
| 18.01 | 1.0 | -1.0 | .49415 |
| 18.05 | 1.0 | -1.0 | .49044 |
| 18.08 | 1.0 | -1.0 | .48676 |
| 18.12 | 1.0 | -1.0 | .48342 |
| 18.16 | 1.0 | -1.0 | .47938 |
| 18.19 | 1.0 | 1.0 | .48090 |
| 18.23 | 1.0 | 1.0 | .48354 |
| 18.27 | 1.0 | 1.0 | .48191 |
| 18.31 | 1.0 | 1.0 | .48790 |
| 18.34 | 1.0 | 1.0 | .48391 |
| 18.38 | 1.0 | 1.0 | .48995 |
| 18.42 | 1.0 | 1.0 | .48651 |
| 18.45 | 1.0 | 1.0 | .47210 |
| 18.49 | 1.0 | 1.0 | .47821 |
| 18.53 | 1.0 | 1.0 | .48465 |
| 18.56 | 1.0 | 1.0 | .48651 |
| 18.60 | 1.0 | 1.0 | .49678 |
| 18.64 | 1.0 | 1.0 | .48261 |
| 18.67 | 1.0 | 1.0 | .54915 |
| 18.71 | 1.0 | 1.0 | .51541 |

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```
ALPHA1 = 18.58   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5365
ALPHA1 = 18.62   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5161
ALPHA1 = 18.66   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .2777
ALPHA1 = 18.90   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .2242
ALPHA1 = 18.93   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .2207
ALPHA1 = 18.97   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5161
ALPHA1 = 19.08   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5124
ALPHA1 = 19.04   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5147
ALPHA1 = 19.08   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5168
ALPHA1 = 19.11   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .4872
ALPHA1 = 19.15   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .4867
ALPHA1 = 19.22   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .4519
ALPHA1 = 19.26   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .7428
ALPHA1 = 19.30   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .4492
ALPHA1 = 19.33   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .4078
ALPHA1 = 19.37   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .4386
ALPHA1 = 19.41   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .4937
ALPHA1 = 19.44   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .5651
ALPHA1 = 19.52   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .5207
ALPHA1 = 19.55   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .5184
ALPHA1 = 19.59   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .5657
ALPHA1 = 19.63   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .5131
ALPHA1 = 19.66   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5777
ALPHA1 = 19.74   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5943
ALPHA1 = 19.77   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5647
ALPHA1 = 19.85   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .2446
ALPHA1 = 19.92   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5842
ALPHA1 = 19.96   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5443
ALPHA1 = 20.00   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5357
ALPHA1 = 20.03   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .2633
ALPHA1 = 20.07   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .5226
ALPHA1 = 20.14   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .4201
ALPHA1 = 20.18   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .4032
ALPHA1 = 20.22   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .4765
ALPHA1 = 20.25   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .4975
ALPHA1 = 20.43   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .5123
ALPHA1 = 20.33   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .5759
ALPHA1 = 20.36   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .5147
ALPHA1 = 20.44   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .2060
ALPHA1 = 20.47   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .2772
ALPHA1 = 20.51   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .5378
ALPHA1 = 20.55   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .5657
ALPHA1 = 20.62   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5342
ALPHA1 = 20.66   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5829
ALPHA1 = 20.73   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5167
ALPHA1 = 20.77   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .2736
ALPHA1 = 20.88   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .2516
ALPHA1 = 20.90   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5648
ALPHA1 = 20.95   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5583
ALPHA1 = 20.98   S1 = 1.0   X2 =   S2 = -1.0   POINT EFFICIENCY = .5673
ALPHA1 = 21.04   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .5506
ALPHA1 = 21.02   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .5778
ALPHA1 = 21.06   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .5214
ALPHA1 = 21.18   S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .4519
ALPHA1 =         S1 = 1.0   X2 =   S2 = 1.0    POINT EFFICIENCY = .5033
```

ALPHA1 = 21.17   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .21575
ALPHA1 = 21.21   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .21575
ALPHA1 = 21.24   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .52868
ALPHA1 = 21.28   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .52602
ALPHA1 = 21.32   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .53115
ALPHA1 = 21.35   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .53828
ALPHA1 = 21.39   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .54141
ALPHA1 = 21.43   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .54854
ALPHA1 = 21.47   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .55166
ALPHA1 = 21.50   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .55678
ALPHA1 = 21.54   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .56190
ALPHA1 = 21.58   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .56702
ALPHA1 = 21.61   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .56116
ALPHA1 = 21.65   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .54527
ALPHA1 = 21.68   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .57342
ALPHA1 = 21.72   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .57356
ALPHA1 = 21.76   S1 = -1.0   X = 1.0   S2 = 4.0   POINT EFFICIENCY = .58778
ALPHA1 = 21.79   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .58199
ALPHA1 = 21.83   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .55622
ALPHA1 = 21.87   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .55474
ALPHA1 = 21.90   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .53903
ALPHA1 = 21.94   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .53336
ALPHA1 = 21.98   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .47806
ALPHA1 = 22.02   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .44598
ALPHA1 = 22.05   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .44874
ALPHA1 = 22.09   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .44254
ALPHA1 = 22.13   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .49791
ALPHA1 = 22.16   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .55238
ALPHA1 = 22.20   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .55765
ALPHA1 = 22.23   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .51242
ALPHA1 = 22.27   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .51779
ALPHA1 = 22.31   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .52235
ALPHA1 = 22.34   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .52772
ALPHA1 = 22.38   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .53288
ALPHA1 = 22.42   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .53763
ALPHA1 = 22.45   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .54259
ALPHA1 = 22.49   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .54734
ALPHA1 = 22.53   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .57894
ALPHA1 = 22.57   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .56623
ALPHA1 = 22.60   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .55953
ALPHA1 = 22.64   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .55385
ALPHA1 = 22.68   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .54819
ALPHA1 = 22.71   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .54255
ALPHA1 = 22.75   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .53694
ALPHA1 = 22.79   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .53134
ALPHA1 = 22.82   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .52819
ALPHA1 = 22.86   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .51605
ALPHA1 = 22.90   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .48692
ALPHA1 = 22.93   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .46574
ALPHA1 = 22.97   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .47655
ALPHA1 = 23.01   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .47537
ALPHA1 = 23.05   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .44018
ALPHA1 = 23.08   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .44599
ALPHA1 = 23.12   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .44900
ALPHA1 = 23.16   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .44451
ALPHA1 = 23.20   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .49991
ALPHA1 = 23.23   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .55421
ALPHA1 = 23.27   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .54901
ALPHA1 = 23.30   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .51380
ALPHA1 = 23.34   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .52239
ALPHA1 = 23.41   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .52818
ALPHA1 = 23.45   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .52807
ALPHA1 = 23.52   S1 = -1.0   X = 1.0   S2 = -1.0   POINT EFFICIENCY = .55558

APPENDIX D

GEOMETRY OF GENERAL CLOCK GEAR TOOTH


The general ogival tooth of thickness t and outside radius $r_o$ consists of a circular arc of radius $\rho$ which blends tangentially into a radial tooth flank, as shown in Figure D-1 (only one center of curvature is indicated). The center of curvature, C, is located at a distance a from the center of the gear or pinion. Frequently this distance a equals the pitch radius $r_p$. The tooth geometry may either be described with the help of the parameters t, $\rho$ and a, or with the combination t, $\rho$ and $r_o$. Both approaches are shown below.


## 1. TOOTH GEOMETRY WITH HELP OF PARAMETERS t, $\rho$ AND a


$C_x$ and $C_y$ represent the coordinates of the center of curvature C. $C_x$ is defined by


$$C_x = \rho - \frac{t}{2} \tag{D-1}$$
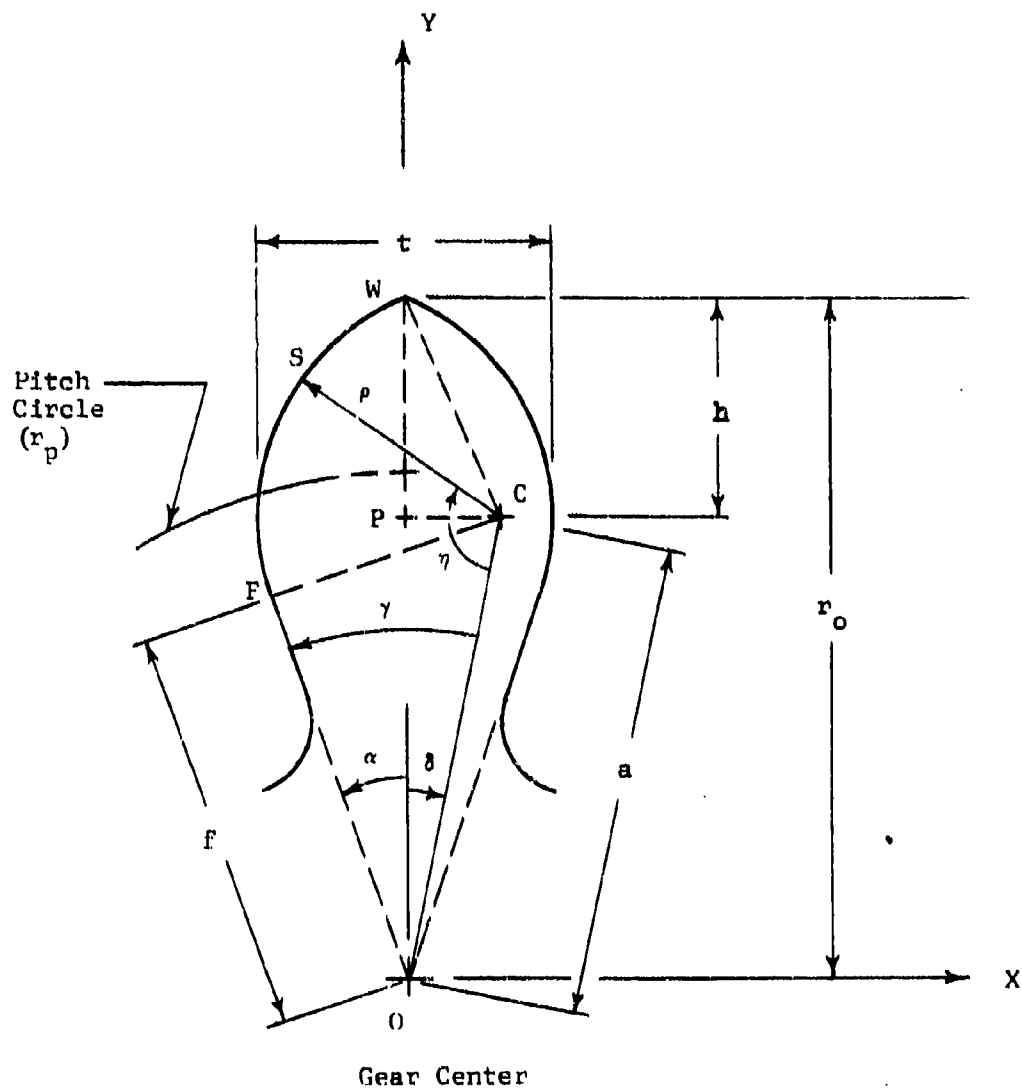
FIGURE D-1

GEOMETRY OF OGIVAL TOOTH (ONLY ONE CENTER OF CURVATURE SHOWN)

D-2

The angle $\delta$ is given by

$$\delta = \sin^{-1} \frac{C_x}{a} \tag{D-2}$$

With the above,

$$C_y = a \cos \delta \tag{D-3}$$

Further, the outside radius $r_o$ may be computed from

$$r_o = C_y + \sqrt{\rho^2 - C_x^2} \tag{D-4}$$

The angle $\gamma$ is obtained from

$$\gamma = \sin^{-1} \frac{\rho}{a} \quad , \tag{D-5}$$

and the flank angle $\alpha$ from

$$\alpha = \gamma - \delta \tag{D-6}$$

D-3

The distance, $f$, from the center of rotation, $O$, to the blend point, $F$, of the flank of the tooth, is given by

$$f = a \cos \gamma \qquad (D-7)$$

The angle $\eta$ defines the tooth contact point $S$ on the ogival, i.e., circular, portion of the tooth with the lines $OC$ and $CS$. The minimum and maximum angles $\eta_{min}$ and $\eta_{max}$ are, respectively,

$$\eta_{min} = \frac{\pi}{2} - \gamma \qquad (D-8)$$

and

$$\eta_{max} = \sin^{-1} \frac{r_0 \sin \delta}{\rho} \qquad \text{(This angle extends into the second quadrant)} \qquad (D-9)$$

## 2. TOOTH GEOMETRY FROM PARAMETERS $\rho$, t AND $r_o$

If the outside radius, $r_o$, is given, distance a must be computed. The length, $C_x$, is still given by Equation (D-1), while

$$a = \sqrt{(r_o - h)^2 + C_x^2} \qquad \text{(D-10)}$$

where, according to Figure D-1,

$$h = \sqrt{\rho^2 - C_x^2} \qquad \text{(D-11)}$$

All other quantities of interest remain as before, i.e.,

$$\delta = \sin^{-1} \frac{C_x}{a} \qquad\qquad \text{See Equation (D-2)}$$

$$\gamma = \sin^{-1} \frac{\rho}{a} \qquad\qquad \text{See Equation (D-5)}$$

$$\alpha = \gamma - \delta \qquad\qquad \text{See Equation (D-6)}$$

$$f = a \cos \gamma \qquad\qquad \text{See Equation (D-7)}$$

$$\eta_{min} = \frac{\pi}{2} - \gamma \qquad\qquad \text{See Equation (D-8)}$$

$$\eta_{max} = \sin^{-1} \frac{r_o \sin \delta}{\rho} \qquad\qquad \text{See Equation (D-9)}$$

D-5

KINEMATICS AND MOMENT INPUT-OUTPUT RELATIONSHIP FOR SINGLE

STEP-UP GEAR MESH WITH CLOCK TEETH

This appendix shows the kinematics of a single step-up gear
mesh with ogival teeth and derives the moment input-output relation-
ship. Both contact and pivot friction are included.

## 1. KINEMATICS OF AN OGIVE MESH

Figure E-1 indicates the condition of initial contact in ogival
meshes when the circular arc portion of the gear tooth drives the
circular arc portion of the pinion tooth.[1] This type of direct
contact will be called "round on round". As the motion progresses,
the circular arc of the gear tooth moves into contact with the
straight flank of the pinion tooth (Figure E-2). During this
"round on flat" phase, distance g at first decreases and then in-
creases again. Before round on round contact can again occur for a
given mesh, i.e., g has become equal to $f_p$ again, the subsequent
mesh comes into engagement with round on round contact. (See below.)

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[1]See section b-VII of this appendix for a check that indicates
whether the initial contact is round on round or whether possibly
the round portion of the pinion touches the flat portion of the gear.
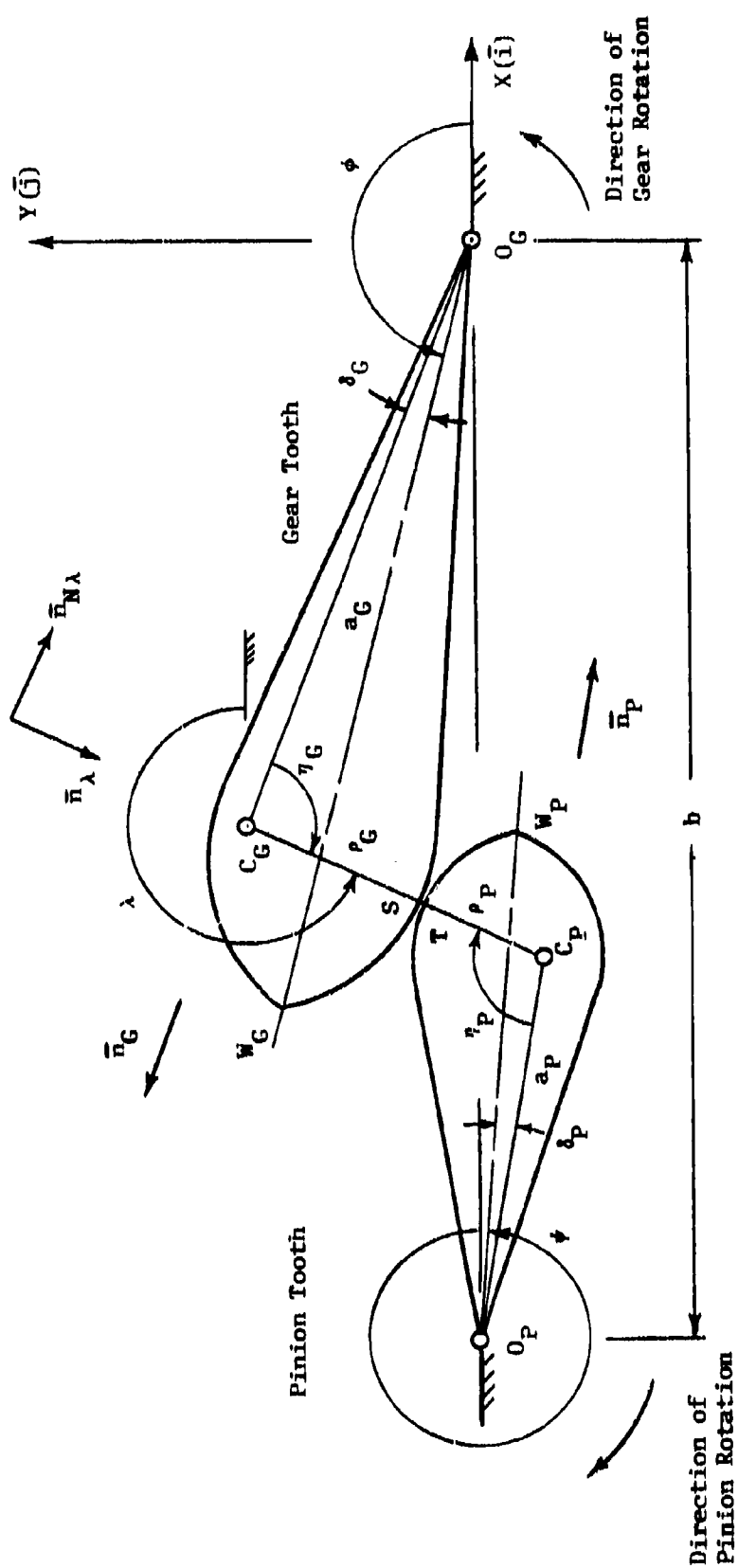
FIGURE E-1

ROUND ON ROUND PHASE OF CONTACT (GEAR DRIVES PINION)

The kinematics of both round on round and round on flat phases will be given first. Further it will be shown how to determine those gear angles,$\phi$, at which the regime changes from round on round to round on flat, and at which contact is taken over by the subsequent set of teeth. Since the ratio of angular velocities between gear and pinion varies with contact point, and the pinion moves faster for a given gear speed for round on round than for round on flat, the original set of teeth disengages as soon as the subsequent one makes contact. Thus, the contact ratio is unity for this type of gearing.

The gear tooth nomenclature given in Appendix D is used throughout. The additional subscripts G and P refer to gear and pinion,respectively.

### a. ROUND ON ROUND PHASE OF MOTION

Figure E-1 shows the input angle $\phi$ as the angle between the x-axis and the center line, $O_G W_G$, of the gear tooth. The output angle,$\psi$, is the angle between the x-axis and the centerline $O_P W_P$. During this phase of motion, the line connecting the centers of curvature $C_G$ and $C_P$ is of constant length $L = \rho_G + \rho_P$ and passes

through the instantaneous contact points S on the gear and T on the pinion. Because of this constant length one may obtain both the output angle $\psi$ and the "coupler" angle $\lambda$ from the equivalent four-bar linkage $O_G$-$C_C$-$C_P$-$O_P$, with ground pivot distance b.

## I.  UNIT VECTORS AND ANGLES $\eta_G$ AND $\eta_P$

The unit vectors associated with round on round phase are given now.

In direction $O_G$-$C_G$

$$\bar{n}_G = \cos(\phi - \delta_G)\bar{I} + \sin(\phi - \delta_G)\bar{J} \qquad (E\text{-}1)$$

In direction $C_G$-$C_P$

$$\bar{n}_\lambda = \cos\lambda\bar{I} + \sin\lambda\bar{J} \qquad (E\text{-}2)$$

The unit vector normal to $\bar{n}_\lambda$ (in the right hand sense) becomes

$$\bar{n}_{N\lambda} = -\sin\lambda\bar{I} + \cos\lambda\bar{J} \qquad (E\text{-}3)$$

In direction $O_p$-$C_p$,

$$\bar{n}_p = \cos(\psi - \delta_p)\bar{I} + \sin(\psi - \delta_p)\bar{J} \qquad (E\text{-}4)$$

Since $\lambda - (\phi - \delta_G) = \pi - \eta_G$

$$\eta_G = \pi + \phi - \lambda - \delta_G \qquad (E\text{-}5)$$

The angle $\eta_p$ is obtained from

$$\eta_p = (\psi - \delta_p) - \lambda \qquad \text{for } \psi - \delta_p > 90^{\circ} \qquad (E\text{-}6a)$$

and

$$\eta_p = 2\pi + (\psi - \delta_p) - \lambda \quad \text{for } \psi - \delta_p < 90^{\circ} \qquad (E\text{-}6b)$$

The above values for $\eta_G$ and $\eta_p$ are only good for the round on round phase of the motion. For the round on flat phase the angle $\eta_{GF}$, of the gear, may be of interest.(See Figure E-2). This angle is given by:

$$\eta_{GF} = (\phi - \delta_G) - (\psi + \alpha_p) + \frac{3\pi}{2} \qquad \text{for } \psi + \alpha_p > 90^{\circ} \qquad (E\text{-}6c)$$

[Pinion flank in 4th quadrant]

and

$$\eta_{GF} = (\phi - \delta_G) - (\psi + \alpha_p) - \frac{\pi}{2} \qquad \text{for } \psi + \alpha_p < 90^{\circ} \qquad (E\text{-}6d)$$

[Pinion flank in first quadrant]

E-5

## II.  DETERMINATION OF OUTPUT ANGLE $\psi$ AND "COUPLER" ANGLE $\lambda$

The loop equation of the equivalent four-bar linkage is given by

$$a_G\bar{n}_G + (\rho_G + \rho_P)\bar{n}_\lambda - a_P\bar{n}_P + b\bar{1} = 0 \qquad (E-7)$$

With appropriate substitutions for the unit vectors, according to Equations (E-1), (E-2), and (E-4), one obtains the following component equation

$$a_G\cos(\phi - \delta_G) + L\cos\lambda + b - a_P\cos(\psi - \delta_P) = 0 \qquad (E-8)$$

$$a_G\sin(\phi - \delta_G) + L\sin\lambda - a_P\sin(\psi - \delta_P) = 0 \qquad (E-9)$$

where

$$L = \rho_G + \rho_P \qquad (E-10)$$

To eliminate $\lambda$, let

$$\sin^2\lambda + \cos^2\lambda = 1 \qquad (E-11)$$

E-6

Substitution of expressions for $\sin\lambda$ and $\cos\lambda$ from Equations (E-9) and (E-8), respectively, leads to

$$A_R \sin\psi \;+\; B_R \cos\psi \;=\; C_R \qquad\qquad (E\text{-}12)$$

where

$$A_R \;=\; \sin(\phi - \delta_G) \;+\; \cos(\phi - \delta_G)\tan\delta_p \;+\; \frac{b}{a_G}\tan\delta_p$$

$$B_R \;=\; \cos(\phi - \delta_G) \;+\; \frac{b}{a_G} \;-\; \sin(\phi - \delta_G)\tan\delta_p$$

$$C_R \;=\; \frac{a_p^2 + a_G^2 + b^2 - L^2}{2a_p a_G \cos\delta_p} \;+\; \frac{b\,\cos(\phi - \delta_G)}{a_p \cos\delta_p}$$

Equation (E-12) is solved for $\psi$ by a method similar to the one given on pg. 296 of R. S. Hartenberg and J. Denavit; Kinematic Synthesis of Linkages, McGraw-Hill Book Co., New York, 1964. Thus,

$$\psi \;=\; 2\tan^{-1}\frac{A_R \;\pm\; \sqrt{A_R^2 \;+\; B_R^2 \;-\; C_R^2}}{B_R \;+\; C_R} \qquad\qquad (E\text{-}13)$$

The correct sign in Equation (E-13) must be found from the geometric conditions of the equivalent four-bar linkage.

E-7

The "coupler" angle $\lambda$ may now be determined either from Equation (E-8) or from Equation (E-9), i.e.,

$$\lambda = \cos^{-1}\left[\frac{a_P\cos(\psi - \delta_P) - a_G\cos(\phi - \delta_G) - b}{L}\right] \qquad \text{(E-14)}$$

or

$$\lambda = \sin^{-1}\left[\frac{a_P\sin(\psi - \delta_P) - a_G\sin(\phi - \delta_G)}{L}\right] \qquad \text{(E-15)}$$

III. DETERMINATION OF OUTPUT PINION ANGULAR VELOCITY $\dot{\psi}$

Differentiation of Equation (E-12) with respect to time leads to

$$\dot{\psi} = \dot{\phi}\left[\frac{A_{RD}\sin\psi + B_{RD}\cos\psi - C_{RD}}{A_R\cos\psi - B_R\sin\psi}\right] \qquad \text{(E-16)}$$

where $A_R$ and $B_R$ are given with Equation (E-12), and where

$$A_{RD} = \tan\delta_P\sin(\phi - \delta_G) - \cos(\phi - \delta_G)$$

$$B_{RD} = \sin(\phi - \delta_G) + \tan\delta_P\cos(\phi - \delta_G)$$

$$C_{RD} = \frac{b}{a_P\cos\delta_P}\sin(\phi - \delta_G)$$

E-8

## IV. RELATIVE VELOCITY AT CONTACT POINT BETWEEN GEAR AND PINION

With point S on the gear and point T on the pinion, as
shown by Figure E-1, the relative velocity between gear and pinion
at the contact point is given by

$$\overline{V}_{S/T} = \overline{V}_{S/O_G} - \overline{V}_{T/O_P} \tag{E-17}$$

This relative velocity is tangent to the contacting surfaces and
can therefore be expressed in terms of the unit vector $\overline{n}_{N\lambda}$
(see Figure E-1). Then, the above becomes

$$\overline{V}_{S/T} = \left[ (\overline{V}_{S/O_G} \cdot \overline{n}_{N\lambda}) - (\overline{V}_{T/O_P} \cdot \overline{n}_{N\lambda}) \right] \overline{n}_{N\lambda} \tag{E-18}$$

or

$$\overline{V}_{S/T} = \left\{ \left[ \dot{\phi}\overline{k} \times (a_G \overline{n}_G + \rho_G \overline{n}_\lambda) \right] \cdot \overline{n}_{N\lambda} - \left[ \dot{\psi}\overline{k} \times (a_P \overline{n}_P - \rho_P \overline{n}_\lambda) \right] \cdot \overline{n}_{N\lambda} \right\} \overline{n}_{N\lambda} \tag{E-19}$$

Appropriate substitution of unit vectors and simplification
results in

$$\overline{V}_{S/T} = \left\{ \dot{\phi} \left[ a_G \cos(\phi - \delta_G - \lambda) + \rho_G \right] - \dot{\psi} \left[ a_P \cos(\psi - \delta_P - \lambda) - \rho_P \right] \right\} \overline{n}_{N\lambda} \tag{E-20}$$

## b. ROUND ON FLAT PHASE OF MOTION

Figure E-2 gives the details of the round on flat contact between the driving gear and the driven pinion. The input angle $\phi$ and the output angle $\psi$ are again defined counter-clockwise between the x-axis and the respective tooth center lines $O_G W_G$ and $O_P W_P$. Since contact is always made on the straight radial flank of the pinion, the line $SC_G$ of the gear is always normal to the flank of the pinion. The contact point is at a distance g from the pinion center, $O_P$, and this distance is always smaller than, or equal to the distance $f_P$ (which is defined by Equation (D-7) in Appendix D). Again, the subscripts G and P are used for gear and pinion tooth parameters, respectively.

## I. UNIT VECTORS

As before, the unit vector in direction $O_G C_G$ is given by

$$\bar{n}_G = \cos(\phi - \delta_G)\bar{I} + \sin(\phi - \delta_G)\bar{J} \tag{E-21}$$

The unit vector in direction $O_P T$, along the flank of the pinion is given by

FIGURE E-2

ROUND ON FLAT PHASE OF CONTACT (GEAR DRIVES PINION)

E-11

$$\bar{n}_F = \cos(\psi + \alpha_p)\bar{i} + \sin(\psi + \alpha_p)\bar{j} \qquad (E\text{-}22)$$

The unit vector in direction $SC_G$ is normal to $\bar{n}_F$ in the right hand sense

$$\bar{n}_{NF} = -\sin(\psi + \alpha_p)\bar{i} + \cos(\psi + \alpha_p)\bar{j} \qquad (E\text{-}23)$$

## II. DETERMINATION OF OUTPUT ANGLE $\psi$ AND DISTANCE $g$

The vector equation for the mechanism loop $O_G\text{-}C_G\text{-}S\text{-}T\text{-}O_p$, which forms the basis of the desired solution, has the following form:

$$a_G\bar{n}_G - \rho_G\bar{n}_{NF} - g\bar{n}_F + b\bar{i} = 0 \qquad (E\text{-}24)$$

Substitution of Equations (E-21) to (E-23) furnishes the component equations

$$a_G\cos(\phi - \delta_G) + \rho_G\sin(\psi + \alpha_p) - g\cos(\psi + \alpha_p) + b = 0$$
$$(E25)$$

$$a_G\sin(\phi - \delta_G) - \rho_G\cos(\psi + \alpha_p) - g\sin(\psi + \alpha_p) = 0$$
$$(E26)$$

From Equation (E-26) one obtains for g

$$g = \frac{a_G \sin(\phi - \delta_G) - \rho_G \cos(\psi + \alpha_P)}{\sin(\psi + \alpha_P)} \qquad (E\text{-}27)$$

This expression for g is now substituted into Equation (E-25). Rearrangement and simplification lead to

$$A_F \sin\psi + B_F \cos\psi = C_F \qquad (E\text{-}28)$$

where

$$A_F = b \cos\alpha_P + a_G \cos(\phi - \delta_G - \alpha_P)$$

$$B_F = b \sin\alpha_P - a_G \sin(\phi - \delta_G - \alpha_P)$$

$$C_F = -\rho_G$$

The solution of Equation (E-28) for $\psi$ is obtained in the same way as that of Equation (E-13), i.e.,

$$\psi = 2\tan^{-1} \frac{A_F \pm \sqrt{A_F^2 + B_F^2 - C_F^2}}{B_F + C_F} \qquad (E\text{-}29)$$

The correct sign in Equation (E-29) depends on the geometric conditions of the mechanism position as in all four-bar linkage solutions of this type.

E-13

III.  DETERMINATION OF PINION ANGULAR VELOCITY $\dot{\psi}$

Differentiation of Equation (E-28) with respect to time gives

$$\dot{\psi} = \dot{\phi}\left[\frac{A_{FD}\sin\psi + B_{FD}\cos\psi}{A_F\cos\psi - B_F\sin\psi}\right] \qquad (E-30)$$

where $A_F$ and $B_F$ are given with Equation (E-28) and where

$$A_{FD} = a_G\sin(\phi - \delta_G - \alpha_P)$$

$$B_{FD} = a_G\cos(\phi - \delta_G - \alpha_P)$$

IV.  RELATIVE VELOCITY AT CONTACT POINT BETWEEN GEAR AND PINION

As for round on round contact, the relative velocity between point S on the gear tooth and point T on the pinion tooth is tangent to the contacting surfaces.  In this case it will have the direction of unit vector $\overline{n}_F$ (see Figure E-2). Then,

$$\bar{V}_{S/T} = \bar{V}_{S/O_G} - \bar{V}_{T/O_P} = \left[ (\bar{V}_{S/O_G} \cdot \bar{n}_F) - (\bar{V}_{T/O_P} \cdot \bar{n}_F) \right] \bar{n}_F$$

<div align="right">(E-31)</div>

Since, because of the radial flank of the pinion, the velocity of the contact point T is normal to unit vector $\bar{n}_F$.

$$\bar{V}_{T/O_P} \cdot \bar{n}_F = 0 \qquad\qquad\text{(E-32)}$$

Therefore,

$$\bar{V}_{S/T} = \left[ \bar{V}_{S/O_G} \cdot \bar{n}_F \right] \bar{n}_F \qquad\qquad\text{(E-33)}$$

or

$$\bar{V}_{S/T} = \left\{ \left[ \dot{\phi}\bar{k} \times (a_G \bar{n}_P - \rho_G \bar{n}_{NF}) \right] \cdot \bar{n}_F \right\} \bar{n}_F \qquad\qquad\text{(E-34)}$$

Appropriate substitution of unit vectors gives

$$\bar{V}_{S/T} = \dot{\phi} \left[ \rho_G + a_G \sin(\psi - \phi + \alpha_P + \delta_G) \right] \bar{n}_F \qquad\qquad\text{(E-35)}$$

<div align="center">E-15</div>

## V. DETERMINATION OF TRANSITION ANGLES FROM ROUND ON ROUND TO ROUND ON FLAT MOTION

The transition angles $\phi_T$ and $\psi_T$, which occur when the round on flat phase follows the round on round one, may be determined with the help of the modified component equations (E-25) and (E-26), i.e., one lets $\phi = \phi_T$, $\psi = \psi_T$ and $g = f_P$. This results in

$$a_G \cos(\phi_T - \delta_G) + \rho_G \sin(\psi_T + \alpha_P) - f_P \cos(\psi_T + \alpha_P) + b = 0 \tag{E-36}$$

and

$$a_G \sin(\phi_T - \delta_G) - \rho_G \cos(\psi_T + \alpha_P) - f_P \sin(\psi_T + \alpha_P) = 0 \tag{E-37}$$

From the above,

$$\cos(\phi_T - \delta_G) = \frac{1}{a_G} \left[ f_P \cos(\psi_T + \alpha_P) - b - \rho_G \sin(\psi_T + \alpha_P) \right] \tag{E-38}$$

and

$$\sin(\phi_T - \delta_G) = \frac{1}{a_G} \left[ f_P \sin(\psi_T + \alpha_P) + \rho_G \cos(\psi_T + \alpha_P) \right] \tag{E-39}$$

The angle $\psi_T$ may now be obtained by first eliminating $\phi_T$ from Equations (E-38) and (E-39). This is accomplished with

$$\sin^2(\phi_T - \delta_G) + \cos^2(\phi_T - \delta_G) = 1 \qquad (E-40)$$

Substitution into the above leads to the following expression in $\psi_T$:

$$A_T \sin\psi_T + B_T \cos\psi_T = C_T \qquad (E-41)$$

where

$$A_T = \rho_G \cos\alpha_P + f_P \sin\alpha_P$$

$$B_T = \rho_G \sin\alpha_P - f_P \cos\alpha_P$$

$$C_T = \frac{a_G^2 - f_P^2 - b^2 - \rho_G^2}{2b}$$

Equation (E-41) is again solved in the manner of Equation (E-13)

$$\psi_T = 2 \tan^{-1} \frac{A_T \pm \sqrt{A_T^2 + B_T^2 - C_T^2}}{B_T + C_T} \qquad (E-42)$$

The correct sign must again be determined from the geometric
conditions.

The associated transition angle $\phi_T$ can now be obtained
either with the help of Equation (E-38) or Equation (E-39):

$$\phi_T = \cos^{-1}\left[\frac{f_P\cos(\psi_T + \alpha_P) - \rho_G\sin(\psi_T + \alpha_P) - b}{a_G}\right] + \delta_G$$

(E-43)

or

$$\phi_T = \sin^{-1}\left[\frac{f_P\sin(\psi_T + \alpha_P) + \rho_G\cos(\psi_T + \alpha_P)}{a_G}\right] + \delta_G$$

(E-44)

# VI. SENSING GEOMETRY FOR THE DETERMINATION OF CONTACT OF SUBSEQUENT TOOTH MESH

The following derives a computer sensing equation which indicates when contact is transferred from one tooth mesh to the succeeding one. Figure E-3 illustrates the case. The active mesh is in the round on flat mode and the subsequent mesh will make its initial contact in the round on round mode. This
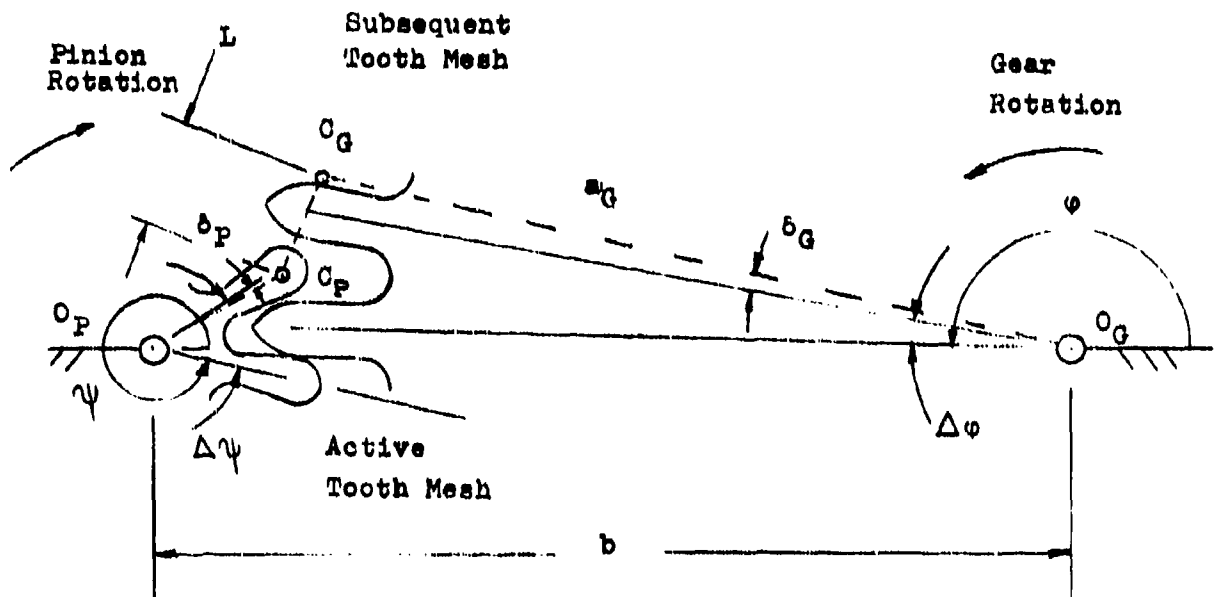


Figure E3
Sensing Geometry for Contact of
Subsequent Tooth Mesh

E-19

assertion is based on experience with the three gear and pinion
combinations of the M125A1 booster. In each of these instances,
the subsequent mesh makes contact before the round of the gear
has left the flat of the pinion in the active mesh, i.e., $g < f_p$.
(See work in section V.) It has also been found that initial
contact of the subsequent mesh is always in the round on round
mode. Generally, contact between the flat of the gear and the
round of the pinion does not occur, and it has not been considered
in this report. Section VII gives a criterion for the existence
of this inverted round on flat mode of contact.

Once contact has been made by the subsequent mesh it becomes
the new active mesh. This can be proven by the fact that, for
a given angular velocity $\dot{\phi}$ of the gear, the angular velocity
of the pinion is always larger in the initial stages of the
round on round mode than in the final stages of the round on
flat one. Thus, once the new mesh has made contact, the old one
separates rapidly and the "contact ratio" is always unity.

The above may be shown
theoretically by the position of the instant center of rotation
between the gear and the pinion on line $O_G O_P$.

If $\Delta\phi$ and $\Delta\psi$ represent the angles between the individual tooth
center lines of the gear and pinion, respectively, (see Figure E-3), the

E-20

closure equation for the subequent mesh may be written in terms
of the active mesh as follows:

$$a_p \left[ \cos(\psi - \delta_p + \Delta\psi)\bar{I} + \sin(\psi - \delta_p + \Delta\psi)\bar{J} \right] + L_x\bar{I} + L_y\bar{J}$$

$$= b\bar{I} + a_G \left[ \cos(\phi - \delta_G - \Delta\phi)\bar{I} + \sin(\phi - \delta_G - \Delta\phi)\bar{J} \right]$$

$$(E\text{-}45)$$

where    $L_x$, $L_y$ = components of the distance $L = C_pC_G$

   $\psi$ = pinion angle determined from round on flat
   mode according to Equation (E-29)

The components $L_x$ and $L_y$ may be obtained from Equation (E-45):

$$L_x = b + a_G\cos(\phi - \delta_G - \Delta\phi) - a_p\cos(\psi - \delta_p + \Delta\psi)$$

$$(E\text{-}46)$$

and

$$L_y = a_G\sin(\phi - \delta_G - \Delta\phi) - a_p\sin(\psi - \delta_p + \Delta\psi) \qquad (E\text{-}47)$$

Contact will have occurred when the distance L becomes equal to,
or slightly smaller than, the sum of the two radii of curvature
$\rho_G$ and $\rho_P$. Thus, the criterion of contact becomes

$$\sqrt{L_x^2 + L_y^2} \leq \rho_G + \rho_P \qquad \text{(E-48)}$$

## VII.  POSSIBILITY OF FLAT OF GEAR MAKING CONTACT WITH THE ROUND PORTION OF THE PINION

Figure E-4 shows a transition configuration in which the flat part of the gear tooth makes contact with the circular arc of the pinion, i.e., $\overline{O_G S} = f_G$.  The radius of curvature $\rho_P$ of of the pinion will only then be normal to the flat of the gear
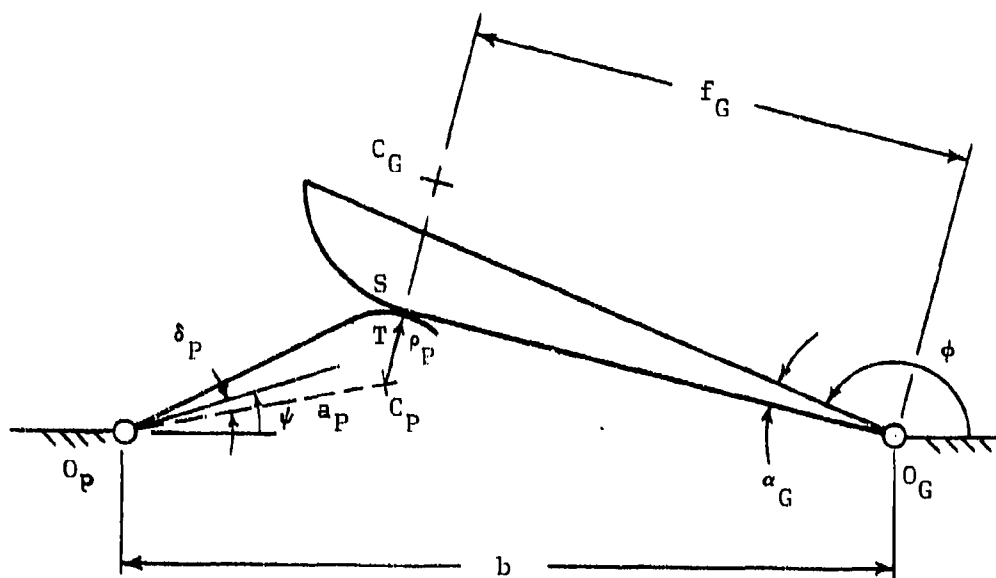


FIGURE E-4

FLAT OF GEAR CONTACTS ROUND OF PINION

E-23

when $\overline{O_G S} \leq f_G$. Accordingly, in order to avoid this type of contact, the following criterion must be met:

$$b \; > \; \left[ a_P \; + \; \sqrt{f_G^2 \; + \; \rho_P^2} \; \right] \qquad\qquad (E-49)$$

The three gear meshes of the M125A1 booster always satisfy equation (E-49), and therefore the initial contact is made in the round on round mode.

## 2. MOMENT INPUT-OUTPUT RELATIONSHIP FOR SINGLE STEP-UP GEAR MESH WITH OGIVAL TEETH

The present section concerns itself with the determination of the equilibrant moment $M_o$ acting on the output pinion of a single ogival mesh, when the input moment $M_{in}$, which acts on the gear, is given. This relationship must be derived both for the round on round and for the round on flat phases of the motion. The directions of the pivot friction forces are again chosen such that the resulting moments oppose the motion of the respective component. (See Appendix A.) The magnitude of these friction moments is expressed in the manner of equation (A-2) of the aforementioned appendix. The direction of the friction force of the gear on the pinion is the same as the direction of the relative velocity $V_{S/T}$, of the gear contact point S with respect to the pinion contact point T. This will be expressed by the appropriate signum convention. It must also be remembered that the kinematic expressions must conform to the motion phase under consideration.

## a. INPUT-OUTPUT RELATIONSHIP FOR THE ROUND ON ROUND PHASE OF MOTION

Figure E-5 shows the free body diagrams for the round on round phase of motion with the gear considered to be component no. 1 and the pinion defined as component no. 2. The contact forces $F_{12}$ and $F_{21}$ are expressed with the help of the unit vector $\bar{n}_\lambda$ [see equation (E-2)]. This unit vector is always normal to both contacting surfaces at points S and T. The unit vector $\bar{n}_{N\lambda}$ is used to describe the direction of the friction forces at the contact point. The sense of these friction forces is determined with the help of

$$s = \frac{V_{S/T}}{\left| V_{S/T} \right|} \qquad (E-50)$$

(See equation (E-20) for $\bar{V}_{S/T}$.) Since the friction force $F_{f12}$ of the gear on the pinion has the same direction as the relative velocity $V_{S/T}$, one may write

$$\bar{F}_{f12} = \mu s F_{12} \bar{n}_{N\lambda} \qquad (E-51)$$

where $\mu$ represents the coefficient of friction at the contact point.

E-26
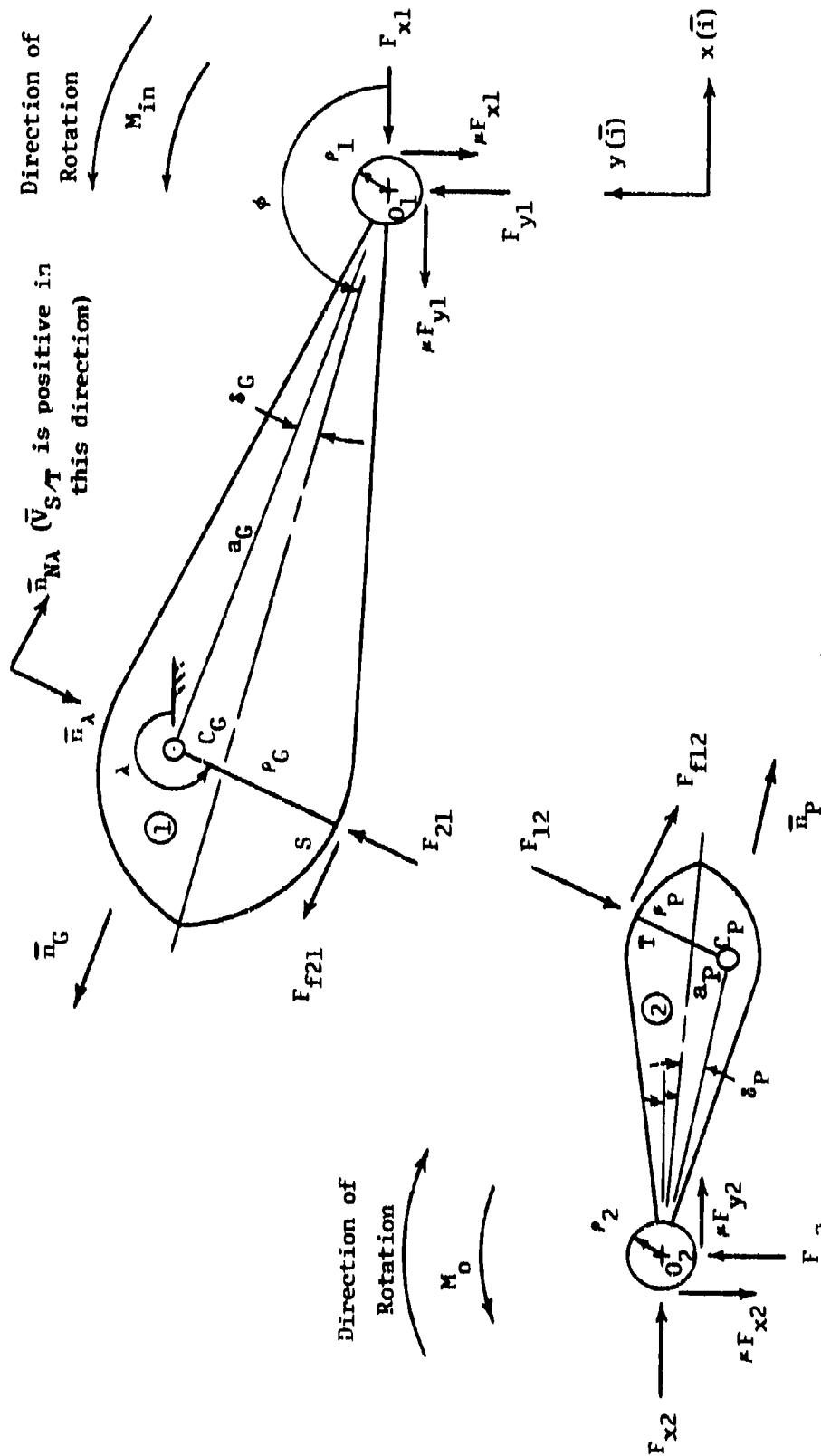
FIGURE E-5

FREE BODY DIAGRAM FOR ROUND ON ROUND PHASE

Since

$$\bar{F}_{21} = - F_{12}\bar{n}_\lambda \qquad\qquad (E\text{-}52)$$

the friction force of the pinion on the gear becomes

$$\bar{F}_{f21} = - \mu s F_{12}\bar{n}_{N\lambda} \qquad\qquad (E\text{-}53)$$

## I.  FORCE AND MOMENT EQUILIBRIUM OF THE GEAR

The force equilibrium of the gear is assured when

$$-F_{12}\bar{n}_\lambda - \mu s F_{12}\bar{n}_{N\lambda} - F_{x1}\bar{i} + F_{y1}\bar{j} - \mu F_{y1}\bar{i} - \mu F_{x1}\bar{j} = 0$$
$$\qquad\qquad (E\text{-}54)$$

In the above $\mu$ also stands for the pivot coefficient of friction.

Moment equilibrium is given by

$$M_{1n}\bar{k} - \mu \rho_1 \sqrt{F_{x1}^2 + F_{y1}^2}\,\bar{k} + \left[a_G\bar{n}_G + \rho_G\bar{n}_\lambda\right] \times \left[-F_{12}\bar{n}_\lambda - \mu s F_{12}\bar{n}_{N\lambda}\right]$$
$$= 0 \qquad\qquad (E\text{-}55)$$

The unit vector $\bar{n}_G$ is defined by equation (E-1).

The following component equations may be obtained from equation (E-54):

$$-F_{12}\cos\lambda \;+\; \mu s F_{12}\sin\lambda \;-\; F_{x1} \;-\; \mu F_{y1} \;=\; 0 \qquad (E-56)$$

and

$$-F_{12}\sin\lambda \;-\; \mu s F_{12}\cos\lambda \;+\; F_{y1} \;-\; \mu F_{x1} \;=\; 0 \qquad (E-57)$$

After substitution and cross-multiplication, one obtains the moment expression, equation (E-55), in scalar form:

$$M_{1n} \;-\; \mu\rho_1\sqrt{F_{x1}^2 + F_{y1}^2} \;+\; F_{12}\Big[-\mu s\rho_G + a_G\sin(\phi - \delta_G - \lambda)$$
$$-\; \mu s a_G\cos(\phi - \delta_G - \lambda)\Big] \;=\; 0 \qquad (E-58)$$

Simultaneous solution of equations (E-56) and (E-57) for $F_{x1}$ and $F_{y1}$ results in

$$F_{x1} \;=\; -F_{12}\left[\frac{(1 + \mu^2 s)\cos\lambda \;+\; \mu(1 - s)\sin\lambda}{1 + \mu^2}\right] \qquad (E-59)$$

and

$$F_{y1} \;=\; F_{12}\left[\frac{(1 + \mu^2 s)\sin\lambda \;-\; \mu(1 - s)\cos\lambda}{1 + \mu^2}\right] \qquad (E-60)$$

E-29

These results are now substituted in equation (E-58). Since $s^2$ is unity and always positive, the resulting expression for $F_{12}$ has the following form:

$$F_{12} = \frac{M_{in}}{\mu(\rho_1 + s\rho_G) + a_G\left[\mu s\cos(\phi - \delta_G - \lambda) - \sin(\phi - \delta_G - \lambda)\right]}$$

(E-61)

## II. FORCE AND MOMENT EQUILIBRIUM OF THE PINION

Force equilibrium of the pinion is assured by

$$F_{12}\bar{n}_\lambda + \mu s F_{12}\bar{n}_{N\lambda} + F_{x2}\bar{i} + F_{y2}\bar{j} + \mu F_{y2}\bar{i} - \mu F_{x2}\bar{j} = 0$$

(E-62)

Moment equilibrium must satisfy

$$M_o\bar{k} + \mu\rho_2\sqrt{F_{x2}^2 + F_{y2}^2}\,\bar{k} + \left[a_P\bar{n}_P - \rho_P\bar{n}_\lambda\right] \times \left[F_{12}\bar{n}_\lambda + \mu s F_{12}\bar{n}_{N\lambda}\right]$$

$$= 0$$

(E-63)

E-30

The unit vector $\bar{n}_p$ is defined by equation (E-4).

Equation (E-62) gives rise to the following component equations:

$$F_{12}\cos\lambda - \mu s F_{12}\sin\lambda + F_{x2} + \mu F_{y2} = 0 \qquad \text{(E-64)}$$

and

$$F_{12}\sin\lambda + \mu s F_{12}\cos\lambda + F_{y2} - \mu F_{x2} = 0 \qquad \text{(E-65)}$$

The moment equation (E-63) becomes, in scalar form,

$$M_o + \mu P_2 \sqrt{F_{x2}^2 + F_{y2}^2} + F_{12}\Big[-\mu s \rho_p - a_p\sin(\psi - \delta_p - \lambda)$$
$$+ \mu s a_p \cos(\psi - \delta_p - \lambda)\Big] = 0$$
$$\text{(E-66)}$$

Simultaneous solution of equations (E-64) and (E-65) for $F_{x2}$ and $F_{y2}$ results in

$$F_{x2} = F_{12}\left[\frac{\mu(1 + s)\sin\lambda - (1 - \mu^2 s)\cos\lambda}{1 + \mu^2}\right] \qquad \text{(E-67)}$$

and

$$F_{y2} = -F_{12}\left[\frac{\mu(1 + s)\cos\lambda + (1 + \mu^2 s)\sin\lambda}{1 + \mu^2}\right] \qquad \text{(E-68)}$$

The above expressions are now substituted into equation (E-66).
Again, $s^2$ is unity and always positive, and the following
expression may be obtained for the contact force $F_{12}$ in terms
of the equilibrant moment $M_o$

$$F_{12} = \frac{M_o}{\mu(s\rho_P - \rho_2) - a_P[\mu s \cos(\psi - \delta_P - \lambda) - \sin(\psi - \delta_P - \lambda)]}$$

(E-69)

III.  MOMENT INPUT-OUTPUT RELATIONSHIP

When equations (E-61) and (E-69) are set equal to each
other, one obtains the following input-output relationship:

$$M_o = M_{in}\left[\frac{\mu(s_R\rho_P - \rho_2) - a_P[\mu s_R\cos(\psi - \delta_P - \lambda) - \sin(\psi - \delta_P - \lambda)]}{\mu(\rho_1 + s_R\rho_G) + a_G[\mu s_R\cos(\phi - \delta_G - \lambda) - \sin(\phi - \delta_G - \lambda)]}\right]$$

(E-70)

To account for the fact that equ.(E-70) is only valid for the
round on round phase of motion,the signum symbol has now been
changed to $s_R$.

E-32

b. __INPUT-OUTPUT RELATIONSHIP FOR ROUND ON FLAT PHASE OF MOTION__

Figure E-6 gives the free body diagrams for the round on flat phase of the motion. Again, the gear is considered to be component no. 1, while the pinion is component no. 2. Using the unit vector $\bar{n}_{NF}$, of equation (E-22), which is normal to the flat side of the pinion to express the force $\bar{F}_{12}$ of the gear on the pinion, one obtains

$$\bar{F}_{12} = -F_{12}\bar{n}_{NF} \tag{E-71}$$

The friction force of the gear on the pinion again has the same direction as the now applicable relative velocity $\bar{V}_{S/T}$ of equation (E-35). With

$$s = \frac{V_{S/T}}{|V_{S/T}|} \tag{E-72}$$

as the applicable signum convention, the friction force $\bar{F}_{f12}$ becomes

$$\bar{F}_{f12} = \mu s F_{12}\bar{n}_{F} \tag{E-73}$$

E-33
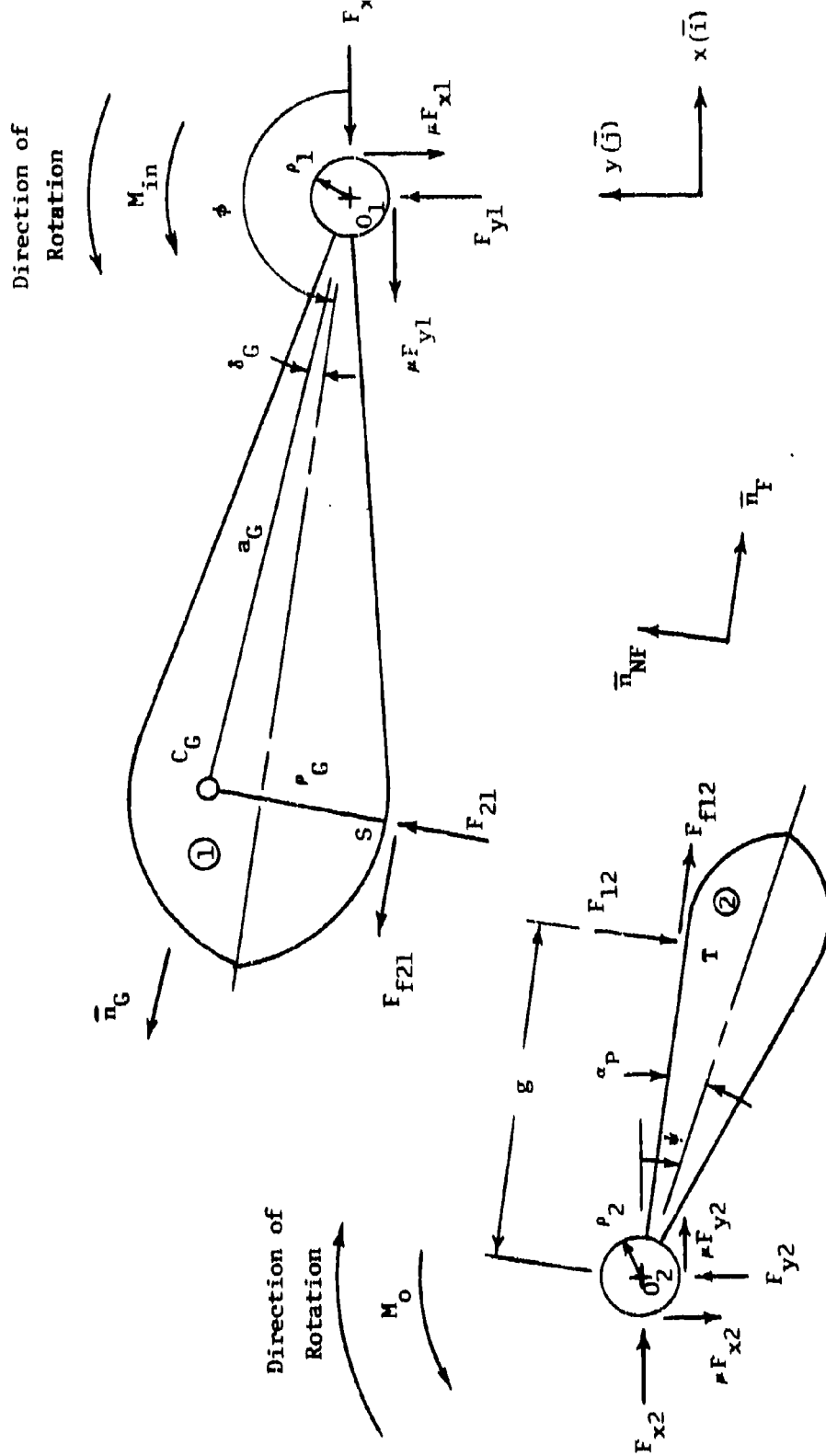
FIGURE E-6

FREE BODY DIAGRAM FOR ROUND ON FLAT PHASE

The contact force $\bar{F}_{21}$ of the pinion on the gear and the associated friction force are equal and opposite to the forces given by equations (E-71) and (E-73) respectively. Thus,

$$\bar{F}_{21} = F_{12}\bar{n}_{NF} \qquad (E-74)$$

and

$$\bar{F}_{f21} = -\mu s F_{12}\bar{n}_F \qquad (E-75)$$

Note that the kinematics of the round on flat phase must now be used.


## I. FORCE AND MOMENT EQUILIBRIUM OF THE GEAR

Force equilibrium of the gear is given by

$$F_{12}\bar{n}_{NF} - \mu s F_{12}\bar{n}_F - F_{x1}\bar{i} + F_{y1}\bar{j} - \mu F_{y1}\bar{i} - \mu F_{x1}\bar{j} = 0$$
$$\qquad (E-76)$$

Moment equilibrium requires

$$M_{in}\bar{k} - \mu\rho_1\sqrt{F_{x1}^2 + F_{y1}^2}\,\bar{k} + \left[a_G\bar{n}_G - \rho_G\bar{n}_{NF}\right] \times \left[F_{12}\bar{n}_{NF} - \mu s F_{12}\bar{n}_F\right]$$
$$= 0 \qquad (E-77)$$

E-35

Equation (E-76) furnishes the following component expressions

$$-F_{12}\sin(\psi + \alpha_p) - \mu s F_{12}\cos(\psi + \alpha_p) - F_{x1} - \mu F_{y1} = 0$$

(E-78)

and

$$F_{12}\cos(\psi + \alpha_p) - \mu s F_{12}\sin(\psi + \alpha_p) + F_{y1} - \mu F_{x1} = 0$$

(E-79)

The scalar form of equation (E-77) is given by

$$M_{in} - \mu \rho_1 \sqrt{F_{x1}^2 + F_{y1}^2} + F_{12}\left[-\mu s \rho_G + a_G \cos(\phi - \delta_G - \psi - \alpha_p)\right.$$
$$\left. + \mu s a_G \sin(\phi - \delta_G - \psi - \alpha_p)\right] = 0$$

(E-80)

Simultaneous solution of equations (E-78) and (E-79) for $F_{x1}$ and $F_{y1}$ furnishes

$$F_{x1} = F_{12}\left[\frac{\mu(1 - s)\cos(\psi + \alpha_p) - (1 + \mu^2 s)\sin(\psi + \alpha_p)}{1 + \mu^2}\right]$$

(E-81)

and

$$F_{y1} = -F_{12}\left[\frac{\mu(1 - s)\sin(\psi + \alpha_p) + (1 + \mu^2 s)\cos(\psi + \alpha_p)}{1 + \mu^2}\right]$$

(E-82)

E-36

The above results are now substituted into equation (E-80). Since $s^2$ is again unity and positive at all times, the resulting expression for $F_{12}$ becomes, in terms of $M_{1n}$,

$$F_{12} = \frac{M_{1n}}{\mu(\rho_1 + s\rho_G) - a_G\left[\cos(\phi - \delta_G - \psi - \alpha_P) + \mu s\sin(\phi - \delta_G - \psi - \alpha_P)\right]}$$

(E-83)

## II . <u>FORCE AND MOMENT EQUILIBRIUM OF THE PINION</u>

Force equilibrium of the pinion is expressed by

$$-F_{12}\bar{n}_{NF} + \mu s F_{12}\bar{n}_F + F_{x2}\bar{I} + F_{y2}\bar{J} + \mu F_{y2}\bar{I} - \mu F_{x2}\bar{J} = 0$$

(E-84)

Moment equilibrium requires that

$$M_o\bar{k} + \mu\rho_2\sqrt{F_{x2}^2 + F_{y2}^2}\,\bar{k} + g\bar{n}_F \times (-F_{12}\bar{n}_{NF}) = 0 \qquad \text{(E-85)}$$

E-37

Equation (E-84) furnishes the following component equations

$$F_{12}\sin(\psi + \alpha_p) + \mu s F_{12}\cos(\psi + \alpha_p) + F_{x2} + \mu F_{y2} = 0$$

and
$$(E-86)$$

$$-F_{12}\cos(\psi + \alpha_p) + \mu s F_{12}\sin(\psi + \alpha_p) + F_{y2} - \mu F_{x2} = 0$$

$$(E-87)$$

The scalar form of the moment equation (E-85) becomes

$$M_o + \mu \rho_2 \sqrt{F_{x2}^2 + F_{y2}^2} - s F_{12} = 0 \qquad (E-88)$$

Simultaneous solution of equations (E-86) and (E-87) for $F_{x2}$ and $F_{y2}$ leads to

$$F_{x2} = -F_{12}\left[\frac{(1 - \mu^2 s)\sin(\psi + \alpha_p) + \mu(1 + s)\cos(\psi + \alpha_p)}{1 + \mu^2}\right]$$

$$(E-89)$$

and

$$F_{y2} = F_{12}\left[\frac{(1 - \mu^2 s)\cos(\psi + \alpha_p) - \mu(1 + s)\sin(\psi + \alpha_p)}{1 + \mu^2}\right]$$

$$(E-90)$$

E-38

The above expressions are now substituted into equation (E-88).
Again, $s^2$ is unity and positive. The following expression for
$F_{12}$ is now obtained

$$F_{12} = \frac{M_o}{\mathcal{C} - \mu P_2} \tag{E-91}$$

III. MOMENT INPUT-OUTPUT RELATIONSHIP

When equations (E-83) and (E-91) are set equal to each
other, one obtains the following input-output relationship:

$$M_o = M_{in}\left[\frac{\mathcal{C} - \mu P_2}{\mu(P_1 + s_F P_G) - a_G[\cos(\phi - \delta_G - \psi - \alpha_P) + \mu s_F \sin(\phi - \delta_G - \psi - \alpha_P)]}\right] \tag{E-92}$$

Note that the signum symbol has been changed to $s_F$ in the above
expression to account for the fact that the expression is only
valid in the round on flat phase of motion.

E-39

# APPENDIX F
## COMPUTER MODELS FOR A SINGLE
## STEP-UP GEAR MESH WITH CLOCK TEETH

The present appendix contains descriptions, listings and sample outputs of the following computer models relating to single step-up gear meshes containing clock (ogival) teeth

1.  Program CLOCK 1:  Kinematics of single pass step-up gear mesh with clock (ogival) teeth
2.  Program CLOCK 2:  Point and cycle efficiencies for single pass step-up gear mesh with clock (ogival) teeth

The relevant background, the input parameters, the manner of the computations, and the form of the output of each program are discussed in detail.  The program proper forms the last part of each section.

1. Program CLOCK 1: Kinematics of Single Pass Step-Up Gear
                    Mesh With Clock (Ogival) Teeth

The program CLOCK 1 is based upon that portion of Appendix E
which deals with the kinematics of single mesh step-up clock
gearing. It may be used to check on the geometric performance
of clock type meshes.

The nomenclature of the program is chosen to
coincide as much as possible with that of Appendix D as well
as that of Appendix E.


   a. Input Parameters (see Program CLOCK 1, below)

The following parameters represent the input data for the
program:


CAPRP = $R_P$, the pitch radius of the gear

RP = $r_P$, the pitch radius of the pinion

AG = $a_G$, the distance from the center of rotation of the gear
            to the center of curvature of the circular arc portion
            of the gear tooth

AP = $a_P$, the distance from the center of rotation of the pinion
            to the center of curvature of the circular arc portion
            of the pinion tooth

RHOG = $\rho_G$, the radius of curvature of the circular arc portion
            of the gear tooth

RHOP = $\rho_P$, the radius of curvature of the circular arc portion
            of the pinion tooth

TG = $t_G$, the maximum thickness of the gear tooth

TP = $t_P$, the maximum thickness of the pinion tooth

NG = $n_G$, the number of teeth of the gear

NP = $n_P$, the number of teeth of the pinion

K = range divisor

PHIDOT = $\dot{\varphi}$ = 1, all velocity computations are based on this value

### b. Computations (see also COMMENT cards in program)

#### I. Computation of Gear Tooth Parameters

The following tooth parameters of the gear as well as of the pinion are first computed:

#### For the gear

CXG = $c_{xG}$ (see eq. (D-1))

DELG = $\delta_G$ (see eq. (D-2))

CYG = $c_{yG}$ (see eq. (D-3))

ROG = $r_{oG}$ (see eq. (D-4))

GAMMG = $\gamma_G$ (see eq. (D-5))

ETMING = $\eta_{minG}$ (see eq. (D-8))

ETMAXG = $\eta_{maxG}$ (see eq. (D-9))

#### For the pinion

CXP = $c_{xP}$ (see eq. (D-1))

DELP = $\delta_P$ (see eq. (D-2))

CYP = $c_{yP}$ (see eq. (D-3))

ROP = $r_{oP}$ (see eq. (D-4))

GAMMP = $\gamma_P$ (see eq. (D-5))

ALPHP = $\alpha_P$ (see eq. (D-6))

ETMINP $= \eta_{min_p}$ (see eq. (D-8))

ETMAXP $= \eta_{max_p}$ (see eq. (D-9))

FP $= f_p$ (see eq. (D-7))


## In general

B $= b$, the center distance between the gear and the pinion

DPHI $= \Delta \varphi$, the angle between the center lines of adjacent
gear teeth

DPSI $= \Delta \psi$, the angle between the center lines of adjacent
pinion teeth

L (see eq. (E-10))


## II. Determination of Transition Angle

In order to define the ranges for the round on round and
the round on flat phases of motion, it is first necessary to
locate the transition angles $\varphi_T$ and $\psi_T$. This is accomplished
with the help of the expression contained in section 1b-V,
Appendix E. The transition angle $\psi_T$ is first solved according
to eq. (E-42). Since the solution furnishes two answers, i.e.,
the angles $\psi_{T1}$ and $\psi_{T2}$, it is necessary to decide which of
these is the desired one. The principal criterion for selecting
the correct value of $\psi_T$ is based on the motion of the contact
point T with respect to the pinion. The first transition
angles $\psi_{T1}$ and $\varphi_{T1}$ occur as the contact point moves from the
round portion of the pinion to the flat one (for increasing
values of $\varphi$). As this motion is continued, the distance g
becomes smaller than the transition parameter $f_p$. Once g has
reached its minimum, the contact point moves outward on the

pinion flat until g theoretically equals $f_p$ once again. This part of the motion is never completed in actuality since the subsequent set of teeth takes over before g reaches this transition value. It is this second transition situation which corresponds to the transition angles $\psi_{T2}$ and $\varphi_{T2}$. Since the round on flat equation does not recognize any limitation on the length of the pinion flat, an increase of $\varphi$ over the value of $\varphi_{T2}$ will have associated with it a theoretical value for g which is larger than $f_p$. Thus, one may identify the correct value of $\varphi_T$ and $\psi_T$ by noting that an increase in the angle $\varphi$ must lead to an associated value of g which is smaller than $f_p$.

The program uses this criterion in the following manner after $\psi_{T1}$ and $\psi_{T2}$ are known:

A. Subroutine TRANS is called and the angle $\varphi_{T1}$, which is associated with $\psi_{T1}$, is computed with the help of eqs. (E-43) and (E-44).

B. The angle $\varphi$ is made slightly larger than $\varphi_{T1}$ to produce PHINEXT, and eq. (E-29) is used to find the associated angle PSINEX. Since there are two such angles, the subroutine selects the one which is closest to the value of transition angle $\psi_{T1}$. Subsequently, the associated value of $g_1$ is computed according to eq. (E-27).

C. Steps A and B are then repeated identically for the second transition angle $\psi_{T2}$. This results in the determination of $g_2$.

D. Control remains with the main program, and that value of $\psi_T$ is chosen for which the associated value of g is smaller

F-5

than $f_p$.

For checking purposes, a subsidiary test for the determination of the transition angle was added to the program. It is based on the idea that, for the correct transition angle $\psi_T$, the line representing the flat portion of the pinion will make a smaller angle with the center line $O_G O_p$ than will be the case for the incorrect one. To this end, TEST1 and TEST2 find both angles with the help of

$$\text{TEST} = 360^\circ - (\psi_T + \alpha_p) \tag{F-1}$$

### III. Determination of Correct Sign for Round on Flat Regime

The sign preceding the square root in eq. (E-29), for the round on flat regime, is determined with the help of the angle $\varphi_T$. The condition which yields that angle $\psi$ which is closest to $\psi_T$ governs. The variable SIGNF is used for the sign under consideration.

### IV. Computation of Final and Initial Values of $\varphi$ and $\psi$

The final and initial values of the gear and pinion angles are found by continuously evaluating the round on flat equation (E-29), using the previously determined value of SIGNF, and simultaneously checking the contact condition for the subsequent set of teeth, as given by eq. (E-48). This loop is initiated at the transition angle and terminated once the condition of eq. (E-48) is met. This allows the simultaneous determination

of both the angles at which the first set of teeth loses contact and at which the second set of teeth comes into engagement. The latter is accomplished by subtracting $\Delta\varphi$ from the "loss of contact" angle $\varphi_F$ and adding $\Delta\psi$ to the "loss of contact" angle $\psi_F$ (see computations following statement label no. 70). PHIF and PSIFF represent the angles when contact is lost for a given mesh, while PHII and PSII stand for the angles when initial contact is made.

## V. Determination of Correct Sign for Round on Round Regime

Eq. (E-13) is used to determine the angle $\psi$ while the gear and pinion are in the round on round regime. The correct sign in eq. (E-13) is obtained by comparing the value of the angle $\psi$, as computed with PHII, with the value of PSII. SIGNR is the variable which furnishes this sign.

## VI. Kinematics

Since the limits as well as the correct signs for both the round on round and the round on flat regimes are known, the various kinematic properties of interest may now be computed.

The angular increment DDPHI of the input angle is found by dividing the range from PHII to PHIF into K parts. (K is the range divisor.) The computational loop begins with statement label no. 110 and ends with the third card from the end of the main program. The computation is terminated when the angle $\varphi$ exceeds PHIF.

## A.  Round on Round Regime

As long as PHI is smaller than the transition angle PHIT, the kinematics of the round on round phase of motion is computed. This results in the following:

PSI $= \psi$ (see eq. (E-13))

LAMDA $= \lambda$ (see eqs. (E-14) and (E-15))

PSIDOT $= \dot{\psi}$ (see eq. (E-16))

VSTR $= \bar{V}_{S/T}$ for round on round (see eq. (E-20))

SR $= s$ (see eq. (E-50) for later use)

ETAPR $= \eta_P$ (see eqs. (E-6a) and (E-6b))

ETAGR $= \eta_G$ (see eq. (E-5).  For discussion of the usefulness of $\eta_P$ and $\eta_G$, see output in section c below.)


## B.  Round on Flat Regime

When PHI is larger than the transition angle PHIT, the kinematics of the round on flat phase is computed.  This computation includes the following:

PSI $= \psi$ (see eq. (E-29))

G $= g$ (see eq. (E-27))

PSIDOT $= \dot{\psi}$ (see eq. (E-30))

VSTF $= \bar{V}_{S/T}$ for round on flat (see eq. (E-35))

SF $= s$ (see eq. (E-72) for later use)

ETAGF $= \eta_{GF}$ (see eqs. (E-6c) and (E-6d))

c. <u>Output (see Program CLOCK 1, below)</u>

As previously, the output of the program is best explained with the help of the sample problem at the end of the program. It contains the following:

I. <u>Input Parameters</u>

CAPRP = $R_p$ = .47725 in.   (1.212 cm)

RP = $r_p$ = .09085 in.   (0.231 cm)

AG = $a_G$ = .47725 in.   (1.212 cm)

AP = $a_p$ = .09085 in.   (0.231 cm)

RHOG = $\rho_G$ = .03870 in.   (0.098 cm)

RHOP = $\rho_p$ = .01740 in.   (0.044 cm)

TG = $t_G$ = .03480 in.   (0.088 cm)

TP = $t_p$ = .02800 in.   (0.071 cm)

NG = $n_G$ = 42

NP = $n_p$ = 8

K = 50

II. <u>Computed Values</u>

The following parameters are printed out:

DELGD = $\delta_G$ = 2.5880°

DELPD = $\delta_p$ = 2.1448°

GAMMPD = $\gamma_p$ = 11.0418°

ALPHPD = $\alpha_p$ = 8.8970°

ETMINGD = $\eta_{min_G}$ = 85.3488°

ETMAXGD = $\eta_{max_G}$ = 144.0484°

$ETMINPD = \eta_{min_P} = 78.9582^\circ$

$ETMAXPD = \eta_{max_P} = 166.5870^\circ$

$FP = f_P = .08917$ in. (0.226 cm)

The printout concerning the transition angles consists of two lines. While the program automatically picks that transition configuration which leads to a decreasing value of g as $\varphi$ is increased, both transition angles are printed out for checking purposes. Thus, one finds

$\varphi_{T1} = 186.36^\circ$   $\psi_{T1} = 308.63^\circ$   $g_1 = .0895$ in. (0.227 cm) TEST1 = 42.47°

$\varphi_{T2} = 178.75^\circ$   $\psi_{T2} = 346.65^\circ$   $g_2 = .0888$ in. (0.226 cm) TEST2 = 4.45°

Clearly, the second line represents the solution since $g_2 = .0888$ for a slight increase in the angle $\varphi$ over $\varphi_T$. This is less than the value of $f_P = .08917$. In addition, TEST2 furnishes the smaller number of degrees. Recall TEST represents the angle between the flat of the pinion the line connecting the pivots.

Furthermore, the program shows the initial and final angles of contact:

PHIID = the angle $\varphi$ at initial contact

PSIID = the angle $\psi$ at initial contact

PHIFD = the angle $\varphi$ at final contact

PSIFD = the angle $\psi$ at final contact

F-10

Note that angle $\varphi$ increases while the angle $\psi$ decreases from the beginning to the end of the contact.

The computational loop begins with PHIID and ends with PHIFD. Further, when PHID reaches the approximate value of PHITD, the output shifts from round on round parameters to round on flat ones. The purpose of the multiple output throughout the motion of the mesh is to gain insight concerning the behavior of the mesh as well as to be able to check geometric values.

The following conclusions may be drawn for either of the phases as well as in general for the gears under consideration.

A. <u>Round on Round Phase ($176.2623^{\circ} < \varphi < 178.6623^{\circ}$)</u>

PSIDOT, the angular velocity of the pinion, is negative, and at all times near the "gear ratio" of $42/8 = 5.25$.

SR is printed here only for checking purposes. It becomes important in the moment input-output analyses of program CLOCK 2.

The angles ETAGRD and ETAPRD are of interest because one needs to make sure that the contact between gear and pinion does not occur too close to the respective tooth tips. This is necessary since the present mathematical model has a pointed tip while in a real situation the tips are rounded. Thus, if contact is sufficiently far from the tips of the teeth, the present model will give valid answers.

The range of ETAGRD is approximately between $86^{\circ}$ and $90^{\circ}$. This is considerably smaller than the ETMAXGD of approximately $144^{\circ}$ and larger than the ETMINGD of approximately $85^{\circ}$. (The

latter shows that the round of the pinion tooth does not touch the flat of the gear tooth.)

The angle ETAPRD of the pinion tooth is always considerably smaller than ETMAXPD. Since contact is transferred to the flat portion of the pinion at the end of this phase of motion, ETAPRD almost equals ETMINPD at that point.

LAMDAD is given for general checking purposes only.

B. <u>Round on Flat Phase</u> $(178.6623^{\circ} < \varphi < 184.834^{\circ})$

The angular velocity $\dot{\psi}$ continues relatively smoothly after the transition. The distance g remains smaller than $f_p$, as expected, and it reaches a minimum of .0822 in. at $\varphi \approx 182.5^{\circ}$. It is further to be noted that g never reaches the value of $f_p$ again since the subsequent set of teeth takes over when $g = .08439$ in. (0.214 cm). For the present program to be valid, it is necessary that contact ends before the round on flat phase is completed.

As before, $s_F$ is given for checking purposes only, and its value has been confirmed, just as was done for $s_R$, by graphical analysis (not shown).

The angle ETAGFD of the gear reaches a maximum of approximately $130^{\circ}$ at the end of this phase. Since this is well below the maximum value ETMAXGD = $144^{\circ}$, there is enough margin for a tip radius on the gear tooth.

## C.   General Considerations

As expected for a direct contact mechanism of this type, the angular velocity ratio, as represented by PSIDOT, is not constant and has a greater absolute value at initial contact than at final contact.   This indicates that the pinion will speed up somewhat as the subsequent set of teeth comes into contact, and that therefore, the original set of teeth loses contact at that instant.   This means that the "effective contact ratio" is unity.

Program CLOCK 1

```
 1           PROGRAM CLOCK 1 (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)

       C
       C  KINEMATICS OF SINGLE PASS STEP-UP GEAR MESH WITH CLOCK (OGIVAL) TEETH
       C
 5             REAL MP,MG,L,LX,LY,X,LAMDA,LAMDAD
               READ(5,1)CAPRP,MP
     1         FORMAT(2F10.5)
               READ(5,1)MG,AP
               READ(5,1)RHOG,RHOP
10             READ(5,1)IG,TP
               READ(5,2)MG,NP
     2         FORMAT(2F10.0)
               READ(5,3)K
     3         FORMAT(F10.0)
15             PI = 3.14159
               PI = PI/180.
               PHIDOT = 1.
               WRITE(6,4)CAPRP,MP,AP,AG,AP
     4         FORMAT(*1*,5X,*CAPRP =*,F7.5,3X,*MP =*,F7.5,3X,*MP =*,F7.5,3X,*AG =*,F7.5,3X,
              *
20             WRITE(6,5)RHOG,RHOP
     5         FORMAT(6X,*RHOG =*,F7.5,3X,*RHOP =*,F7.5/)
               WRITE(6,6)IG,TP
     6         FORMAT(6X,*TG =*,F7.5,3X,*TP =*,F7.5/)
               WRITE(6,7)MG,NP
25     7         FORMAT(6X,*MG =*,F7.5,3X,*NP =*,F4.0/)
               WRITE(6,8)PHIDOT
     8         FORMAT(6X,*PHIDOT =*,F5.1/)
               WRITE(6,9)K
     9         FORMAT(6X,*K =*,F5.0////)
30     C
       C  COMPUTATION OF GEAR TOOTH PARAMETERS
       C
               CXG = RHOG - TG/2
               DELG = ASIN(CXG/RG)
35             DELGD = DELG/Z
               CYG = AG*COS(DELG)
               RUG = CYG - SQRT(RHOG*RHOG - CXG*CXG)
               GAMMG = ASIN(RHOG/AG)
               GAMMGD = GAMMG/Z
40             ETAIN5D = 90. - GAMMGD
               ETMAX6D = 180. - ASIN(RHOG*SIN(DELG)/RHOG)/Z
               CXP = RHOP - TP/2.
               DELP = ASIN(CXP/RP)
               DELPD = DELP/Z
45             CYP = AP*COS(DELP)
               RUP = CYP - SQRT(RHOP*RHOP - CXP*CXP)
               GAMMP = ASIN(RHOP/AP)
               GAMMPD = GAMMP/Z
               ALPHP = GAMMP - DELP
50             ALPHPD = ALPHP/Z
               ETMINPD = 90. - GAMMPD
               ETMAXPD = 180. - ASIN(RGP*SIN(DELP)/RHOP)/Z
```

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```
        FP = AP*COS(GAMMP)
        B = CAPHP + RH

        DPHI = 360./RG*2
        DPSI = 360./RP*2
        L = RHOG + RHOP
        WRITE(6,10)DELGD,DELPD,GAMMPD,ALPHPD,ETRIMGD,ETMAXGD,ETMINPD,
     1 ETMAXPD,FP
 10     FORMAT(6X,7DELGD =*,F9.4,3X,*DELPD =*,F9.4,3X,*GAMMPD =*,F9.4,3X
     1,*ALPHPD =*,F9.4//2X,*ETMINGD =*,F9.4,3X,*ETMAXGD =*,F9.4,3X,*ETMI
        CAPD =*,F9.4,3X,*ETMAXPD =*,F9.4//6X,*FP =*,F1.5///)

C
C    DETERMINATION OF TRANSITION ANGLE
C
        AT = RHOG*COS(ALPHP) - FP*SIN(ALPHP)
        BT = RHOG*SIN(ALPHP) - FP*COS(ALPHP)
        CT = (AG*AG - FP*FP - H*B - RHOG*RHOG)/(2.*B)
        ROOTT = AT*AT + BT*BT - CT*CT
        TI1 = AT + SQRT(ROOTT)
        TI2 = AT - SQRT(ROOTT)
        XT = BT - CT
        PSIT1 = 2.*ATAN2(TI1,XT)
        PSIT2 = 2.*ATAN2(TI2,XT)
        IF(PSIT1 .LT. 0.)PSIT1 = PSIT1 + 2.*PI
        IF(PSIT2 .LT. 0.)PSIT2 = PSIT2 + 2.*PI
        PSIT1D = PSIT1/Z
        PSIT2D = PSIT2/Z
        CALL TRANS(RHOG,ALPHP,FP,AG,B,DELG,PSIT1,PHIT1,G1)
        CALL TRANS(RHOG,ALPHP,FP,AG,B,DELG,PSIT1,PHIT2,G2)
        PHIT1D = PHIT1/Z
        PHIT2D = PHIT2/Z
        TEST1 = 360. - (PSIT1D + ALPHPD)
        TEST2 = 360. - (PSIT2D + ALPHPD)
        WRITE(6,11)PHIT1D,PSIT1D,G1,TEST1
        WRITE(6,12)PHIT2D,PSIT2D,G2,TEST2
 11     FORMAT(3X,*PHIT1U =*,F9.4,3X,*PSIT1D =*,F9.4,3X,*G1 =*,F7.4,3X,
     1 *TEST1 =*,F7.4//)
 12     FORMAT(3X,*PHIT2U =*,F9.4,3X,*PSIT2D =*,F9.4,3X,*G2 =*,F7.4,3X,
     1 *TEST2 =*,F7.4//)
        IF(G1 .LT. FP)GO TO 28
        PHIT = PHIT2
        PSIT = PSIT2
        GO TO 30
 20     PHIT = PHIT1
        PSIT = PSIT1
        PHITD = PHIT/Z
 30     PSITD = PSIT/Z

C
C    DETERMINATION OF CORRECT SIGN FOR ROUND ON FLAT REGIME
C
        AF = B*COS(ALPHP) - AG*COS(PHIT-DELG-ALPHP)
        BF = B*SIN(ALPHP) - AG*SIN(PHIT-DELG-ALPHP)
        CF = -RHOG
        ROOTF = AF*AF + BF*BF - CF*CF
```

                                                          F-16

```
        TF1 = AF + SQRT(ADSIF)

        TF2 = AF - SQRT(ADSIF)

        AF = NF + CF
110     PSIF1 = 2.*ATAN2(TF1,XF)
        PSIF2 = 2.*ATAN2(TF2,XF)

        IF(PSIF1 .LT. 0.)PSIF1 = PSIF1 - 2.*PI
        IF(PSIF2 .LT. 0.)PSIF2 = PSIF2 - 2.*PI

        IF(ABS(PSI1-PSITF .LT. 2*STPSIF-2-PSITF)GO TO 48
        SIGM = -1.
        GO TO 50
115
48      SIGM = 1.

C
C       COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI
C

        DO 60 I=1,1000
120     PHID = PHID + (I-1.)/IG6.

        PHI = PHID+Z

        AF = B*COS(ALPHP) + AG*COS(PHI-DELG-ALPHP)
125     BF = B*SIN(ALPHP) - AG*SIN(PHI-DELG-ALPHP)

        CF = HROG
        ROOTF = AF*AF + BF*BF - CF*CF
        TF = AF - SIGF*SQRT(ROOTF)
        XF = BF + CF

        PSIF = 2.*ATAN2(TF,XF)
130     IF(PSIF .LT. 0.)PSIF = PSIF + 2.*PI

        LX = B*COS(PHI-DELG-PHI) - AG*COS(PSIF-DPSI-DELP)
        LY = AG*SIN(PHI-DELG-DPHI) - AG*SIN(PSIF-DELP-DPSI)
        IF(SQRT(LX*LX + LY*LY) .LE. L*50 TO 78
135     CONTINUE

78      PHIF = PHI
        PSIFF = PSIF

        PHII = PHIF + DPHI
        PSII = PSIFF + DPSI
        IF(PSII-G-2.*PI)PSII=PSII-2.*PI

        PHIID = PHII/Z
140     PSIID = PSII/Z
        PHIFC = PHIF/Z
        PSIFC = PSIFF/Z

        WRITE (6,80)PHIID,PSIID,PHIFD,PSIFD
145     80 FORMAT(PHIF=PHIID =,F9.4,3X,PSIID =,F9.4,3X,PHIFD =,F9.4,3X,
       1 PSIFC =,F9.4///)
C
C       DETERMINATION OF CORRECT SIGN FOR POUND OR ROUND REGIME
C
150     AR = SIN(PHII-DELG) + COS(PHII-DELG)*TAN(DELP) + B*TAN(DELP)/AG
        BR = COS(PHII-DELG) - B/AG - SIN(PHII-DELG)*TAN(DELP)
        CR = (AP+AD + AG*AG + B+d - L*L)/(2.*AP*AG*COS(DELP))
        TF = AR*COS(PHII-DELG)/(AP*COS(DELP))
        ROOTR = AR*AR + BR*BR - CR*CR
155     TR1 = BR + SQRT(ROOTR)
        TR2 = AR - SQRT(ROOTR)
        RN = BR - CR
        PSIR1 = 2.*ATAN2(TR1,XR)
```

```
140        PSIK2 = 2.*ATAN2(Y,2.*X)
           IF(PSIK1.LT.0.)PSIK1=PSIK1-2.*PI
           IF(PSIK2.LT.0.)PSIK2 = 2.*PI
           IF(ABS(PSI1-PSI2X).LT.ABS(PSI1-PSIK2)GO TO 90
           SIGN = -1.
           GO TO 100
165     90 SIGN = 1.

C  KINEMATICS
C
170    100 DDPHI = (PHIF-PHII)/K
           PHI = PHII + DDPHI
       110 PHI = PHI + DDPHI
           IF(PHI.GE. PHIF) GO TO 203
           PHIO = PHI/Z
175        IF(PHI.GE. PHIF)GO TO 126
           AM = SIN(PHI-DELG) - COS(PHI-DELG)*TAN(DELP) + B*TAN(DELP)/A6
           BX = COS(PHI-DELP)+B/A6 - TAN(DELP)*SIN(PHI-DELG)
           CX = (AP*AP + A6*A6 - L*L)/(2.*AP*A6*COS(DELP))
       1 = ACOS(PHI-DELG)/(A6*COS(DELP))
180        ROOTK = AM*AM + BM*BM - CM*CM
           TH = A6-SIGN*SQRT(ROOTK)
           AM = BM + CM
185        PSI = 2.*ATAN2(TH,AM)
           IF(PSI.LT.0.)PSI = PSI + 2.*PI
           PSIO = PSI/Z
           SLAM = (AP*SIN(PSI-DELG) - A6*SIN(PHI-DELG))/Z
           CLAM = (AP*COS(PSI-DELG) - A6*COS(PHI-DELG)) - B)/L
           LAMDA = ATAN2(SLAM,CLAM)
190        IF(LAMDA.LT.0.)LAMDA = LAMDA + 2.*PI
           LAMDA = LAMDA//
           AMD = TAN(PSI-DELP)*(PHI-DELG) - COS(PHI-DELG)
           AMG = SIN(PHI-DELG) - TAN(DELP)*COS(PHI-DELG)
           CMD = B*SIN(PHI-DELG)-(AP-COS(DELP))
195        PSIDOT = (APD*COS(PSI) - CMD)/(AP*COS(PSI) - BM*
           SIN(PSI))*PHIDOT
           VSTW = PHIDOT*(A6*COS(PHI-DELG-LAMDA) - PSIDOT*(APP
           ...PSI*PSI-DELP-LAMDA) - - W*G,)
           SW = VSTW/ABS(PSIN)
200        IF(ABS(B-DELP0 .LT. 90.)ETAPR0 = (2.*PI - PSI - LAMDA--DELP)/Z
           IF(PSID-DELP0 .GE. 90.)ETAPR0 = (PSI-LAMDA-DELP)/Z
           ETAPR0 = (PI - PHI - LAMDA - DELP)//
           WTITE(6,115)PHIO-PSIO,PSIDOT-PI,ETAPR0,ETAPR0,LAMDA0
205    115 FORMAT(10*F6.3,3X*ETAG00 =*,F9*4,3X*CTAPR0 =*,F8*4,3X.
           1*SW =*F6.1,3X*ETAG00 =*,F9*4,3X*ETAPR0 =*,F9*4,3X.*LAMDA0 =*.
           2F9*4)
           GO TO 110
       203 AF = A*COS(R-PHI) + A6*COS(PHI-DELG-ALPHA)
           BF = B*SIN(ALPHA) - A6*SIN(PHI-DELG-ALPHA)
210        ROOTF = AF*AF - BF*BF - CF*CF
           YF = AF - SIGN*SQRT(ROOTF)
           XF = BF + CF
```

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```
           PSI = 2.*ATAN2(YF,XF)
           IF(PSI.LT.0.)PSI = PSI + 2.*PI
215
           PSID = PSI/2.
        6 = (A6*SIN(PHI-DELG) - RHOG*COS(PSI*ALPHP))/SIN(PSI*ALPHP)
           AFD = A6*SIN(PHI-DELG-ALPHP)
           AFU = A6*COS(PHI-DELG-ALPHP)
220        PSIDOT = PHIDOT*(AFD*SIN(PSI) - BFD*COS(PSI))/(AF*COS(PSI) - BF*
          &SIN(PSI))
           VSTF = PHIDOT*(RHOG + A6*SIN(PSI-PHI*ALPHP+DELG))
           VF = VSTF/ABS(VSTF)
           IF(PSID-ALPHP).GE.0.)ETAGFD=(PHI-DELG-PSI-ALP+P-3.*PI/2.)/2
225        IF(PSID-ALPHP).LT.0.)ETAGFD=(PHI-DELG-PSI-ALP-P-PI)/2
           WRITE(6,13C)PHID,PSID,PSIDOT,6,SF,ETAGFU
13,    FORMAT(10X,PHID =*,F9.4,3X,PSID =*,F9.4,3X,PSIDOT =*,F8'.4,3X,
           &G =*,F7.5,3X,SF =*,F4.1,3X,ETAGFD =*,F9.4)
           GO TO 110
20*    STOP
           END
230
```

```
         SUBROUTINE TRANS(RMAG,ALPHP,FP,AG,B,DELG,C,PSIT,PHIT,G)
         PI = 3.14159
   1
         ST = (FP*SIN(PSIT-ALPHP) - RMAG*COS(PSIT+ALPHP))/AG
         CT = (FP*COS(PSIT-ALPHP) - RMAG*SIN(PSIT+ALPHP) - B)/AG
         PHIT = ATAN2(ST,CT) + DELG
   5     IF(PHIT .LT. 0.)PHIT = PHIT + 2.*PI
         PHINEXT = PHIT + .162
         BF = B*COS(ALPHP) + AG*COS(PHINEXT+DELG - ALPHP)
         HF = G*SIN(ALPHP) - AG*SIN(PHINEXT + DELG - ALPHP)
  10     CF = HF/BF
         HOOTF = BF*BF - CF*CF
         YF1 = BF + SQRT(HOOTF)
         YF2 = BF - SQRT(HOOTF)
         XF = BF - CF
         PSINE1 = 2.*ATAN(YF1,XF)
  15     PSINE2 = 2.*ATAN(YF2,XF)
         IF(PSINE1 .LT. 0.)PSINE1 = PSINE1 + 2.*PI
         IF(PSINE2 .LT. 0.)PSINE2 = PSINE2 + 2.*PI
         IF(ABS(PSINE1-PSII) .LT. ABS(PSINE2-PSII))GO TO 1
         PSINEXT = PSINE2
  20     GO TO 2
    1    PSINEXT = PSINE1
    2    G = (AG*SIN(PHINEXT-DELG) - RMAG*COS(PSINEXT+ALPHP))/SIN(PSINEXT)
  25     1-ALPHP)
         RETURN
         END
```

| PHID | PSID | PSIDOT | G | SF | ETAGFD |
|---|---|---|---|---|---|
| PHID = 181.4824 | PSID = 332.8464 | PSIDOT = -5.4823 | G = .84286 | SF = 1.0 | ETAGFD = 107.8542 |
| PHID = 181.5766 | PSID = 331.1581 | PSIDOT = -5.4591 | G = .84268 | SF = 1.0 | ETAGFD = 108.9635 |
| PHID = 181.7468 | PSID = 330.2245 | PSIDOT = -5.4328 | G = .84252 | SF = 1.0 | ETAGFD = 110.8695 |
| PHID = 181.9194 | PSID = 329.2959 | PSIDOT = -5.4011 | G = .84249 | SF = 1.0 | ETAGFD = 111.1695 |
| PHID = 182.0901 | PSID = 328.3728 | PSIDOT = -5.3463 | G = .84238 | SF = 1.0 | ETAGFD = 112.2629 |
| PHID = 182.2623 | PSID = 327.4542 | PSIDOT = -5.3279 | G = .84224 | SF = 1.0 | ETAGFD = 113.3511 |
| PHID = 182.4337 | PSID = 326.5464 | PSIDOT = -5.2059 | G = .84220 | SF = 1.0 | ETAGFD = 114.4323 |
| PHID = 182.6852 | PSID = 325.6441 | PSIDOT = -5.2403 | G = .84224 | SF = 1.0 | ETAGFD = 115.5068 |
| PHID = 182.7766 | PSID = 324.7499 | PSIDOT = -5.1915 | G = .84222 | SF = 1.0 | ETAGFD = 116.5717 |
| PHID = 182.9481 | PSID = 323.8644 | PSIDOT = -5.1394 | G = .84227 | SF = 1.0 | ETAGFD = 117.6296 |
| PHID = 183.1194 | PSID = 322.9886 | PSIDOT = -5.0843 | G = .84235 | SF = 1.0 | ETAGFD = 118.6784 |
| PHID = 183.2909 | PSID = 322.1213 | PSIDOT = -5.0261 | G = .84244 | SF = 1.0 | ETAGFD = 119.7165 |
| PHID = 183.4623 | PSID = 321.2649 | PSIDOT = -4.9656 | G = .84268 | SF = 1.0 | ETAGFD = 120.7425 |
| PHID = 183.6337 | PSID = 320.4198 | PSIDOT = -4.9023 | G = .84277 | SF = 1.0 | ETAGFD = 121.7597 |
| PHID = 183.8952 | PSID = 319.5842 | PSIDOT = -4.8367 | G = .84297 | SF = 1.0 | ETAGFD = 122.7660 |
| PHID = 183.9766 | PSID = 318.7688 | PSIDOT = -4.7689 | G = .84329 | SF = 1.0 | ETAGFD = 123.7699 |
| PHID = 184.1488 | PSID = 317.9492 | PSIDOT = -4.6994 | G = .84346 | SF = 1.0 | ETAGFD = 124.7638 |
| PHID = 184.3194 | PSID = 317.1459 | PSIDOT = -4.6274 | G = .84374 | SF = 1.0 | ETAGFD = 125.7166 |
| PHID = 184.4909 | PSID = 316.3628 | PSIDOT = -4.5542 | G = .84405 | SF = 1.0 | ETAGFD = 126.6731 |
| PHID = 184.6623 | PSID = 315.5685 | PSIDOT = -4.4795 | G = .84439 | SF = 1.0 | ETAGFD = 127.6188 |
| PHID = 184.8337 | PSID = 314.8278 | PSIDOT = -4.4036 | G = .84476 | SF = 1.0 | ETAGFD = 128.5517 |

2. Program Clock 2: Point and Cycle Efficiencies for Single
Pass Step-Up Gear Mesh With Clock
(Ogival) Teeth

The program CLOCK 2 furnishes the moment input-output
relationship for a single step-up gear mesh with clock (ogival)
teeth as derived in section 2 of Appendix E. This program uses
the same kinematics as program CLOCK 1, and it differs from that
program only in that it also determines both point and cycle
efficiencies. Because of the above, sections a to c below will
only contain discussions on those portions of the program which
are different from CLOCK 1. The last section shows the complete
program CLOCK 2.

    a.  Input Parameters (see Program CLOCK 2, below)

In addition to the input parameters of CLOCK 1, the following
data must be supplied to the program:

MU = $\mu$, the coefficient of friction at the gear and pinion pivots
     as well as at the contact point between the gear and the
     pinion

RHO1 = $\rho_1$, the pivot radius of the gear

RHO2 = $\rho_2$, the pivot radius of the pinion

    b.  Computations (See also COMMENT cards in program)

      I.  Computation of Gear Tooth Parameters

The computation of the gear tooth parameters is identical

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with that given in program CLOCK 1.


II.  Determination of Transition Angle

The transition angle is determined in the same manner as described in program CLOCK 1.


III.  Determination of Correct Sign for Round on
      Flat Regime

This computation is also identical with that given in program CLOCK 1.


IV.  Computation of Final and Initial Values of $\varphi$ and $\psi$

This computation is also identical with that given in program CLOCK 1.


V.  Determination of Correct Sign for Round on
    Round Regime

This computation is also identical with that given in program CLOCK 1.


VI.  Kinematics, Point and Cycle Efficiencies

The angular increment DDPHI of the input angle is found in the same manner as shown in program CLOCK 1. While the initial, transition and final angles of the mesh are obtained in the same manner as shown in CLOCK 1, the computational loop of the program ranges from the initial angle PHII to one increment before the final angle PHIF. This is necessary in

order to accommodate the numerical integration for the cycle efficiency.

## A. Round on Round Regime

As before for CLOCK 1, as long as PHI is smaller than the transition angle PHIT, the kinematics of the round on round phase of the motion is computed for each increment. These kinematic computations are identical with those given in CLOCK 1. The point efficiency in this phase of motion is obtained with the help of eq. (E-70) and takes the form of eq. (3), i.e.,

$$\epsilon_p = K_{ratio} \frac{M_o}{M_{in}} \qquad (F-2)$$

Since the transmission ratio depends on the angular velocity ratio, and the input angular velocity is unity, eq. (F-2) becomes

$$\epsilon_p = \frac{M_o}{M_{in}} \left| \dot{\psi} \right| \qquad (POINTEF) \qquad (F-3)$$

## B. Round on Flat Regime

When PHI is larger than the transition angle PHIT, the kinematics of the round on flat phase becomes applicable. While the computed kinematic quantities are identical with those of CLOCK 1, the present program contains an expression for the point efficiency for the round on flat phase of the motion. This expression is obtained with the help of eq. (E-92), and it

is computed in the manner of eq. (F-3) with the now current values of the output angular velocity PSIDOT.

### C. Computation of Cycle Efficiency

Once the computational loop is terminated, the cycle efficiency is computed in the manner of eq. (C-10) of Appendix C, i.e.,

$$\epsilon_C = \frac{\Delta\varphi \; \Sigma \epsilon_p}{(\varphi_{FIN} - \varphi_{IN})} \quad \text{(CYCLEFF)} \tag{F-4}$$

The summation was accomplished inside the loop in terms of

$$MTOT = MTOT + POINTEF \tag{F-5}$$

Further,

$\Delta\varphi$ = DDPHI in the program, and similarly

$\varphi_{FIN}$ = PHIF and

$\varphi_{IN}$ = PHII

### c. Output (See section d below)

The output of CLOCK 2 is identical with that of CLOCK 1 with the exception that the point and cycle efficiencies are printed out. The identical geometric parameters are used, and therefore, the same kinematic output is obtained.

Program CLOCK 2

```
1          PROGRAM CLOCK 2 (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)

      C    POINT AND CYCLE EFFICIENCIES FOR SINGLE PASS STEP-UP
      C    GEAR MESH WITH CLOCK LOGIVAL TEETH
      C
           REAL MP,NG,L,LV,LY,R,RM,MTOT,LAMDA
           READ(5,1)C,PHP,HP
5     1    FORMAT(2F10.5)
           READ(5,1)AG,AP
           READ(5,1)RHOG,RHOP
           READ(5,1)G,TP
10         READ(5,2)MG,MP
      2    FORMAT(2F10.5)
           READ(5,1)RHO1,RHO2
           READ(5,3)K
15    3    FORMAT(F10.8)
           READ(5,3)MU
           PI = 3.14159
           Z = PI/1.G
20         PHIGT = 1.
           WRITE(6,4)CAPPP,HP,AG,AP
      4    FORMAT(1X,5X,*CAP=P  =*,F7.5,3X,*PP  =*,F7.5,3X,*AG =*,F7.5,3X,
          1AP =*,F7.5/)
           WRITE(6,5)RHOG,RHOP
25    5    FORMAT(6X,*RHOG =*,F7.5,3X,*RHOP =*,F7.5/)
           WRITE(6,6)TG,TP
      6    FORMAT(6X,*TG =*,F7.5,3X,*TP =*,F7.5/)
           WRITE(6,7)MG,MP
      7    FORMAT(6X,*MG =*,F4.0,3X,*MP =*,F4.0/)
30         WRITE(6,8)RHO1,RHO2
      8    FORMAT(6X,*RHO1 =*,F6.4,3X,*RHO2 =*,F6.4/)
           WRITE(6,9)PHIGT,K,MU
      9    FORMAT(6X,*PHIDO1 =*,F5.1//6X,*K =*,F5.9/6X,*MU =*,F5.2///)
           MIOT = 0.
35    C
      C    COMPUTATION OF GEAR TOOTH PARAMETERS
      C
           CKG = RHOG - TG/2.
           DELG = ASIN(CKG/AG)
40         CYG = AG*COS(DELG)
           ROG = CYG * SQRT(RHOG*RHOG - CKG*CKG)
           DELGD = DELG/Z
           EAMPG = ASIN(RHOG/AG)
           EAMPGD = EAMPG/Z
45         EIMINGD = 90. - GAMMGD
           EIMAXGD = 180. - ASIN(RHOG*SIN(DELG)/GMMOG1/Z
           CKP = RHOP - TP/2.
           DELP = ASIN(CKP/AP)
           CYP = AP*COS(DELP)
50         ROP = CYP + SQRT(RHOP*RHOP - CKP*CKP)
           DELPD = DELP/Z
           GAMP = ASIN(RHOP/AP)
           GAMPD = GAMP/Z
```

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```
55    ALPHP = GAMMP - DELP
      ALPHPD = ALPHP/Z
      ETMINPD = 90. - GAMMPD
      ETMAXPD = 180. - ASIN(RHOP*SIN(DELP)/RHOP)/Z
      FP = AP*COS(GAMMP)
60    B = CAPRP - RP
      DAPHI = 360./NG*Z
      DPSI = 360/NG*Z
      L = RHOG + RHOP
      WRITE(6,10)DELGD,DELPD,GAMMPD,ALPHPD,ETMINGD,ETMAXGD,
     1ETMINPD,ETMAXPD,FP
65 10 FORMAT(16X,8HDELGD =,F9.4,3X,8HDELPD =,F9.4,3X,8HGAMMPD =,F9.4,
     1*,ALPHPD =,F9.4,/6X,8HETMINGD =,F9.4,3X,8HETMAXGD =,F9.4,
     2//6X,8HETMINPD =,F9.4,3X,8HETMAXPD =,F9.4,6X,4HFP =,F7.5,///)
C
C     DETERMINATION OF TRANSITION ANGLE
C
70    AT = RHOG*COS(ALPHP) + FP*SIN(ALPHP)
      BT = RHOG*SIN(ALPHP) - FP*COS(ALPHP)
      CT = (RHOG - FP*FP - B - RHOG*RHOG)/(2.*B)
      ROOT = AT*AT + BT*BT - CT*CT
75    YT1 = AT + SQRT(ROOT)
      YT2 = AT - SQRT(ROOT)
      AT = BT + CT
      PSIT1 = 2.*ATAN2(YT1,AT)
      PSIT2 = 2.*ATAN2(YT2,AT)
80    IF(PSIT1 .LT. 0.)PSIT1 = PSIT1 - 2.*PI
      IF(PSIT2 .LT. 0.)PSIT2 = PSIT2 - 2.*PI
      PSIT1D = PSIT1/Z
      PSIT2D = PSIT2/Z
      CALL TRANS(K-DG,ALPHP,FP,AG,B,DFLG,7,PSIT1,PHIT1,G1)
85    CALL TRANS(RHOK,ALPHP,FP,AG,B,DFLG,2,PSIT2,PHIT2,G2)
      PHIT1D = PHIT1/Z
      PHIT2D = PHIT2/Z
      TEST1 = 360. - (PSIT1D + ALPHPD)
90    TEST2 = 360. - (PSIT2D + ALPHPD)
      WRITE(6,11)PHIT1D,PSIT1D,PHIT2D,G1,TEST1
   11 FORMAT(16X,8HPHIT1D =,F9.4,3X,8HPSIT1D =,F9.4,3X,8HG1 =,F7.4,3X,
     1*TEST1 =,F7.2/)
   12 FORMAT(16X,8HPHIT2D =,F9.4,3X,8HPSIT2D =,F9.4,3X,8HG2 =,F7.4,3X,
95    1*TEST2 =,F7.2///)
      IF(G1 .LT. FP)GO TO 20
      PHIT = PHIT1
      PSIT = PSIT1
      GO TO 30
100 20 PHIT = PHIT1
   30 PSIT = PSIT1
      PHITD = PHIT/Z
      PSITD = PSIT/Z
C
105 C     DETERMINATION OF CORRECT SIGN FOR ROUND-ON FLAT-REGIME
C
```

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```
          AF = B*COS(ALPHP) - AG*COS(PHIT-DELG-ALPHP)
          BF = B*SIN(ALPHP) - AG*SIN(PHIT-DELG-ALPHP)
          CF = -HHOG
          HDOTF = AF*AF + BF*BF - CF*CF
          YF1 = AF + SQRT(HDOTF)
          YF2 = AF - SQRT(HDOTF)
          XF = BF - CF
110       PSIF1 = 2.*ATAN2(YF1,XF)
          PSIF2 = 2.*ATAN2(YF2,XF)
          IF(PSIF1 .LT. 0.)PSIF1 = PSIF1 + 2.*PI
115       IF(PSIF2 .LT. 0.)PSIF2 = PSIF2 + 2.*PI
          IF(ABS(PSIF1-PSIT) .LT. ABS(PSIF2-PSIT))GO TO 45
          SIGNF = -1.
          GO TO 50
120    45 SIGNF = 1.
C
C      COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI
C
125    50 DO 60 I=1,1000
          PHIO = PHIIO + (I-1)/180.
          PHI = PHIO*Z
          AF = B*COS(ALPHP) - AG*COS(PHI-DELG-ALPHP)
          BF = B*SIN(ALPHP) - AG*SIN(PHI-DELG-ALPHP)
          CF = -HHOG
130       HDOTF = AF*AF + BF*BF - CF*CF
          YF = AF + SIGNF*SQRT(HDOTF)
          XF = BF - CF
          PSIF = 2.*ATAN2(YF,XF)
135       IF(PSIF .LT. 0.)PSIF = PSIF + 2.*PI
          LF = B - AG*COS(PHI-DELG-DPHI) - AP*COS(PSIF+DPSI+DPSI-DELP)
          LY = AG*SIN(PHI-DELG-DPHI) - AP*SIN(PSIF-DELP-DPSI)
          IF(SQRT(LF*LF + LY*LY) .LE. L)GO TO 70
       60 CONTINUE
140    70 PHIF = PHI
          PSIFF = PSIF
          PHII = PHIF - DPHI
          PSII = PSIFF + DPSI
          IF(I-GT.2.*PI)PHII=PHII-2.*PI
145       PHIIO = PHII/Z
          PSIIO = PSII/Z
          PHIFO = PHIF/Z
          PSIFO = PSIFF/Z
          WRITE(6,88)PHIIO,PSIID,PHIFD,PSIFD
150    88 FORMAT(//,1PE9.4,3X,1PE9.4,3X,1PE9.4,3X,1PE9.4,3X,
          1PSIFD =0,F9.4,//)
C
C      DETERMINATION OF CORRECT SIGN FOR ROUND OR ROUND REGIME
          AR = SIN(PHII-DELG) - COS(PHII-DELG)*TAN(UELP) + B*TAN(DELP)/AG
          BR = COS(PHII-DELG) - B/AG - SIN(PHII-DELG)*TAN(DELP)
155       CR = TAN(DELP) + AG*AG - L*L/Z*AG + AP*AG*COS(DELP))
          1 + A*COS(PHII-DELG)/(AP*COS(DELP))
          HDOTR = AR*AR + BR*BR - CR*CR
          YR1 = AR + SQRT(HDOTR)
```

```
160          TM2 = AH - SQRT(HOUTR)
             AK = BA + CM
             PSIR1 = 2.*ATAN2(TM1,AK)
             PSIR2 = 2.*ATAN2(TM2,AK)
165          IF(PSIR1 .LT. 0.)PSIP1 = PSIR1 - 2.*PI
             IF(PSIR2 .LT. 0.)PSIP2 = PSIR2 - 2.*PI
             IF(ABS(PSI1-PSIN1) .LT. ABS(PSI1-PSIN2))GO TO 98
             SIGN = -1.
             GO TO 108
             98 SIGN = 1.
170    C
       C   KINEMATICS
       C
       100 DDPHI = TM+JF+PHI1+A
             PHI = PHII - DDPHI
       110 PHI = PHI - DDPHI
175          IF(PHI .GE. PHIF - DDPHI)GO TO 288
             PHID = PHI/2.
             IF(PHI .GE. PHII)GO TO 128
             BM = SIN(PHI-DELG) - COS(PHI-DELG)*TAN(DELG) - B*TAN(DELP)/AG
180          BA = COS(PHI-DELG) - B/AG - (AM(DELP)*SIN(PHI-DELG)
             CM = (AP*AP - AG*AG - B*B - (AG)/(2.*AP*AG*ACOS(DELP))
             1. B*COS(PHI-DELG)/(AP*COS(DELP))
             HOUTR = BA*BA - CM*CM
             YK = AH*SIGN*SQRT(HOUTR)
185          BK = BA + CM
             PSI = 2.*ATAN2(YK,AK)
             IF(PSI .LT. 0.)PSI = PSI - 2.*PI
             PSID = PSI/2
             SLP = (AP*SIN(PSI-DELP) - AG*SIN(PHI-DELG))/L
190          CLP = (AP*COS(PSI-DELP) - AG*COS(PHI-DELG) - B)/L
             LPDA = B*AG*SLP*M*CLP
             IF(LAMDA .LT. 0.)LAMDA = LAMDA + 2.*PI
             AMD = TAN(DELP)+SIN(PHI-DELG) - COS(PHI-DELG)
             BMD = SIN(PHI-DELG) + TAN(DELP)+COS(PHI-DELG)
195          CMD = B*SIN(PHI-DELG)/(AP*COS(DELP))
             PSIDCT = (AMD*SIN(PSI) + BMD*COS(PSI) - CMD)/(AP*COS(PSI) - BMD
             SIH = (V-SI)*PHIDOT
200          VSTH = -PHIDOT*(AG*COS(PHI-DELG-LAMDA) + BM*B) - PSIDOT*(AP*
             ACOS(PSI-DELP-LAMDA)    +AG*B)
             SH = VSTH/ABS(VSIM)
             IF(PSID-DELP0 .LT. 0.)FAP=0   2.*PI - PSI - LAMDA - DELP)/L
             IF(*SID-DELP0 .GE. 90.)FAP=3 = (PSI-LAMDA-DELP)/L
205          ETADG = *PHI - LAMDA - DELG*/2
             EFR = AP*(SP*PHCM-BM02) - AP*(MP*SP*COS(PSI-DELP-LAMDA) - SIH(PSI
             1-DELP-LAMDA))
             EFWD = AP*(BM-0) + S*BMDG: + AG*(MP*SP*COS(PHI-DELG-LAMDA) - SIN(
             1PHI-DELG-LAMDA))
             POINTEF = EFGA/(FKG+ABS(PSIDOT))
210    115 FORMAL(6X,*P-10 =*,F9.,+3X,*PSID =*,F9.4,3X,*PSIDOT =*,F8.,+3X,
             1*SR =*,F6.13X,*ETAGRD =*,F9.4,3X,*ETAPRO =*,F9.4,3X,*POINTEF =*,
             1CF6.3)
```

```
         GO TO 158
     128 AF = B*COS(ALPHP) + A6*COS(PHI-DELG-ALPHP)
         BF = B*SIN(ALPHP) - A6*SIN(PHI-DELG-ALPHP)
         CF = -RHOG
         HOOIF = AF*AF + BF*BF - CF*CF
         YF = AF + SIGN*SQRT(HOOIF)
         AF = BF + CF
 220     PSI = 2.*ATAN2(YF,XF)
         IF(PSI .LT. 0.)PSI = PSI + 2.*PI
         PSIO = PSI/2.
         G = (A6*SIN(PHI-DELG) - RHOG*COS(PSI+ALPHP))/SIN(PSI-ALPHP)
 225     AFD = A6*SIN(PHI-DELG-ALPHP)
         BFD = A6*COS(PHI-DELG-ALPHP)
         PSIDOT = PHIDOT*(AFD*SIN(PSI) + BFD*COS(PSI)) + (G*COS(PSI) + BF*
        1SIN(PSI))
         VSTF = PHIDOT*(RHOG + A6*SIN(PSI-PHI+ALPHP+DELE))
         W = VSTF/ABS(VSTF)
 230     IF(PSIO-ALPHP.GE.98.)ETAGFD=(PHI-DELG-PSI-ALPHP+3.*PI/2.)/2
         IF(PSIO-ALPHP.LT.98.)ETAGFD=(PHI-DELG-PSI-ALPHP-.5*PI)/2
         EFD = G - RHOG*W
         EFFU = W*(G*SIN(PSI) - SF*A6*COS(PHI-DELG-PSI-ALPHP))
        1+ W*G*SIN(PHI-DELG-PSI-ALPHP))
         POINTEF = (FFD/EFD)*ABS(PSIDOT)
 235     WRITE(6,136)PHID,PSID,PSIOOT+6,SF,ETAGFD,POINTEF
     136 FORMAT(6X,PHID =,F8.4,3X,PSID =,F8.4,3X,PSIDOT =,F8.4,3X,
        1G =,F7.5,3X,SF =,F4.1,3X,ETAGFD =,F9.4,3X,POINTEF =,F6.3)
 150     NTOT = NTOT + POINTEF
         GO TO 11
 200     CYCLEFF = NTOT*RHOM1/(NMIF*PHI1)
         WRITE(6,210)CYCLEFF
 210     FORMAT(//6,5X,CYCLE EFFICIENCY =,F6.3)
         STOP
         END
```

F-32

```
        SUBROUTINE TRANS(RHOG,ALPHP,FP,AG,B,DELS,C,PSIT,PHIT,G)
 1      PI = 3.14159
        SI = (FP*SIN(PSIT+ALPHP) + RHOG*COS(PSIT+ALPHP))/AG
        CI = (FP*COS(PSIT+ALPHP) - RHOG*SIN(PSIT+ALPHP) - B)/AG
        PHIT = ATAN2(SI,CI) - DELG
 5      IF(PHIT .LT. 0.)PHIT = PHIT + 2.*PI
        PHINXT = PHIT + .1*2
        AF = B*COS(PHP) - AG*COS(PHINXT - DELG - ALPHP)
        BF = B*SIN(ALPHP) - AG*SIN(PHINXT - DELG - ALPHP)
10      CF = RHOG
        ROOTF = AF*AF + BF*BF - CF*CF
        RF = AF * CF
        YF1 = AF + SQRT(ROOTF)
        YF2 = AF - SQRT(ROOTF)
15      PSINX1 = 2.*ATAN2(YF1,RF)
        PSINX2 = 2.*ATAN2(YF2,RF)
        IF(PSINX1 .LT. 0.)PSINX1 = PSINX1 + 2.*PI
        IF(PSINX2 .LT. 0.)PSINX2 = PSINX2 + 2.*PI
        IF(ABS(PSINX1-PSIT) .LT. ABS(PSINX2-PSIT))GO TO 1
20      PSINXT = PSINX2
        GO TO 2
 1      PSINXT = PSINX1
 2      G = (AG*SIN(PHINX1-DELG) - RHOG*COS(PSINXT+ALPHP))/SIN(PSINXT)
25      C = ALPHP
        RETURN
        END
```

F-33

| | | | | | | |
|---|---|---|---|---|---|---|
| PHID = 180.5480 | PSID = 336.8763 | PSIDOT = -5.5366 | G = .08420 | SF = 1.0 | ETAGFD = 102.2667 | POINTEF = .855 |
| PHID = 180.7194 | PSID = 335.8771 | PSIDOT = -5.5353 | G = .08300 | SF = 1.0 | ETAGFD = 103.3874 | POINTEF = .852 |
| PHID = 180.8944 | PSID = 334.9787 | PSIDOT = -5.5286 | G = .08358 | SF = 1.0 | ETAGFD = 104.5071 | POINTEF = .848 |
| PHID = 181.6623 | PSID = 333.9828 | PSIDOT = -5.5168 | G = .08331 | SF = 1.0 | ETAGFD = 105.6253 | POINTEF = .844 |
| PHID = 181.2337 | PSID = 333.0375 | PSIDOT = -5.5015 | G = .08307 | SF = 1.0 | ETAGFD = 106.7412 | POINTEF = .841 |
| PHID = 181.4052 | PSID = 332.0965 | PSIDOT = -5.4823 | G = .08286 | SF = 1.0 | ETAGFD = 107.8542 | POINTEF = .837 |
| PHID = 181.5756 | PSID = 331.1581 | PSIDOT = -5.4591 | G = .08268 | SF = 1.0 | ETAGFD = 108.9635 | POINTEF = .834 |
| PHID = 181.7480 | PSID = 330.2245 | PSIDOT = -5.4328 | G = .08252 | SF = 1.0 | ETAGFD = 110.0665 | POINTEF = .830 |
| PHID = 181.9174 | PSID = 329.2959 | PSIDOT = -5.4011 | G = .08248 | SF = 1.0 | ETAGFD = 111.1685 | POINTEF = .827 |
| PHID = 182.0909 | PSID = 328.3729 | PSIDOT = -5.3663 | G = .08238 | SF = 1.0 | ETAGFD = 112.2629 | POINTEF = .823 |
| PHID = 182.2623 | PSID = 327.4552 | PSIDOT = -5.3279 | G = .08224 | SF = 1.0 | ETAGFD = 113.3511 | POINTEF = .819 |
| PHID = 182.4317 | PSID = 326.5464 | PSIDOT = -5.2858 | G = .08228 | SF = 1.0 | ETAGFD = 114.4323 | POINTEF = .816 |
| PHID = 182.6052 | PSID = 325.6441 | PSIDOT = -5.2403 | G = .08228 | SF = 1.0 | ETAGFD = 115.5090 | POINTEF = .812 |
| PHID = 182.7766 | PSID = 324.7494 | PSIDOT = -5.1915 | G = .08222 | SF = 1.0 | ETAGFD = 116.5717 | POINTEF = .809 |
| PHID = 182.9480 | PSID = 323.8644 | PSIDOT = -5.1394 | G = .08227 | SF = 1.0 | ETAGFD = 117.6286 | POINTEF = .805 |
| PHID = 183.1195 | PSID = 322.9886 | PSIDOT = -5.0843 | G = .08235 | SF = 1.0 | ETAGFD = 118.6764 | POINTEF = .802 |
| PHID = 183.2909 | PSID = 322.1213 | PSIDOT = -5.0263 | G = .08246 | SF = 1.0 | ETAGFD = 119.7145 | POINTEF = .798 |
| PHID = 183.4623 | PSID = 321.2648 | PSIDOT = -4.9656 | G = .08268 | SF = 1.0 | ETAGFD = 120.7425 | POINTEF = .795 |
| PHID = 183.6337 | PSID = 320.4194 | PSIDOT = -4.9023 | G = .08277 | SF = 1.0 | ETAGFD = 121.7597 | POINTEF = .791 |
| PHID = 183.8052 | PSID = 319.5842 | PSIDOT = -4.8367 | G = .08297 | SF = 1.0 | ETAGFD = 122.7668 | POINTEF = .788 |
| PHID = 183.9766 | PSID = 318.7609 | PSIDOT = -4.7689 | G = .08320 | SF = 1.0 | ETAGFD = 123.7608 | POINTEF = .784 |
| PHID = 184.1480 | PSID = 317.9492 | PSIDOT = -4.6998 | G = .08346 | SF = 1.0 | ETAGFD = 124.7638 | POINTEF = .781 |
| PHID = 184.3194 | PSID = 317.1498 | PSIDOT = -4.0274 | G = .08374 | SF = 1.0 | ETAGFD = 125.7246 | POINTEF = .777 |
| PHID = 184.4909 | PSID = 316.3628 | PSIDOT = -4.5542 | G = .08405 | SF = 1.0 | ETAGFD = 126.6731 | POINTEF = .774 |
| PHID = 184.6623 | PSID = 315.5885 | PSIDOT = -4.4795 | G = .08439 | SF = 1.0 | ETAGFD = 127.6188 | POINTEF = .770 |

CYCLE EFFICIENCY = .850

F-35

## KINEMATICS OF TWO AND THREE STEP-UP GEAR TRAINS WITH CLOCK TEETH

Figure G-1 shows the basic configuration of a three step-up gear train used in certain fuze applications. The general layout is identical to that shown in Figure A-5 of Appendix A. Now, ogival type gear teeth are used instead of involute type ones. Again, it is required to find the equilibrant moment, $M_{o4}$, acting on pinion no. 4 which holds the input moment, $M_{in}$, acting on gear no. 1, in equilibrium when both pivot and contact friction are taken into account, and when the fuze body spins. Appendix H gives the force and moment analyses for the determination of this moment input-output relationship. The same appendix also shows the derivation of such an input-output relationship for a two step-up gear train with ogival gear teeth which must operate in a spin environment. (Figure A-10 of Appendix A shows this type of configuration with involute teeth.) The present appendix lays the groundwork for the moment relationships of Appendix H by providing the kinematics of the three ogival gear meshes involved.

In each case both round on round and round on flat phases
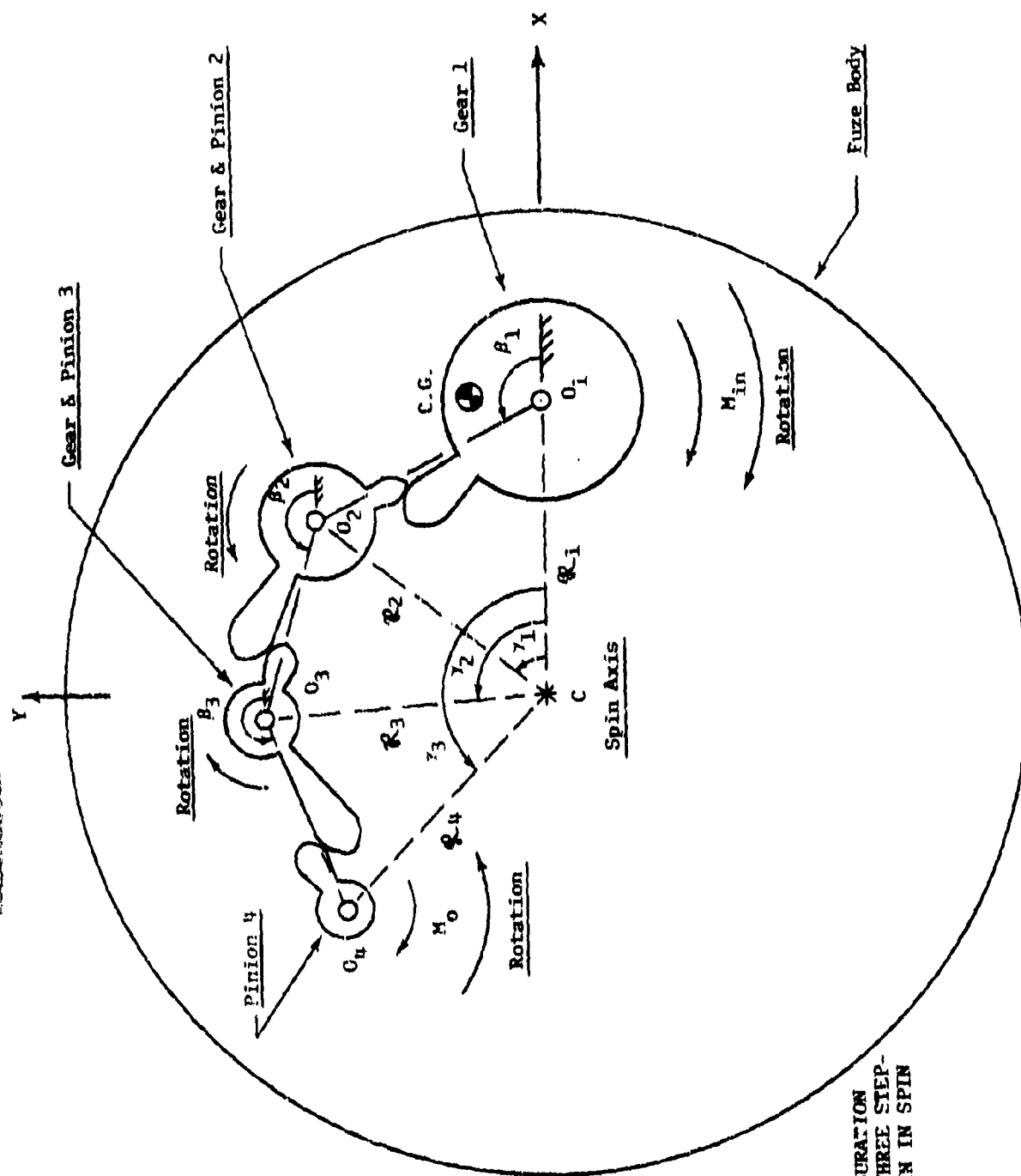
FIGURE G-1

BASIC CONFIGURATION
FOR OGIVAL THREE STEP-
UP GEAR TRAIN IN SPIN
ENVIRONMENT

G-2

of motion have to be considered.  All derivations follow the
pattern set in Section 1 of Appendix E.  The derivations must
take into account that the driving gears of meshes no. 1 and
no. 3, i.e., between gear no. 1 and pinion no. 2 and between
gear no. 3 and pinion no. 4, respectively, have clockwise
rotations.  The driving gear of mesh no. 2, i.e., between gear no. 2
and pinion no. 3, has counterclockwise rotation.

Finally, the inclinations of the various pivot to pivot
centerlines with respect to the body-fixed X-axes, as represented
by the angles $\beta_1$, $\beta_2$ and $\beta_3$, must be considered.

For the sake of simplicity, the notation will in most cases
not differentiate between round on round and round on flat phases
of motion.  For example, the output angle, $\psi_1$, and its derivatives
will have the same symbol for both phases.

For definitions of angles $\beta_1$ and $\gamma_1$ as well as the distances
$\mathbb{Q}_1$, see Appendix A-6.

## 1. KINEMATICS OF MESH NO. 1 (GEAR NO. 1 AND PINION NO. 2)

### a. ROUND ON ROUND PHASE OF MOTION

Figure G-2 shows the round on round phase of the motion of mesh no. 1 in a schematic manner. Only the contacting faces of the gear and the pinion are indicated.

### I. UNIT VECTORS

The unit vector in the direction $O_1 C_{G1}$ of the gear is given by

$$\bar{n}_{G1} = \cos(\phi_1 + \delta_{G1})\bar{1} + \sin(\phi_1 + \delta_{G1})\bar{j} \qquad (G-1)$$

The unit vector in the direction $C_{G1} C_{P1}$ is given by

$$\bar{n}_{\lambda 1} = \cos\lambda_1 \bar{1} + \sin\lambda_1 \bar{j} \qquad (G-2)$$

The unit vector normal to $\bar{n}_{\lambda 1}$ (in the right hand sense) becomes

$$\bar{n}_{N\lambda 1} = -\sin\lambda_1 \bar{1} + \cos\lambda_1 \bar{j} \qquad (G-3)$$
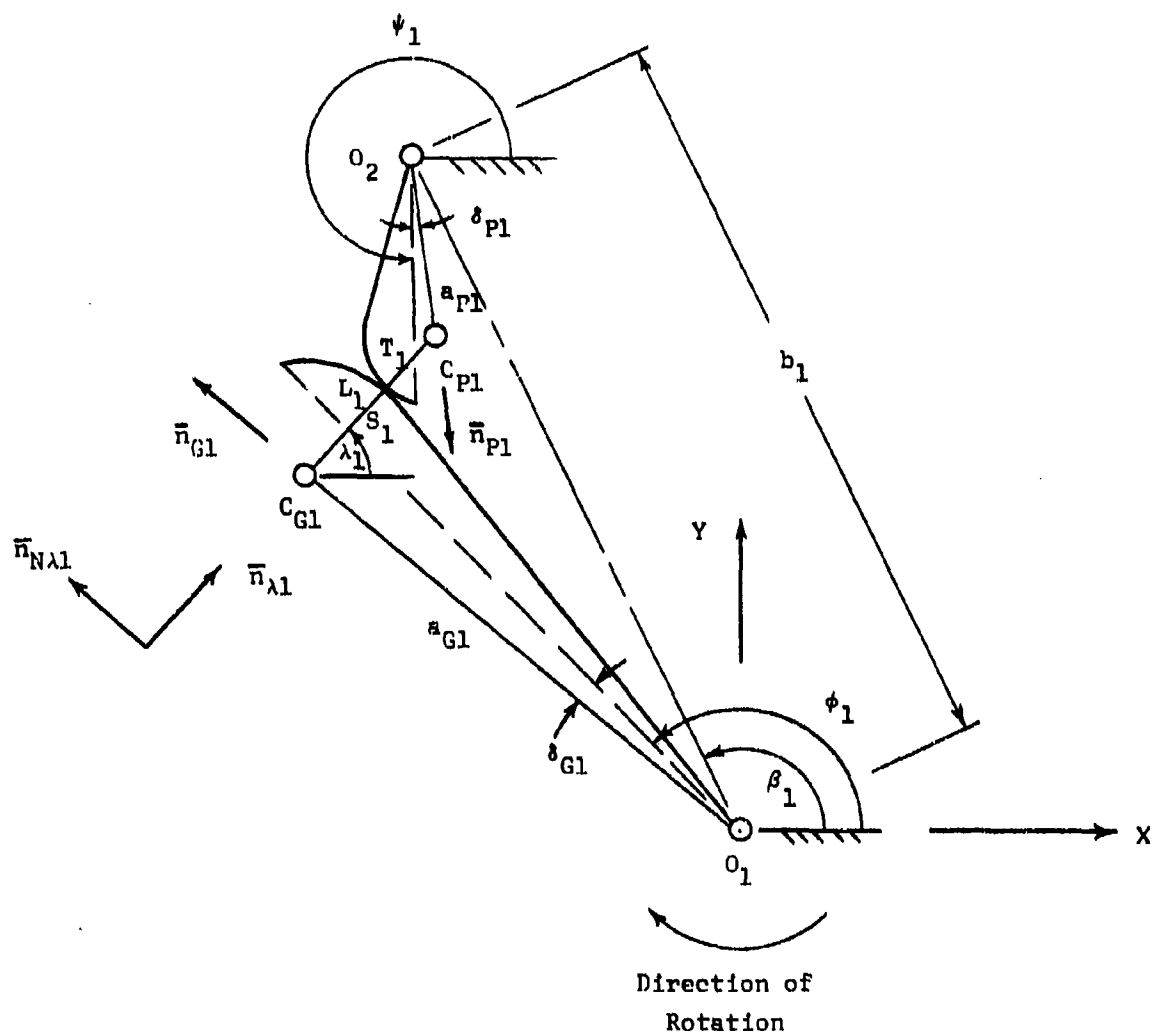
FIGURE G-2

ROUND ON ROUND PHASE FOR MESH NO. 1

G-5

The unit vector in the direction $O_2C_{P1}$ is given by

$$\bar{n}_{P1} = \cos(\psi_1 + \delta_{P1})\bar{I} + \sin(\psi_1 + \delta_{P1})\bar{J} \qquad (G-4)$$

Finally, the unit vector along the centerline, $O_1O_2$, is given by

$$\bar{n}_{\beta 1} = \cos\beta_1\bar{I} + \sin\beta_1\bar{J} \qquad (G-5)$$

## II.  DETERMINATION OF OUTPUT ANGLE $\psi_1$ AND "COUPLER" ANGLE $\lambda_1$

The loop equation of the equivalent four-bar linkage is given by

$$a_{G1}\bar{n}_{G1} + L_1\bar{n}_{\lambda 1} - a_{P1}\bar{n}_{P1} - b_1\bar{n}_{\beta 1} = 0 \qquad (G-6)$$

where $\quad L_1 = \rho_{G1} + \rho_{P1}$ $\qquad\qquad\qquad\qquad$ (G-7)

With the appropriate substitution for the unit vectors, one obtains the following component equations:

G-6

$$a_{G1}\cos(\phi_1 + \delta_{G1}) + L_1\cos\lambda_1 - b_1\cos\beta_1 - a_{P1}\cos(\psi_1 + \delta_{P1}) = 0$$

$$(G-8)$$

and

$$a_{G1}\sin(\phi_1 + \delta_{G1}) + L_1\sin\lambda_1 - b_1\sin\beta_1 - a_{P1}\sin(\psi_1 + \delta_{P1}) = 0$$

$$(G-9)$$

To eliminate $\lambda_1$, let

$$\sin^2\lambda_1 + \cos^2\lambda_1 = 1 \qquad (G-10)$$

The above trigonometric functions are obtained from equations (G-8) and (G-9), respectively. Substitution into equation (G-10) furnishes

$$A_{1R}\sin\psi_1 + B_{1R}\cos\psi_1 = C_{1R} \qquad (G-11)$$

where

$$A_{1R} = a_{G1}\sin(\phi_1 + \delta_{G1} - \delta_{P1}) - b_1\sin(\beta_1 - \delta_{P1})$$

$$B_{1R} = a_{G1}\cos(\phi_1 + \delta_{G1} - \delta_{P1}) - b_1\cos(\beta_1 - \delta_{P1})$$

$$C_{1R} = \frac{a_{P1}^2 + a_{G1}^2 + b_1^2 - L_1^2 - 2a_{G1}b_1\cos(\phi_1 + \delta_{G1} - \beta_1)}{2a_{P1}}$$

G-7

Equation (G-11) is now solved for $\psi_1$ in the manner described in Appendix E in connection with equation (E-12), i.e.,

$$\psi_1 = 2 \tan^{-1} \frac{A_{1R} \pm \sqrt{A_{1R}^2 + B_{1R}^2 - C_{1R}^2}}{B_{1R} + C_{1R}} \qquad (G-12)$$

The correct sign on equation (G-12) must be found from geometric considerations.

The coupler angle $\lambda_1$ may now be determined either from equation (G-8) or from equation (G-9). Thus,

$$\lambda_1 = \cos^{-1} \left[ \frac{b_1 \cos\beta_1 + a_{P1} \cos(\psi_1 + \delta_{P1}) - a_{G1} \cos(\phi_1 + \delta_{G1})}{L_1} \right]$$

$$\qquad (G-13)$$

or

$$\lambda_1 = \sin^{-1} \left[ \frac{b_1 \sin\beta_1 + a_{P1} \sin(\psi_1 + \delta_{P1}) - a_{G1} \sin(\phi_1 + \delta_{G1})}{L_1} \right]$$

$$\qquad (G-14)$$

Differentiation of equation (G-11) with respect to time gives

$$\dot{\psi}_1 = \dot{\phi}_1 \left[ \frac{B_{1RD}\cos\psi_1 - A_{1RD}\sin\psi_1 + C_{1RD}}{A_{1R}\cos\psi_1 - B_{1R}\sin\psi_1} \right] \qquad \text{(G-15)}$$

where

$$A_{1RD} = a_{G1}\cos(\phi_1 + \delta_{G1} - \delta_{P1})$$

$$B_{1RD} = a_{G1}\sin(\phi_1 + \delta_{G1} - \delta_{P1})$$

$$C_{1RD} = \frac{a_{G1}b_1\sin(\phi_1 + \delta_{G1} - \beta_1)}{a_{P1}}$$

## IV. RELATIVE VELOCITY AT THE CONTACT POINT

The relative velocity $\overline{V}_{S1/T1_R}$ of the contact point $S_1$ on gear no. 1 with respect to point $T_1$ on pinion no. 2, represents the vectorial difference between the absolute velocities of these points. Thus,

$$\overline{V}_{S1/T1_R} = \overline{V}_{S1/C} - \overline{V}_{T1/C} \qquad \text{(G-16)}$$

where C represents the spin center of the fuze body. If $\overline{\omega}$ stands for the angular velocity of the fuze body, then appropriate substitution into equation (G-16) gives (See also Figures G-1 and G-2)

$$\overline{V}_{S1/T1_R} = \left[ \overline{\omega} \times \overline{\ell}_1 + (\overline{\omega} + \overline{\dot{\phi}}_1) \times (\overline{a}_{G1} + \overline{\rho}_{G1}) \right]$$
$$- \left[ \overline{\omega} \times \overline{\ell}_2 + (\overline{\omega} + \dot{\psi}_1) \times (\overline{a}_{P1} + \overline{\rho}_{P1}) \right]$$

$$\text{(G-17)}$$

Since

$$\overline{\omega} \times \left[ \overline{\ell}_1 + \overline{a}_{G1} + \overline{\rho}_{G1} \right] = \overline{\omega} \times \left[ \overline{\ell}_2 + \overline{a}_{P1} + \overline{\rho}_{P1} \right] \qquad \text{(G-18)}$$

because the position vectors in brackets are equal, equation (G-17) may be written

$$\bar{V}_{S1/T1_R} = \bar{V}_{S1/O_1} - \bar{V}_{T1/O_2}$$

$$= \bar{\dot{\phi}}_1 \times (\bar{a}_{G1} + \bar{\rho}_{G1}) - \bar{\dot{\psi}}_1 \times (\bar{a}_{P1} + \bar{\rho}_{P1}) \qquad (G\text{-}19)$$

Note that $\bar{V}_{S1/T1_R}$ becomes the vectorial difference of the contact point velocities with respect to the fuze body.

Since this relative velocity is tangent to the contacting surfaces, it may be written as the vectorial difference of the velocity components along these surfaces. Accordingly, equation (G-19) becomes, with the help of the unit vector $\bar{n}_{N\lambda1}$,

$$\bar{V}_{S1/T1_R} = \left\{ \left[ \dot{\phi}_1 \bar{k} \times (a_{G1}\bar{n}_{G1} + \rho_{G1}\bar{n}_{\lambda1}) \right] \cdot \bar{n}_{N\lambda1} \right.$$

$$\left. - \left[ \dot{\psi}_1 \bar{k} \times (a_{P1}\bar{n}_{P1} - \rho_{P1}\bar{n}_{\lambda1}) \right] \cdot \bar{n}_{N\lambda1} \right\} \bar{n}_{N\lambda1}$$

$$(G\text{-}20)$$

Appropriate substitution of unit vectors given earlier in Section I and simplification results in

$$\nabla_{S1/T1_R} = \left\{ \dot{\phi}_1 \left[ a_{G1} \cos(\phi_1 + \delta_{G1} - \lambda_1) + \rho_{G1} \right] \right.$$

$$\left. - \dot{\psi}_1 \left[ a_{P1} \cos(\psi_1 + \delta_{P1} - \lambda_1) - \rho_{P1} \right] \right\} \bar{n}_{N\lambda 1}$$

<div align="right">(G-21)</div>

## b. ROUND ON FLAT PHASE OF MOTION

Figure G-3 shows a schematic view of mesh no. 1 in the round on flat phase of the motion.

## I. UNIT VECTORS

The unit vector in the direction $O_2 T_1$ is given by

$$\bar{n}_{F1} = \cos(\psi_1 - \alpha_{P1})\bar{I} + \sin(\psi_1 - \alpha_{P1})\bar{J} \qquad \text{(G-22)}$$

The unit vector $\bar{n}_{NF1}$, in the direction $C_{G1}S_1$, is always normal to $\bar{n}_{F1}$

$$\bar{n}_{NF1} = -\sin(\psi_1 - \alpha_{P1})\bar{I} + \cos(\psi_1 - \alpha_{P1})\bar{J} \qquad \text{(G-23)}$$
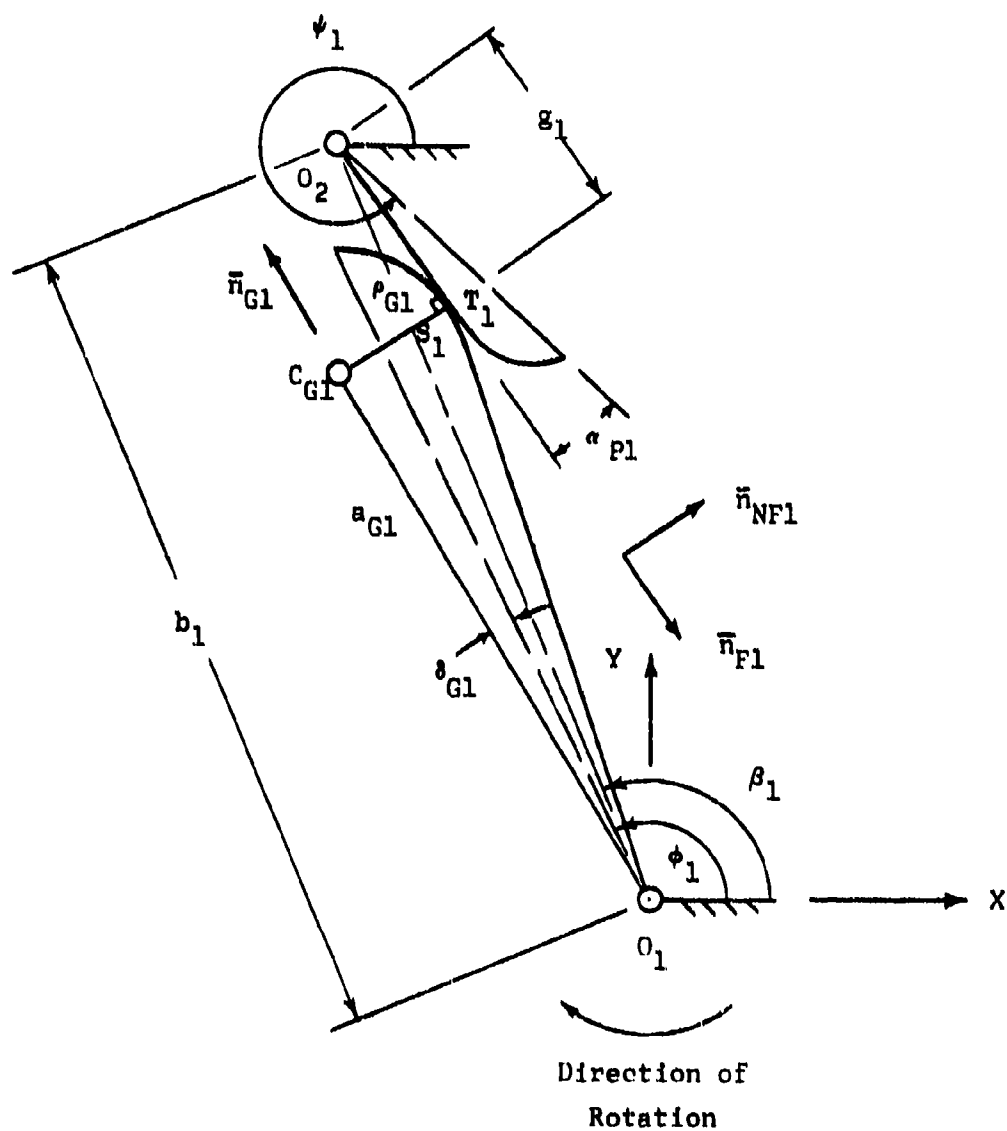
FIGURE G-3

ROUND ON FLAT PHASE FOR MESH NO. 1

G-13

## II. DETERMINATION OF OUTPUT ANGLE $\psi_1$ AND DISTANCE $g_1$

The vector equation for the mechanism loop $O_1-C_{G1}-S_1-T_1-O_2$ has the form:

$$a_{G1}\bar{n}_{G1} + \rho_{G1}\bar{n}_{NF1} - g_1\bar{n}_{F1} - b_1\bar{n}_{\beta1} = 0 \qquad (G-24)$$

Substitution of equations (G-1), (G-5), (G-22) and (G-23) leads to the following component equations.

$$a_{G1}\cos(\phi_1 + \delta_{G1}) - \rho_{G1}\sin(\psi_1 - \alpha_{P1}) - b_1\cos\beta_1 - g_1\cos(\psi_1 - \alpha_{P1})$$
$$= 0 \qquad (G-25)$$

and

$$a_{G1}\sin(\phi_1 + \delta_{G1}) + \rho_{G1}\cos(\psi_1 - \alpha_{P1}) - b_1\sin\beta_1 - g_1\sin(\psi_1 - \alpha_{P1})$$
$$= 0 \qquad (G-26)$$

From equation (G-26) one obtains for the quantity $g_1$

$$g_1 = \frac{a_{G1}\sin(\phi_1 + \delta_{G1}) + \rho_{G1}\cos(\psi_1 - \alpha_{P1}) - b_1\sin\beta_1}{\sin(\psi_1 - \alpha_{P1})}$$

$$(G-27)$$

This expression is now substituted in equation (G-25). This leads to the following:

$$A_{1F} \sin\psi_1 \; + \; B_{1F} \cos\psi_1 \; = \; C_{1F} \tag{G-28}$$

where

$$A_{1F} \; = \; a_{G1} \cos(\phi_1 + \delta_{G1} + \alpha_{P1}) \; - \; b_1 \cos(\beta_1 + \alpha_{P1})$$

$$B_{1F} \; = \; -\, a_{G1} \sin(\phi_1 + \delta_{G1} + \alpha_{P1}) \; + \; b_1 \sin(\beta_1 + \alpha_{P1})$$

$$C_{1F} \; = \; \rho_{G1}$$

Equation (G-28) is solved for $\psi_1$ in the now customary manner:

$$\psi_1 \; = \; 2 \tan^{-1} \frac{A_{1F} \; \pm \; \sqrt{A_{1F}^2 + B_{1F}^2 - C_{1F}^2}}{B_{1F} \; + \; C_{1F}} \tag{G-29}$$

The appropriate sign is again found from geometric considerations.

## III. DETERMINATION OF ANGULAR VELOCITY $\dot{\psi}_1$ DURING ROUND ON FLAT
## PHASE OF MOTION

Implicit differentiation of equation (G-28) with respect to time gives, for $\dot{\psi}_1$:

$$\dot{\psi}_1 = \dot{\phi}_1 \left[ \frac{A_{1FD}\sin\psi_1 + B_{1FD}\cos\psi_1}{A_{1F}\cos\psi_1 - B_{1F}\sin\psi_1} \right] \qquad \text{(G-30)}$$

where

$$A_{1FD} = a_{G1}\sin(\phi_1 + \delta_{G1} + \alpha_{P1})$$

$$B_{1FD} = a_{G1}\cos(\phi_1 + \delta_{G1} + \alpha_{P1})$$

## IV. RELATIVE VELOCITY $\overline{V}_{S1/T1_F}$ AT CONTACT POINT DURING ROUND

## ON FLAT PHASE OF MOTION

For the round on flat phase, the relative velocity $\overline{V}_{S1/T1_F}$ may be expressed by

$$\overline{V}_{S1/T1_F} = \overline{V}_{S1/O_1} - \overline{V}_{T1/O_2} \tag{G-31}$$

Now, this velocity has the direction of the unit vector $\pm \overline{n}_{F1}$. Since there is no velocity component along the pinion flank, equation (G-31) becomes

$$\begin{aligned}
\overline{V}_{S1/T1_f} &= \overline{V}_{S1/O_1} \cdot \overline{n}_{F1} \\
&= \left\{ \left[ \dot{\phi}_1 \times (a_{G1}\overline{E}_{G1} + \rho_{G1}\overline{n}_{NF1}) \right] \cdot \overline{n}_{F1} \right\} \overline{n}_{F1}
\end{aligned} \tag{G-32}$$

Appropriate substitution of unit vectors furnishes

$$\overline{V}_{S1/T1_F} = \dot{\phi}_1 \left[ a_{G1}\sin(\psi_1 - \alpha_{P1} - \phi_1 - \delta_{G1}) - \rho_{G1} \right] \overline{n}_{F1} \tag{G-33}$$

## V. DETERMINATION OF TRANSITION ANGLES

The transition angle, $\phi_{1T}$, and the corresponding angle, $\psi_{1T}$, are reached when the round or round phase is followed by the round on flat one. They are obtained by letting $g_1 = f_{P1}$ in the component equations (G-25) and (G-26). This gives

$$a_{G1}\cos(\phi_{1T} + \delta_{G1}) - \rho_{G1}\sin(\psi_{1T} - \alpha_{P1}) - b_1\cos\beta_1 - f_{P1}\cos(\psi_{1T} - \alpha_{P1})$$

$$= 0 \qquad (G-34)$$

and

$$a_{G1}\sin(\phi_{1T} + \delta_{G1}) + \rho_{G1}\cos(\psi_{1T} - \alpha_{P1}) - b_1\sin\beta_1 - f_{P1}\sin(\psi_{1T} - \alpha_{P1})$$

$$= 0 \qquad (G-35)$$

From the above, one obtains

$$\cos(\phi_{1T} + \delta_{G1}) = \frac{1}{a_{G1}}\left[\rho_{G1}\sin(\psi_{1T} - \alpha_{P1}) + b_1\cos\beta_1 + f_{P1}\cos(\psi_{1T} - \alpha_{P1})\right]$$

$$(G-36)$$

$$\sin(\phi_{1T} + \delta_{G1}) = \frac{1}{a_{G1}}\left[-\rho_{G1}\cos(\psi_{1T} - \alpha_{P1}) + b_1\sin\beta_1 + f_{P1}\sin(\psi_{1T} - \alpha_{P1})\right]$$

$$(G-37)$$

The angle $\psi_{1T}$ is now obtained by letting

$$\sin^2(\phi_{1T} + \delta_{G1}) + \cos^2(\phi_{1T} + \delta_{G1}) = 1$$

This results in

$$A_{1T}\sin\psi_{1T} + B_{1T}\cos\psi_{1T} = C_{1T} \qquad (G-38)$$

where

$$A_{1T} = \rho_{G1}\cos(\beta_1 + \alpha_{P1}) + f_{P1}\sin(\beta_1 + \alpha_{P1})$$

$$B_{1T} = -\rho_{G1}\sin(\beta_1 + \alpha_{P1}) + f_{P1}\cos(\beta_1 + \alpha_{P1})$$

$$C_{1T} = \frac{a_{G1}^2 - \rho_{G1}^2 - b_1^2 - f_{P1}^2}{2b_1}$$

Finally,

$$\psi_{1T} = 2\tan^{-1}\frac{A_{1T} \pm \sqrt{A_{1T}^2 + B_{1T}^2 - C_{1T}^2}}{B_{1T} + C_{1T}} \qquad (G-39)$$

The appropriate sign must be found from geometric considerations. The associated angle $\phi_{1T}$ may be found with the help of either equation (G-36) or equation (G-37):

$$\phi_{1T} = \cos^{-1}\left[\frac{\rho_{G1}\sin(\psi_{1T} - \alpha_{P1}) + f_{P1}\cos(\psi_{1T} - \alpha_{P1}) + b_1\cos\beta_1}{a_{G1}}\right]$$

$$- \delta_{G1} \qquad\qquad (G\text{-}40)$$

or

$$\phi_{1T} = \sin^{-1}\left[\frac{-\rho_{G1}\cos(\psi_{1T} - \alpha_{P1}) + f_{P1}\sin(\psi_{1T} - \alpha_{P1}) + b_1\sin\beta_1}{a_{G1}}\right]$$

$$- \delta_{G1} \qquad\qquad (G\text{-}41)$$

## VI.   <u>SENSING EQUATION FOR THE DETERMINATION OF CONTACT ON SUBSEQUENT TOOTH MESH</u>

The following contact sensing equation is derived with the assumption that subsequent contact is made in the round on round phase of the motion, in the manner shown in Section VI of Appendix E.  Now, the configuration is that of Figure G-2 where gear no. 1 rotates in a clockwise direction.

Before contact is made, the distance between the centers of curvature $C_{G1}$ and $C_{P1}$ is given by

$$\overline{C_{G1}C_{P1}} = L_{x1}\overline{i} + L_{y1}\overline{j} \tag{G-42}$$

If $\Delta\phi_1$ and $\Delta\psi_1$ represent the tooth spacing angles of gear no. 1 and pinion no. 2, respectively, the associated loop equation becomes (see Figures E-3 and G-2).

$$a_{G1}\left[\cos(\phi_1 + \Delta\phi_1 + \delta_{G1})\overline{I} + \sin(\phi_1 + \Delta\phi_1 + \delta_{G1})\overline{j}\right] + L_{x1}\overline{i} + L_{y1}\overline{j}$$

$$= b_1(\cos\beta_1\overline{I} + \sin\beta_1\overline{J}) + a_{P1}\left[\cos(\psi_1 - \Delta\psi_1 + \delta_{P1})\overline{I} + \sin(\psi_1 - \Delta\psi_1 + \delta_{P1})\overline{J}\right]$$

$$\tag{G-43}$$

where

$\psi_1$ = angle of pinion no. 2 as determined for the round on flat mode with equation (G-29)

The magnitudes of $L_{x1}$ and $L_{y1}$ are determined from the component form of equation (G-43), i.e.,

$$L_{x1} = b_1\cos\beta_1 + a_{P1}\cos(\psi_1 - \Delta\psi_1 + \delta_{P1}) - a_{G1}\cos(\phi_1 + \Delta\phi_1 + \delta_{G1})$$

and $$\tag{G-44}$$

$$L_{y1} = b_1\sin\beta_1 + a_{P1}\sin(\psi_1 - \Delta\psi_1 + \delta_{P1}) - a_{G1}\sin(\phi_1 + \Delta\phi_1 + \delta_{G1})$$

$$\tag{G-45}$$

G-21

Contact will have occurred as soon as

$$\sqrt{L_{x1}^2 + L_{y1}^2} \leq \rho_{G1} + \rho_{P1} \qquad (G-46)$$

## 2. KINEMATICS OF MESH NO. 2 (GEAR NO. 2 AND PINION NO. 3)

### a. ROUND ON ROUND PHASE OF MOTION

Figure G-4 gives a schematic representation of the round on round phase of the motion. Only the contacting faces of the gear and the pinion are shown. It is to be noted that the input gear rotates in the counter-clockwise direction, and that the output angle $\psi_1$ of mesh no. 1 is identical to the input angle $\phi_2$ of mesh no. 2.

### I. UNIT VECTORS

The unit vector in the direction $O_2 C_{G2}$ of the gear is given by

$$\bar{n}_{G2} = \cos(\phi_2 - \delta_{G2})\bar{i} + \sin(\phi_2 - \delta_{G2})\bar{j} \qquad (G-47)$$

The unit vector in the direction $C_{G2}C_{P2}$ is given by

$$\bar{n}_{\lambda 2} = \cos\lambda_2\bar{i} + \sin\lambda_2\bar{j} \qquad (G-48)$$
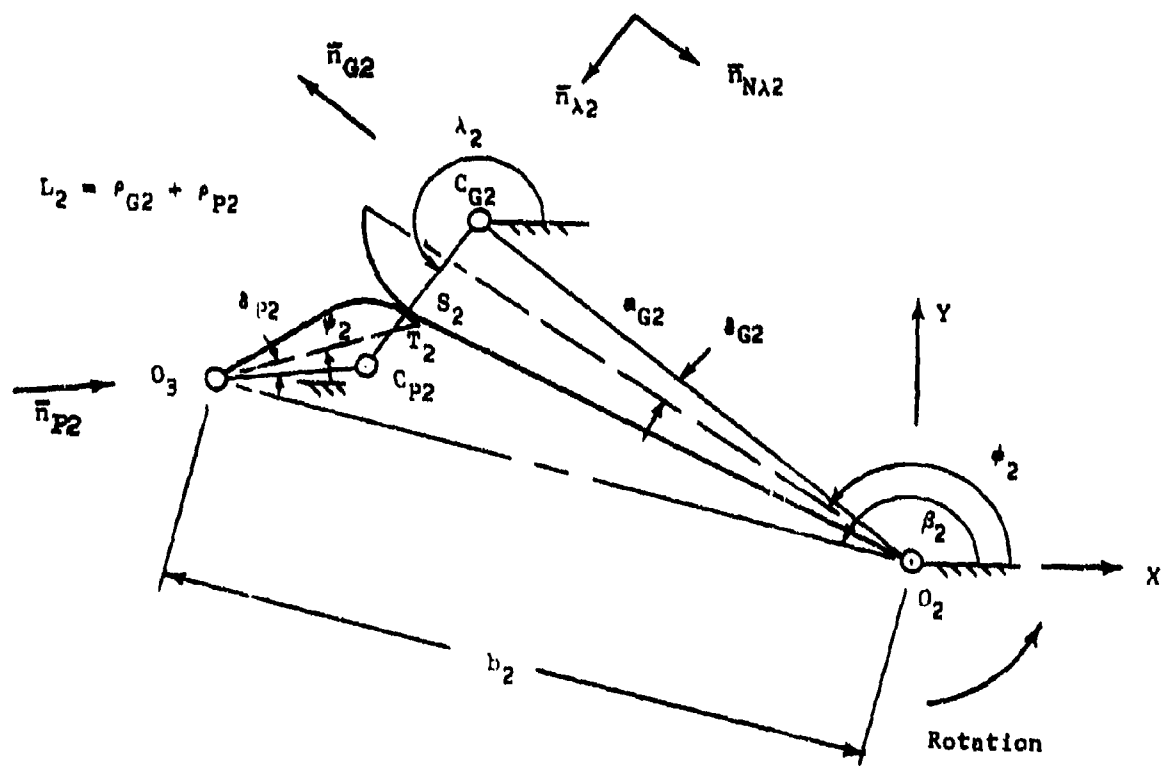
FIGURE G-4

ROUND ON ROUND PHASE FOR MESH NO. 2

The unit vector normal to $\bar{n}_{\lambda 2}$ in the right hand sense becomes

$$\bar{n}_{N\lambda 2} \;\; = \;\; - \sin\lambda_2 \bar{I} \;\; + \;\; \cos\lambda_2 \bar{J} \tag{G-49}$$

The pinion unit vector $\bar{n}_{P2}$, in the direction $O_3 C_{P2}$, is represented by

$$\bar{n}_{P2} \;\; = \;\; \cos(\psi_2 - \delta_{P2})\bar{I} \;\; + \;\; \sin(\psi_2 - \delta_{P2})\bar{J} \tag{G-50}$$

Finally, the unit vector along the centerline $O_2 O_3$ is given by

$$\bar{n}_{\beta 2} \;\; = \;\; \cos\beta_2 \bar{I} \;\; + \;\; \sin\beta_2 \bar{J} \tag{G-51}$$

## II.  DETERMINATION OF OUTPUT ANGLE $\psi_2$ AND "COUPLER" ANGLE $\lambda_2$

The loop equation of the equivalent four-bar linkage is given by

$$a_{G2}\bar{n}_{G2} \;\; + \;\; L_2\bar{n}_{\lambda 2} \;\; - \;\; a_{P2}\bar{n}_{P2} \;\; - \;\; b_2\bar{n}_{\beta 2} \;\; = \;\; 0 \tag{G-52}$$

where

$$L_2 = \rho_{G2} + \rho_{P2} \qquad (G\text{-}53)$$

After substitution of the unit vector, as given earlier, one obtains the following component equations

$$a_{G2}\cos(\phi_2 - \delta_{G2}) + L_2\cos\lambda_2 - a_{P2}\cos(\psi_2 - \delta_{P2}) - b_2\cos\beta_2 = 0$$

$$(G\text{-}54)$$

and

$$a_{G2}\sin(\phi_2 - \delta_{G2}) + L_2\sin\lambda_2 - a_{P2}\sin(\psi_2 - \delta_{P2}) - b_2\sin\beta_2 = 0$$

$$(G\text{-}55)$$

To solve for the output angle $\psi_2$ in terms of the input angle $\phi_2$, substitute the expressions for $\sin\lambda_2$ and $\cos\lambda_2$, as obtained from the component equations (G-54) and (G-55), into

$$\sin^2\lambda_2 + \cos^2\lambda_2 = 1 \qquad (G\text{-}56)$$

This leads to

$$A_{2R}\sin\psi_2 + B_{2R}\cos\psi_2 = C_{2R} \qquad (G\text{-}57)$$

where

$$A_{2R} = b_2\sin(\beta_2 + \delta_{P2}) - a_{G2}\sin(\phi_2 - \delta_{G2} + \delta_{P2})$$

$$B_{2R} = b_2\cos(\beta_2 + \delta_{P2}) - a_{G2}\cos(\phi_2 - \delta_{G2} + \delta_{P2})$$

$$C_{2R} = \frac{L_2^2 - b_2^2 - a_{G2}^2 - a_{P2}^2 + 2a_{G2}b_2\cos(\phi_2 - \delta_{G2} - \beta_2)}{2a_{P2}}$$

Equation (G-57) is then solved for $\psi_2$ in the manner discussed
in Appendix E

$$\psi_2 = 2\tan^{-1}\frac{A_{2R} \pm \sqrt{A_{2R}^2 + B_{2R}^2 - C_{2R}^2}}{B_{2R} + C_{2R}} \qquad \text{(G-58)}$$

The correct sign must again be determined from geometric considerations

The angle $\lambda_2$ may now be determined either from equation (G-54)
or equation (G-55)

$$\lambda_2 = \cos^{-1}\left[\frac{b_2\cos\beta_2 + a_{P2}\cos(\psi_2 - \delta_{P2}) - a_{G2}\cos(\phi_2 - \delta_{G2})}{L_2}\right]$$

$$\text{(G-59)}$$

or

$$\lambda_2 = \sin^{-1}\left[\frac{b_2\sin\beta_2 + a_{P2}\sin(\psi_2 - \delta_{P2}) - a_{G2}\sin(\phi_2 - \delta_{G2})}{L_2}\right]$$

$$\text{(G-60)}$$

## III. DETERMINATION OF OUTPUT ANGULAR VELOCITY $\dot{\psi}_2$

Implicit differentiation of equation (G-57) with respect to time leads to

$$\dot{\psi}_2 = \dot{\phi}_2 \left[ \frac{A_{2RD}\sin\psi_2 - B_{2RD}\cos\psi_2 - C_{2RD}}{A_{2R}\cos\psi_2 - B_{2R}\sin\psi_2} \right] \qquad (G-61)$$

where

$$A_{2RD} = a_{Q2}\cos(\phi_2 - \delta_{Q2} + \delta_{P2})$$

$$B_{2RD} = a_{Q2}\sin(\phi_2 - \delta_{Q2} + \delta_{P2})$$

$$C_{2RD} = \frac{a_{Q2}b_2\sin(\phi_2 - \delta_{Q2} - \beta_2)}{a_{P2}}$$

## IV. RELATIVE VELOCITY AT THE CONTACT POINT

The relative velocity $\overline{V}_{S2/T2_R}$, of point $S_2$ on gear no. 2 with respect to point $T_2$ on pinion no. 3 has the direction of the unit vector $\bar{n}_{N\lambda 2}$. Thus, in the manner of equation (G-20).

$$\overline{V}_{S2/T2_R} = \left\{ \left[ \dot{\phi}_2 \bar{k} \times (a_{G2}\bar{n}_{G2} + \rho_{G2}\bar{n}_{\lambda 2}) \right] \cdot \bar{n}_{N\lambda 2} \right.$$

$$\left. - \left[ \dot{\psi}_2 \bar{k} \times (a_{P2}\bar{n}_{P2} - \rho_{P2}\bar{n}_{\lambda 2}) \right] \cdot \bar{n}_{N\lambda 2} \right\} \bar{n}_{N\lambda 2}$$

$$(G\text{-}62)$$

Substitution of unit vectors yields

$$\overline{V}_{S2/T2_R} = \left\{ \dot{\phi}_2 \left[ a_{G2}\cos(\phi_2 - \delta_{G2} - \lambda_2) + \rho_{G2} \right] \right.$$

$$\left. - \dot{\psi}_2 \left[ a_{P2}\cos(\psi_2 - \delta_{P2} - \lambda_2) - \rho_{P2} \right] \right\} \bar{n}_{N\lambda 2}$$

$$(G\text{-}63)$$

## b. ROUND ON FLAT PHASE OF MOTION

Figure G-5 shows a schematic view of mesh no. 2 in the round on flat phase of the motion. Again, only the contacting sides of the gear teeth are indicated.

### I. UNIT VECTORS

The unit vector in the direction $O_3T_2$, along the flank of pinion no. 3, is given by

$$\bar{n}_{F2} = \cos(\psi_2 + \alpha_{P2})\bar{I} + \sin(\psi_2 + \alpha_{P2})\bar{J} \qquad (G-64)$$

The unit vector $\bar{n}_{NF2}$, in the direction $S_2C_{G2}$, is normal to $\bar{n}_{F2}$ in the right hand sense

$$\bar{n}_{NF2} = -\sin(\psi_2 + \alpha_{P2})\bar{I} + \cos(\psi_2 + \alpha_{P2})\bar{J} \qquad (G-65)$$

FIGURE G-5

ROUND ON FLAT PHASE OF MOTION OF MESH NO. 2

## II. <u>DETERMINATION OF OUTPUT ANGLE $\psi_2$ AND DISTANCE $g_2$</u>

The vector equation for the mechanism loop $O_2$-$C_{G2}$-$S_2$-$T_2$-$O_3$ has the form

$$a_{G2}\bar{n}_{G2} - \rho_{G2}\bar{n}_{NF2} - g_2\bar{n}_{F2} - b_2\bar{n}_{\beta2} = 0 \qquad (G\text{-}66)$$

Appropriate substitutions for the unit vectors furnish the following component equations.

$$a_{G2}\cos(\phi_2 - \delta_{G2}) + \rho_{G2}\sin(\psi_2 + \alpha_{P2}) - b_2\cos\beta_2 - g_2\cos(\psi_2 + \alpha_{P2})$$
$$= 0 \qquad (G\text{-}67)$$

and

$$a_{G2}\sin(\phi_2 - \delta_{G2}) - \rho_{G2}\cos(\psi_2 + \alpha_{P2}) - b_2\sin\beta_2 - g_2\sin(\psi_2 + \alpha_{P2})$$
$$= 0 \qquad (G\text{-}68)$$

From equation (G-68) one obtains the following expression for $g_2$

$$g_2 = \frac{a_{G2}\sin(\phi_2 - \delta_{G2}) - \rho_{G2}\cos(\psi_2 + \alpha_{P2}) - b_2\sin\beta_2}{\sin(\psi_2 + \alpha_{P2})} \qquad (G\text{-}69)$$

G-32

This expression is now substituted into equation (G-67), and
one obtains the following:

$$A_{2F}\sin\psi_2 \; + \; B_{2F}\cos\psi_2 \; = \; C_{2F} \tag{G-70}$$

where

$$A_{2F} \; = \; a_{G2}\cos(\phi_2 - \delta_{G2} - \alpha_{P2}) \; - \; b_2\cos(\beta_2 - \alpha_{P2})$$

$$B_{2F} \; = \; - \, a_{G2}\sin(\phi_2 - \delta_{G2} - \alpha_{P2}) \; + \; b_2\sin(\beta_2 - \alpha_{P2})$$

$$C_{2F} \; = \; - \, \rho_{G2}$$

Equation (G-70) is solved in the customary manner:

$$\psi_2 \; = \; 2\,\tan^{-1} \frac{A_{2F} \; \pm \; \sqrt{A_{2F}^2 \, + \, B_{2F}^2 \, - \, C_{2F}^2}}{B_{2F} \; + \; C_{2F}} \tag{G-71}$$

The appropriate sign is again found from geometric considerations.

## III. DETERMINATION OF ANGULAR VELOCITY $\dot{\psi}_2$ DURING ROUND ON FLAT
## PHASE OF MOTION

Implicit differentiation of equation (G-70) with respect to time gives the following expression for $\dot{\psi}_2$:

$$\dot{\psi}_2 = \dot{\phi}_2 \left[ \frac{A_{2FD}\sin\psi_2 + B_{2FD}\cos\psi_2}{A_{2F}\cos\psi_2 - B_{2F}\sin\psi_2} \right] \qquad (G-72)$$

where

$$A_{2FD} = a_{G2}\sin(\phi_2 - \delta_{G2} - \alpha_{P2})$$

$$B_{2FD} = a_{G2}\cos(\phi_2 - \delta_{G2} - \alpha_{P2})$$

## IV. RELATIVE VELOCITY $\bar{V}_{S2/T2_F}$ AT CONTACT POINT DURING ROUND
## ON FLAT PHASE OF MOTION

Again, the relative velocity $\bar{V}_{S2/T2_F}$ consists only of that component of $\bar{V}_{S2/O_2}$ which is directed along the pinion flank. Thus,

G-34

$$\overline{V}_{S2/T2_F} = \left[ \overline{V}_{S2/O_2} \cdot \overline{n}_{F2} \right] \overline{n}_{F2}$$

$$= \left\{ \left[ \dot{\phi}_2 \overline{k} \times (a_{G2}\overline{n}_{G2} - \rho_{G2}\overline{n}_{NF2}) \right] \cdot \overline{n}_{F2} \right\} \overline{n}_{F2}$$

$$(G-73)$$

Appropriate substitution of unit vectors gives

$$\overline{V}_{S2/T2_F} = \dot{\phi}_2 \left[ a_{G2}\sin(\psi_2 + \alpha_{P2} - \phi_2 + \delta_{G2}) + \rho_{G2} \right] \overline{n}_{F2}$$

$$(G-74)$$

## V. DETERMINATION OF TRANSITION ANGLES

The transition angle $\phi_{2T}$ and the angle $\psi_{2T}$, which correspond to the transition from the round on round to the round on flat phase of motion, are obtained by letting $g_2 = f_{P2}$ in the component equations (G-67) and (G-68). From this one finds the following with $\phi_2 = \phi_{2T}$ and $\psi_2 = \psi_{2T}$:

$$\cos(\phi_{2T} - \delta_{G2}) = \frac{1}{a_{G2}} \left[ - \rho_{G2}\sin(\psi_{2T} + \alpha_{P2}) + b_2\cos\beta_2 + f_{P2}\cos(\psi_{2T} + \alpha_{P2}) \right]$$

$$(G-75)$$

and

$$\sin(\phi_{2T} - \delta_{G2}) = \frac{1}{a_{G2}}\left[\rho_{G2}\cos(\psi_{2T} + \alpha_{P2}) + b_2\sin\beta_2 + f_{P2}\sin(\psi_{2T} + \alpha_{P2})\right]$$

(G-76)

The angle $\psi_{2T}$ is now found by substituting the above expressions into

$$\sin^2(\phi_{2T} - \delta_{G2}) + \cos^2(\phi_{2T} - \delta_{G2}) = 1 \qquad (G-77)$$

This results in

$$A_{2T}\sin\psi_{2T} + B_{2T}\cos\psi_{2T} = C_{2T} \qquad (G-78)$$

where

$$A_{2T} = -\rho_{G2}\cos(\beta_2 - \alpha_{P2}) + f_{P2}\sin(\beta_2 - \alpha_{P2})$$

$$B_{2T} = \rho_{G2}\sin(\beta_2 - \alpha_{P2}) + f_{P2}\cos(\beta_2 - \alpha_{P2})$$

$$C_{2T} = \frac{a_{G2}^2 - \rho_{G2}^2 - b_2^2 - f_{P2}^2}{2 b_2}$$

G-36

Finally, in the usual way

$$\psi_{2T} = 2 \tan^{-1} \frac{A_{2T} \pm \sqrt{A_{2T}^2 + B_{2T}^2 - C_{2T}^2}}{B_{2T} + C_{2T}} \tag{G-79}$$

Again, the sign must be decided from geometric considerations.

The associated angle $\phi_{2T}$ may be found with the help of either equation (G-75) or equation (G-76), i.e.,

$$\phi_{2T} = \cos^{-1}\left[\frac{-\rho_{G2}\sin(\psi_{2T} + \alpha_{P2}) + b_2\cos\beta_2 + f_{P2}\cos(\psi_{2T} + \alpha_{P2})}{a_{G2}}\right] + \delta_{G2} \tag{G-80}$$

or

$$\phi_{2T} = \sin^{-1}\left[\frac{\rho_{G2}\cos(\psi_{2T} + \alpha_{P2}) + b_2\sin\beta_2 + f_{P2}\sin(\psi_{2T} + \alpha_{P2})}{a_{G2}}\right] + \delta_{G2} \tag{G-81}$$

G-37

# VI. SENSING EQUATION FOR THE DETERMINATION OF CONTACT ON SUBSEQUENT TOOTH MESH

The contact sensing equation for mesh no. 2 is derived similarly to that for mesh no. 1   before contact is made in the round on round mode. The distance between the centers of curvature $C_{G2}$ and $C_{P2}$ is given by

$$\overline{C_{G2}C_{P2}} = L_{x2}\bar{i} + L_{y2}\bar{j} \tag{G-82}$$

If $\Delta\phi_2$ and $\Delta\psi_2$ represent the tooth spacing angles of gear no. 2 and pinion no. 3 respectively, the associated loop equation becomes (see Figures E-3 and G-4)

$$a_{G2}\left[\cos(\phi_2 - \Delta\phi_2 - \delta_{G2})\bar{i} + \sin(\phi_2 - \Delta\phi_2 - \delta_{G2})\bar{j}\right] + L_{x2}\bar{i} + L_{y2}\bar{j}$$

$$- a_{P2}\left[\cos(\psi_2 + \Delta\psi_2 - \delta_{P2})\bar{i} + \sin(\psi_2 + \Delta\psi_2 - \delta_{P2})\bar{j}\right]$$

$$- b_2\left[\cos\beta_2\bar{i} + \sin\beta_2\bar{j}\right] = 0 \tag{G-83}$$

Note that for mesh no. 2, the angular increment $\Delta\phi_2$ is negative while $\Delta\psi_2$ is positive. Further, as before, the angle $\psi_2$ must be

determined for the round on flat phase of the motion.

The magnitudes of $L_{x2}$ and $L_{y2}$ are determined from the components of equation (G-83), i.e.,

$$L_{x2} = b_2\cos\beta_2 + a_{P2}\cos(\psi_2 + \Delta\psi_2 - \delta_{P2}) - a_{G2}\cos(\phi_2 - \Delta\phi_2 - \delta_{G2})$$

(G-84)

while

$$L_{y2} = b_2\sin\beta_2 + a_{P2}\sin(\psi_2 + \Delta\psi_2 - \delta_{P2}) - a_{G2}\sin(\phi_2 - \Delta\phi_2 - \delta_{G2})$$

(G-85)

Contact will occur as soon as

$$\sqrt{L_{x2}^2 + L_{y2}^2} \leq \rho_{G2} + \rho_{P2}$$

(G-86)

## 3. KINEMATICS OF MESH NO. 3 (GEAR NO. 3 AND PINION NO. 4)

Since mesh no. 3 is kinematically equivalent to mesh no. 1, the kinematic equations for mesh no. 3 may be obtained from those for mesh no. 1. The angle $\beta_3$ must replace the angle $\beta_1$ and the center distance $b_3$ is used instead of $b_1$. All parameters of gear no. 1 are replaced by those of gear no. 3 and the pinion parameters of pinion no. 4 are substituted for those of pinion no. 2.

It is to be noted that the input angle $\phi_3$ of mesh no. 3 is identical to the output angle $\psi_2$ of mesh no. 2.

### a. ROUND ON ROUND PHASE OF MOTION

The ouput angle $\psi_3$ is obtained with the help of equation (G-12)

$$\psi_3 = 2 \tan^{-1} \frac{A_{3R} \pm \sqrt{A_{3R}^2 + B_{3R}^2 - C_{3R}^2}}{B_{3R} + C_{3R}} \qquad (G-87)$$

where

$$A_{3R} = u_{G3}\sin(\phi_3 + \delta_{G3} - \delta_{P3}) - b_3\sin(\beta_3 - \delta_{P3})$$

G-40

$$B_{3R} = a_{G3}\cos(\phi_3 + \delta_{G3} - \delta_{P3}) - b_3\cos(\beta_3 - \delta_{P3})$$

$$C_{3R} = \frac{a_{P3}^2 + a_{G3}^2 + b_3^2 - L_3^2 - 2a_{G3}b_3\cos(\phi_3 + \delta_{G3} - \beta_3)}{2\,a_{P3}}$$

and

$$L_3 = \rho_{G3} + \rho_{P3} \qquad\qquad\qquad\text{(G-88)}$$

The angle $\lambda_3$ may be found with the help of equation (G-13) or equation (G-14)

$$\lambda_3 = \cos^{-1}\left[\frac{b_3\cos\beta_3 + a_{P3}\cos(\psi_3 + \delta_{P3}) - a_{G3}\cos(\phi_3 + \delta_{G3})}{L_3}\right]$$
<div align="right">(G-89)</div>

or

$$\lambda_3 = \sin^{-1}\left[\frac{b_3\sin\beta_3 + a_{P3}\sin(\psi_3 + \delta_{P3}) - a_{G3}\sin(\phi_3 + \delta_{G3})}{L_3}\right]$$
<div align="right">(G-90)</div>

The angular velocity $\dot{\psi}_3$ is obtained from equation (G-15)

$$\dot{\psi}_3 = \dot{\phi}_3\left[\frac{B_{3RD}\cos\psi_3 - A_{3RD}\sin\psi_3 + C_{3RD}}{A_{3R}\cos\psi_3 - B_{3R}\sin\psi_3}\right]$$
<div align="right">(G-91)</div>

where

$$A_{3RD} = a_{G3}\cos(\phi_3 + \delta_{G3} - \delta_{P3})$$

$$B_{3RD} = a_{G3}\sin(\phi_3 + \delta_{G3} - \delta_{P3})$$

$$C_{3RD} = \frac{a_{G3}b_3\sin(\phi_3 + \delta_{G3} - \beta_3)}{a_{P3}}$$

The relative velocity $\bar{V}_{S3/T3_R}$ becomes, according to Equation (G-21),

$$\bar{V}_{S3/T3_R} = \left\{ \dot{\phi}_3\left[a_{G3}\cos(\phi_3 + \delta_{G3} - \lambda_3) + \rho_{G3}\right] \right.$$
$$\left. - \dot{\psi}_3\left[a_{P3}\cos(\psi_3 + \delta_{P3} - \lambda_3) - \rho_{P3}\right] \right\}\bar{n}_{N\lambda3}$$

$$(G-92)$$

where, according to equation (G-3),

$$\bar{n}_{N\lambda3} = -\sin\lambda_3\bar{i} + \cos\lambda_3\bar{j} \tag{G-93}$$

**b.  ROUND ON FLAT PHASE OF MOTION**

The output angle $\psi_3$ is obtained from equation (G-29)

$$\psi_3 = 2 \tan^{-1} \frac{A_{3F} \pm \sqrt{A_{3F}^2 + B_{3F}^2 - C_{3F}^2}}{B_{3F} + C_{3F}} \qquad \text{(G-94)}$$

where

$$A_{3F} = a_{G3}\cos(\phi_3 + \delta_{G3} + \alpha_{P3}) - b_3\cos(\beta_3 + \alpha_{P3})$$

$$B_{3F} = - a_{G3}\sin(\phi_3 + \delta_{G3} + \alpha_{P3}) + b_3\sin(\beta_3 + \alpha_{P3})$$

$$C_{3F} = \rho_{G3}$$

The distance $g_3$ becomes, according to equation (G-27),

$$g_3 = \frac{a_{G3}\sin(\phi_3 + \delta_{G3}) + \rho_{G3}\cos(\psi_3 - \alpha_{P3}) - b_3\sin\beta_3}{\sin(\psi_3 - \alpha_{P3})}$$

$$\text{(G-95)}$$

The angular velocity $\dot{\psi}_3$ for the round on flat phase of the motion is found from equation (G-30)

$$\dot{\psi}_3 = \dot{\phi}_3 \left[ \frac{A_{3FD}\sin\psi_3 + B_{3FD}\cos\psi_3}{A_{3F}\cos\psi_3 - B_{3F}\sin\psi_3} \right] \qquad (G-96)$$

where

$$A_{3FD} = a_{G3}\sin(\phi_3 + \delta_{G3} + \alpha_{P3})$$

$$B_{3FD} = a_{G3}\cos(\phi_3 + \delta_{G3} + \alpha_{P3})$$

The relative velocity $\overline{V}_{S3/T3_F}$ for the round on flat phase comes from equation (G-33)

$$\overline{V}_{S3/T3_F} = \dot{\phi}_3 \left[ a_{G3}\sin(\psi_3 - \alpha_{P3} - \phi_3 - \delta_{G3}) - \rho_{G3} \right] \bar{n}_{F3}$$

$$(G-97)$$

where

$$\bar{n}_{F3} = \cos(\psi_3 - \alpha_{P3})\bar{i} + \sin(\psi_3 - \alpha_{P3})\bar{j} \qquad (G-98)$$

according to equation (G-22).

The transition angle $\psi_{3T}$ is obtained by way of equation (G-39)

$$\psi_{3T} = 2\tan^{-1}\frac{A_{3T} \pm \sqrt{A_{3T}^2 + B_{3T}^2 - C_{3T}^2}}{B_{3T} + C_{3T}} \qquad (G-99)$$

where

$$A_{3T} = \rho_{G3}\cos(\beta_3 + \alpha_{P3}) + f_{P3}\sin(\beta_3 + \alpha_{P3})$$

$$B_{3T} = -\rho_{G3}\sin(\beta_3 + \alpha_{P3}) + f_{P3}\cos(\beta_3 + \alpha_{P3})$$

$$C_{3T} = \frac{a_{G3}^2 - \rho_{G3}^2 - b_3^2 - f_{P3}^2}{2 b_3}$$

The associated angle $\phi_{3T}$ may be obtained from equation (G-40) or from equation (G-41)

$$\phi_{3T} = \cos^{-1}\left[\frac{\rho_{G3}\sin(\psi_{3T} - \alpha_{P3}) + f_{P3}\cos(\psi_{3T} - \alpha_{P3}) + b_3\cos\beta_3}{a_{G3}}\right] - \delta_{G3}$$
$$(G-100)$$

or

$$\phi_{3T} = \sin^{-1}\left[\frac{-\rho_{G3}\cos(\psi_{3T} - \alpha_{P3}) + f_{P3}\sin(\psi_{3T} - \alpha_{P3}) + b_3\sin\beta_3}{a_{G3}}\right] - \delta_{G3}$$
$$(G-101)$$

G-45

Finally, the contact sensing equation is based on equations (G-44) - (G-46). Contact will occur, when

$$\sqrt{L_{x3}^2 + L_{y3}^2} \leq \rho_{G3} + \rho_{P3} \qquad \text{(G-102)}$$

where with the tooth spacing angles $\Delta\phi_3$ and $\Delta\psi_3$,

$$L_{x3} = b_3\cos\beta_3 + a_{P3}\cos(\psi_3 - \Delta\psi_3 + \delta_{P3}) - a_{G3}\cos(\phi_3 + \Delta\phi_3 + \delta_{G3}) \qquad \text{(G-103)}$$

and

$$L_{y3} = b_3\sin\beta_3 + a_{P3}\sin(\psi_3 - \Delta\psi_3 + \delta_{P3}) - a_{G3}\sin(\phi_3 + \Delta\phi_3 + \delta_{G3}) \qquad \text{(G-104)}$$

## APPENDIX H

### MOMENT INPUT-OUTPUT RELATIONSHIPS FOR TWO AND THREE STEP-UP GEAR TRAINS WITH TEETH OPERATING IN A SPIN ENVIRONMENT

The following gives the derivations for the moment input-output relationships of two and three step-up gear trains which operate in a spin environment.

Figure G-1 of Appendix G shows the basic configuration of a three step-up gear train. The input moment, $M_{in}$, which acts on gear no. 1, is held in equilibrium by the moment, $M_{o4}$, which acts on pinion no. 4.

Since in all three meshes there may either be round on round or round on flat type of contact, the force and moment analyses must account for various contact combinations. Table H-1 shows the eight different phase combinations which may occur in a three step-up gear train, and for which input-output relationships must be found. The two step-up gear train, which is shown in Figure A-10 of Appendix A for involute gearing, does not contain pinion 4 and gear no. 3. Here, the input moment $M_{in}$, which acts on gear no. 1, is held in equilibrium by moment $M_{o3}$, which acts on pinion no. 3.

| Case no. | Mesh No. 3 (Gear 3 & Pinion 4) | Mesh No. 2 (Gear 2 & Pinion 3) | Mesh No. 1 (Gear 1 & Pinion 2) |
|---|---|---|---|
| 1 | R | R | R |
| 2 | R | R | F |
| 3 | R | F | F |
| 4 | R | F | R |
| 5 | F | F | F |
| 6 | F | F | R |
| 7 | F | R | R |
| 8 | F | R | F |

TABLE H-1

POSSIBLE COMBINATIONS OF PHASES FOR THREE STEP-UP GEAR TRAIN AS SHOWN IN FIGURE G-1

R = Round on Round

F = Round on Flat

When ogival teeth are involved, there are four possible phase combinations of the two remaining meshes. These are shown in Table H-2. Again input-output relationships must be obtained for each of them.

| Case No. | Mesh No. 2 (Gear 2 & Pinion 3) | Mesh No. 1 (Gear 1 & Pinion 2) |
|---|---|---|
| 1 | R | R |
| 2 | R | F |
| 3 | F | F |
| 4 | F | R |

TABLE H-2

POSSIBLE COMBINATIONS OF PHASES FOR TWO STEP-UP GEAR TRAIN AS
SHOWN IN FIGURE A-10
R = Round on Round
F = Round on Flat

The unit vectors, mechanism angles and kinematic terms
necessary for the following analyses were derived in Appendix G.
(See also Appendix D for a description of the geometry of ogival
teeth.) Certain terms used in connection with mesh no. 3 may
be obtained from expressions derived for mesh no. 1 in Appendix G
by the replacement of the appropriate subscript numbers, since
the kinematics of these meshes are identical. The following
additional nomenclature is used:

$\mathcal{R}_i$ = distance from spin axis C to pivot points $O_i$ of individual gears. ($i$ = 1, 2, 3, 4 as applicalbe)

$\gamma_i$ = angle of lines $\mathcal{R}_i = CO_i$ with respect to the body-fixed X-axis

$\omega$ = spin velocity of fuze body

$m_i$ = mass of various gears, pinions and gear-pinion combinations

$Q_i$ = $m_i \mathcal{R}_i \omega^2$ , centrifugal force acting on individual gear components. (Now called $Q_i$ to differentiate it from the pinion contact point $T_i$.)

$\rho_i$ = pivot radius

$\rho_{Pi}$ = radius of curvature of pinion tooth (ogival)

$\rho_{Gi}$ = radius of curvature of gear tooth (ogival)

$\mu$ = coefficient of friction at pivots as well as at contact point between gears and pinions

The pivot friction moments are obtained according to equation (A-3b) of Appendix A. They always oppose motion regardless of the assumption of direction of the pivot reactions $F_{xi}$ and $F_{yi}$. To this end the pivot forces $\widetilde{F}_{xi}$ and $\widetilde{F}_{yi}$, which represent the sums of the absolute values of their component parts, are added

H-4

algebraically. The algebraic addition of such modified reactions provides a conservative, i.e., a somewhat overstated friction moment.

The directions of the friction forces of the gears on the pinions are always those of the relative velocities $\bar{V}_{S1/T1}$, where points $S_1$ and $T_1$ are located at the contact points of the gears and pinions, respectively. This allows the introduction of a signum convention. For the round on round phases,

$$s_{1R} = \frac{V_{S1/T1_R}}{\left|V_{S1/T1_R}\right|} \tag{H-1}$$

For the round on flat phase, the convention becomes

$$s_{1F} = \frac{V_{S1/T1_F}}{\left|V_{S1/T1_F}\right|} \tag{H-2}$$

The expressions for the above relative velocities, which are different for round on round and for round on flat, are given in Appendix G.

# 1. INPUT-OUTPUT ANALYSIS OF THREE STEP-UP GEAR TRAIN

## a. CASE NO. 1: RRR

### I. FORCE AND MOMENT EQUILIBRIA OF PINION NO. 4

Figure H-1 shows a schematic free body diagram of pinion no. 4 in the round on round mode of contact. The equivalent four-bar linkage associated with mesh no. 3 is also indicated. The pinion is acted upon by the equilibrant moment $M_{o4}$ in the direction opposite to its counterclockwise rotation. The contact force of gear no. 3 on the pinion is given by

$$\bar{F}_{34} = F_{34} \bar{n}_{\lambda 3} \tag{H-3}$$

The associated friction force exerted by the gear on the pinion has the direction of the relative velocity $\bar{V}_{S3/T3_R}$, as given by equation (G-92). With the use of the signum convention of equation (H-1) the friction force $\bar{F}_{f34}$ becomes

$$\bar{F}_{f34} = \mu s_{3R} F_{34} \bar{n}_{N\lambda 3} \tag{H-4}$$

Note: $\psi$ is positive in ccw direction
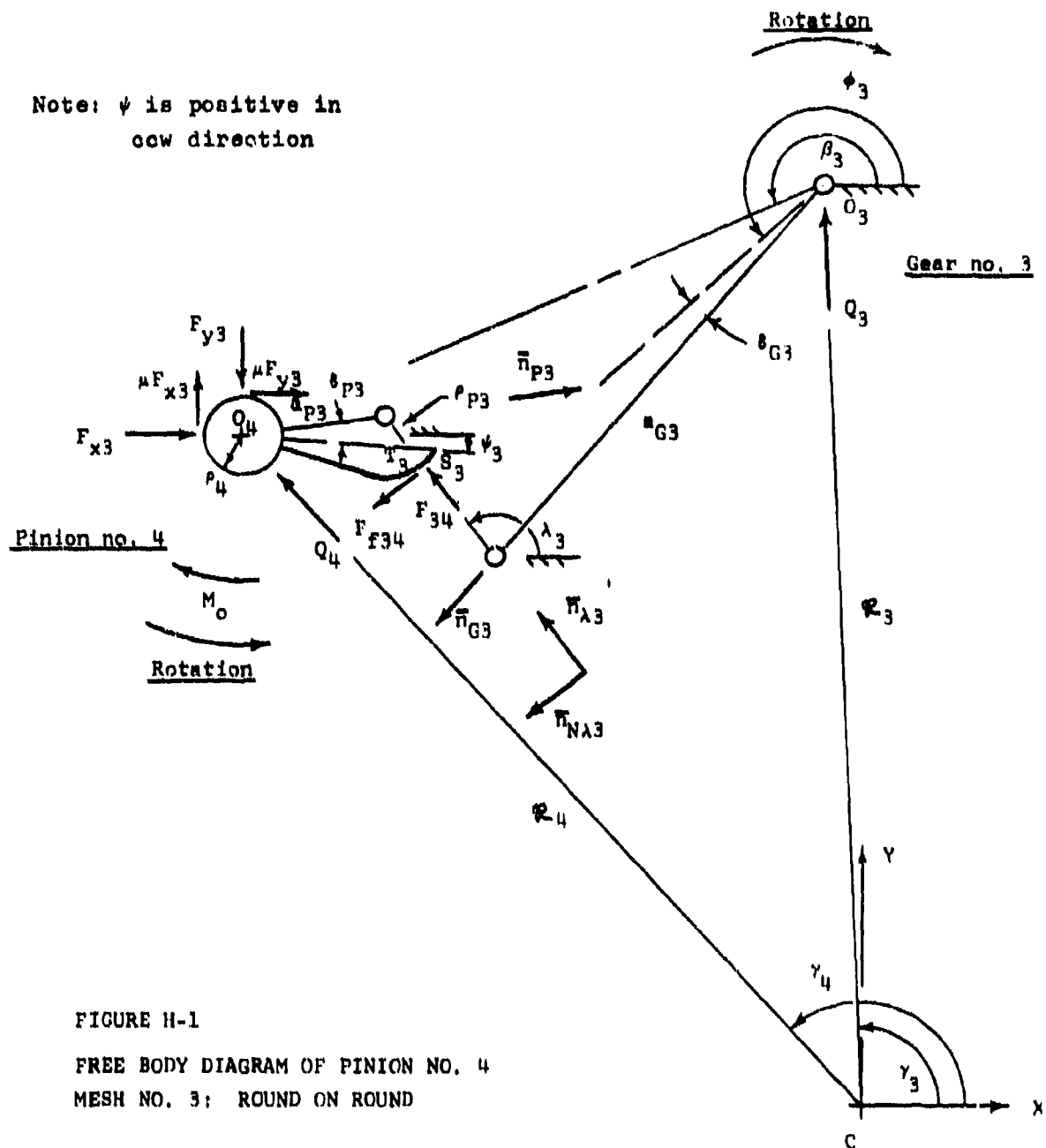
FIGURE H-1

FREE BODY DIAGRAM OF PINION NO. 4
MESH NO. 3: ROUND ON ROUND

The centrifugal force, due to the pinion mass, is given by

$$\overline{Q}_4 = Q_4(\cos\gamma_4\overline{i} + \sin\gamma_4\overline{j}) \qquad (H\text{-}5)$$

where

$$Q_4 = R_4 m_4 \omega^2 \qquad (H\text{-}6)$$

Force equilibrium is given by

$$F_{34}\overline{n}_{\lambda3} + \mu s_{3R}F_{34}\overline{n}_{N\lambda3} + F_{x4}\overline{I} + \mu F_{y4}\overline{I} - F_{y4}\overline{J} + \mu F_{x4}\overline{J} + \overline{Q}_4 = 0 \qquad (H\text{-}7)$$

Moment equilibrium requires the following:

$$-M_{o4}\overline{k} - \mu \rho_4(\widetilde{F}_{x4} + \widetilde{F}_{y4})\overline{k} + (a_{P3}\overline{n}_{P3} - \rho_{P3}\overline{n}_{\lambda3}) \times (F_{34}\overline{n}_{\lambda3} + \mu s_{3R}F_{34}\overline{n}_{N\lambda3})$$
$$= 0 \qquad (H\text{-}8)$$

Equation (H-7) furnishes the following component equations:

$$F_{34}\cos\lambda_3 - \mu s_{3R}F_{34}\sin\lambda_3 + F_{x4} + \mu F_{y4} + Q_4\cos\gamma_4 = 0 \qquad (H\text{-}9)$$

and

$$F_{34}\sin\lambda_3 + \mu s_{3R}F_{34}\cos\lambda_3 - F_{y4} + \mu F_{x4} + Q_4\sin\gamma_4 = 0 \qquad (H\text{-}10)$$

H-8

The scalar form of the moment equation becomes

$$-M_{o4} - \mu \rho_4 (\widetilde{F}_{x4} + \widetilde{F}_{y4}) - F_{34} \left\{ a_{P3} \left[ \sin(\psi_3 + \delta_{P3} - \lambda_3) - \mu a_{3R} \cos(\psi_3 + \delta_{P3} - \lambda_3) \right] \right.$$
$$\left. + \mu a_{3R} \rho_{P3} \right\} = 0 \qquad (H-11)$$

Simultaneous solution of equations (H-9) and (H-10) for $F_{x4}$ and $F_{y4}$ gives

$$F_{x4} = \frac{F_{34} \left[ \mu(a_{3R} - 1)\sin\lambda_3 - (1 + \mu^2 a_{3R})\cos\lambda_3 \right] - Q_4(\mu\sin\gamma_4 + \cos\gamma_4)}{1 + \mu^2}$$

$$(H-12)$$

and

$$F_{y4} = \frac{F_{34} \left[ (1 + \mu^2 a_{3R})\sin\lambda_3 + \mu(a_{3R} - 1)\cos\lambda_3 \right] + Q_4(\sin\gamma_4 - \mu\cos\gamma_4)}{1 + \mu^2}$$

$$(H-13)$$

The sum $\widetilde{F}_{x4} + \widetilde{F}_{y4}$ of equation (H-11) is now made up of equations (H-12) and (H-13) in the sense of equation (A-3b) of Appendix A

$$\widetilde{F}_{x4} + \widetilde{F}_{y4} = Q_4 A_1 + F_{34} A_2 + Q_4 A_3 + F_{34} A_4 \qquad (H-14)$$

$$A_1 = \left| \frac{\mu \sin\gamma_4 + \cos\gamma_4}{1 + \mu^2} \right| \qquad \text{(H-15)}$$

$$A_2 = \left| \frac{\mu(s_{3R} - 1)\sin\lambda_3 - (1 + \mu^2 s_{3R})\cos\lambda_3}{1 + \mu^2} \right| \qquad \text{(H-16)}$$

$$A_3 = \left| \frac{\sin\gamma_4 - \mu\cos\gamma_4}{1 + \mu^2} \right| \qquad \text{(H-17)}$$

$$A_4 = \left| \frac{(1 + \mu^2 s_{3R})\sin\lambda_3 + \mu(s_{3R} - 1)\cos\lambda_3}{1 + \mu^2} \right| \qquad \text{(H-18)}$$

Equation (H-14) is now substituted into the moment equation (H-11) and the latter is solved for the contact force $F_{34}$

$$F_{34} = \frac{M_{o4}}{C_2} + \frac{Q_4 C_1}{C_2} \qquad \text{(H-19)}$$

where

$$C_1 = \mu \rho_4 (A_1 + A_3)$$

$$C_2 = s_{P3}\left[\mu s_{3R}\cos(\psi_3 + \delta_{P3} - \lambda_3) - \sin(\psi_3 + \delta_{P3} - \lambda_3)\right]$$
$$- \mu\left[\rho_{P3}s_{3R} + \rho_4(A_2 + A_4)\right]$$

## II. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 3

Figure H-2 gives a schematic free body diagram of the gear and pinion combination no. 3. Mesh no. 2 is also indicated to obtain the directions of the forces of gear no. 2 on pinion no. 3. The forces of pinion no. 4 on gear no. 3 are opposite to those given by equations (H-3) and (H-4) respectively. Thus, the contact force becomes

$$\overline{F}_{43} = -\overline{F}_{34} = -F_{34}\overline{n}_{\lambda3} \qquad \text{(H-20)}$$

The friction force of pinion no. 4 on gear no. 3 becomes:

$$\overline{F}_{f43} = -\overline{F}_{f34} = -\mu s_{3R}F_{34}\overline{n}_{N\lambda3} \qquad \text{(H-21)}$$

The contact force of gear no. 2 on pinion no. 3 is given by

$$\overline{F}_{23} = F_{23}\overline{n}_{\lambda2} \qquad \text{(H-22)}$$

while the associated friction force of gear no. 2 on pinion no. 3 becomes

H-11

FIGURE H-2

FREE BODY DIAGRAM OF GEAR & PINION NO. 3
MESH NO. 3:   ROUND ON ROUND
MESH NO. 2:   ROUND ON ROUND

H-12

$$\overline{F}_{f23} = \mu s_{2R} F_{23} \overline{n}_{N\lambda 2} \qquad (H-23)$$

The pivot reactions $F_{x3}$ and $F_{y3}$ as well as the associated friction forces are drawn in a separate diagram in Figure H-2. As was the case earlier, the friction moment due to the friction forces again opposes rotation.

The centrifugal force, due to the mass of the combined gear and pinion no. 3, is given by

$$\overline{Q}_3 = Q_3 (\cos \gamma_3 \overline{I} + \sin \gamma_3 \overline{J}) \qquad (H-24)$$

where

$$Q_3 = \mathcal{R}_3 m_3 \omega^2 \qquad (H-25)$$

The force equilibrium of the combination is given by

$$-F_{34} \overline{n}_{\lambda 3} - \mu s_{3R} F_{34} \overline{n}_{N\lambda 3} + F_{23} \overline{n}_{\lambda 2} + \mu s_{2R} F_{23} \overline{n}_{N\lambda 2} + F_{x3} \overline{I} + \mu F_{y3} \overline{I}$$

$$+ F_{y3} \overline{J} - \mu F_{x3} \overline{J} + \overline{Q}_3 = 0 \qquad (H-26)$$

H-13

The moment equation is given by

$$\mu \rho_3 (\tilde{F}_{x3} + \tilde{F}_{y3})\bar{k} + [a_{G3}\bar{n}_{G3} + \rho_{G3}\bar{n}_{\lambda 3}] \times [-F_{34}\bar{n}_{\lambda 3} - \mu s_{3R}F_{34}\bar{n}_{N\lambda 3}]$$

$$+ [a_{P2}\bar{n}_{P2} - \rho_{P2}\bar{n}_{\lambda 2}] \times [F_{23}\bar{n}_{\lambda 2} + \mu s_{2R}F_{23}\bar{n}_{N\lambda 2}] = 0 \qquad \text{(H-27)}$$

Equation (H-26) gives the following component expressions:

$$-F_{34}\cos\lambda_3 + \mu s_{3R}F_{34}\sin\lambda_3 + Q_3\cos\gamma_3 + F_{x3} + \mu F_{y3} + F_{23}\cos\lambda_2$$

$$- \mu s_{2R}F_{23}\sin\lambda_2 = 0 \qquad \text{(H-28)}$$

and

$$-F_{34}\sin\lambda_3 - \mu s_{3R}F_{34}\cos\lambda_3 + Q_3\sin\gamma_3 + F_{y3} - \mu F_{x3} + F_{23}\sin\lambda_2$$

$$+ \mu s_{2R}F_{23}\cos\lambda_2 = 0 \qquad \text{(H-29)}$$

The scalar form of the moment equation (H-27) becomes

$$\mu \rho_3 (\tilde{F}_{x3} + \tilde{F}_{y3}) - \mu s_{3R}\rho_{G3}F_{34} - \mu s_{2R}\rho_{P2}F_{23} + a_{G3}F_{34}[\sin(\phi_3 + \delta_{G3} - \lambda_3)$$

$$- \mu s_{3R}\cos(\phi_3 + \delta_{G3} - \lambda_3)] + a_{P2}F_{23}[-\sin(\psi_2 - \delta_{P2} - \lambda_2)$$

$$+ \mu s_{2R}\cos(\psi_2 - \delta_{P2} - \lambda_2)] = 0 \qquad \text{(H-30)}$$

H-14

Simultaneous solution of equations (H-28) and (H-29) for the pivot reactions $F_{x3}$ and $F_{y3}$ gives

$$F_{x3} = \frac{1}{1 + \mu^2} \left\{ F_{34}\left[(1 - \mu^2 s_{3R})\cos\lambda_3 - \mu(1 + s_{3R})\sin\lambda_3\right] + \right.$$
$$F_{23}\left[\mu(1 + s_{2R})\sin\lambda_2 - (1 - \mu^2 s_{2R})\cos\lambda_2\right] +$$
$$\left. Q_3\left[\mu\sin\gamma_3 - \cos\gamma_3\right] \right\} \qquad \text{(H-31)}$$

and

$$F_{y3} = \frac{1}{1 + \mu^2} \left\{ F_{34}\left[(1 - \mu^2 s_{3R})\sin\lambda_3 + \mu(1 + s_{3R})\cos\lambda_3\right] + \right.$$
$$F_{23}\left[(\mu^2 s_{2R} - 1)\sin\lambda_2 - \mu(1 + s_{2R})\cos\lambda_2\right] -$$
$$\left. Q_3\left[\sin\gamma_3 + \mu\cos\gamma_3\right] \right\} \qquad \text{(H-32)}$$

The sum $\widetilde{F}_{x3} + \widetilde{F}_{y3}$ of equation (H-30) is now made up of equations (H-31) and (H-32) in the sense of equation (A-3b).

$$\widetilde{F}_{x3} + \widetilde{F}_{y3} = F_{34}A_5 + F_{23}A_6 + Q_3A_7 + F_{34}A_8 + F_{23}A_9 + Q_3A_{10}$$

$$\text{(H-33)}$$

where

$$A_5 = \left| \frac{(1 - \mu^2 s_{3R})\cos\lambda_3 - \mu(1 + s_{3R})\sin\lambda_3}{1 + \mu^2} \right| \qquad (H-34)$$

$$A_6 = \left| \frac{\mu(1 + s_{2R})\sin\lambda_2 - (1 - \mu^2 s_{2R})\cos\lambda_2}{1 + \mu^2} \right| \qquad (H-35)$$

$$A_7 = \left| \frac{\mu\sin\gamma_3 - \cos\gamma_3}{1 + \mu^2} \right| \qquad (H-36)$$

$$A_8 = \left| \frac{(1 - \mu^2 s_{3R})\sin\lambda_3 + \mu(1 + s_{3R})\cos\lambda_3}{1 + \mu^2} \right| \qquad (H-37)$$

$$A_9 = \left| \frac{(\mu^2 s_{2R} - 1)\sin\lambda_2 - \mu(1 + s_{2R})\cos\lambda_2}{1 + \mu^2} \right| \qquad (H-38)$$

$$A_{10} = \left| \frac{\sin\gamma_3 + \mu\cos\gamma_3}{1 + \mu^2} \right| \qquad (H-39)$$

Equation (H-33) is now substituted into equation (H-30) and the resulting expression is solved for the contact force $F_{23}$. Thus,

$$F_{23} = \frac{-F_{34}C_3 - Q_3 C_4}{C_5} \qquad (H-40)$$

where

$$C_3 = \mu\rho_3(A_5 + A_8) - \mu s_{3R}\rho_{G3} + s_{G3}\left[\sin(\phi_3 + \delta_{G3} - \lambda_3)\right.$$
$$\left. - \mu s_{3R}\cos(\phi_3 + \delta_{G3} - \lambda_3)\right]$$

$$C_4 = \mu\rho_3(A_7 + A_{10})$$

$$C_5 = \mu\rho_3(A_6 + A_9) - \mu s_{2R}\rho_{P2} + s_{P2}\left[\mu s_{2R}\cos(\psi_2 - \delta_{P2} - \lambda_2)\right.$$
$$\left. - \sin(\psi_2 - \delta_{P2} - \lambda_2)\right]$$

## III. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 2

Figure H-3 gives the free body diagram of the gear and pinion combination no. 2. In addition, mesh no. 1 is indicated to obtain the directions of the forces of gear no. 1 on pinion no. 2.

The forces of pinion no. 3 on gear no. 2 have directions opposite to those given by equation (H-22) and (H-23) respectively. Thus, the normal force is given by

$$\bar{F}_{32} = - F_{23}\bar{n}_{\lambda 2} \tag{H-41}$$

The friction force of pinion no. 3 on gear no. 2 becomes

$$\bar{F}_{f32} = - \mu s_{2R} F_{23}\bar{n}_{N\lambda 2} \tag{H-42}$$

The contact force of gear no. 1 on pinion no. 2 is given by

$$\bar{F}_{12} = F_{12}\bar{n}_{\lambda 1} \tag{H-43}$$

while the associated friction force of gear no. 1 on pinion no. 2 becomes

H-18
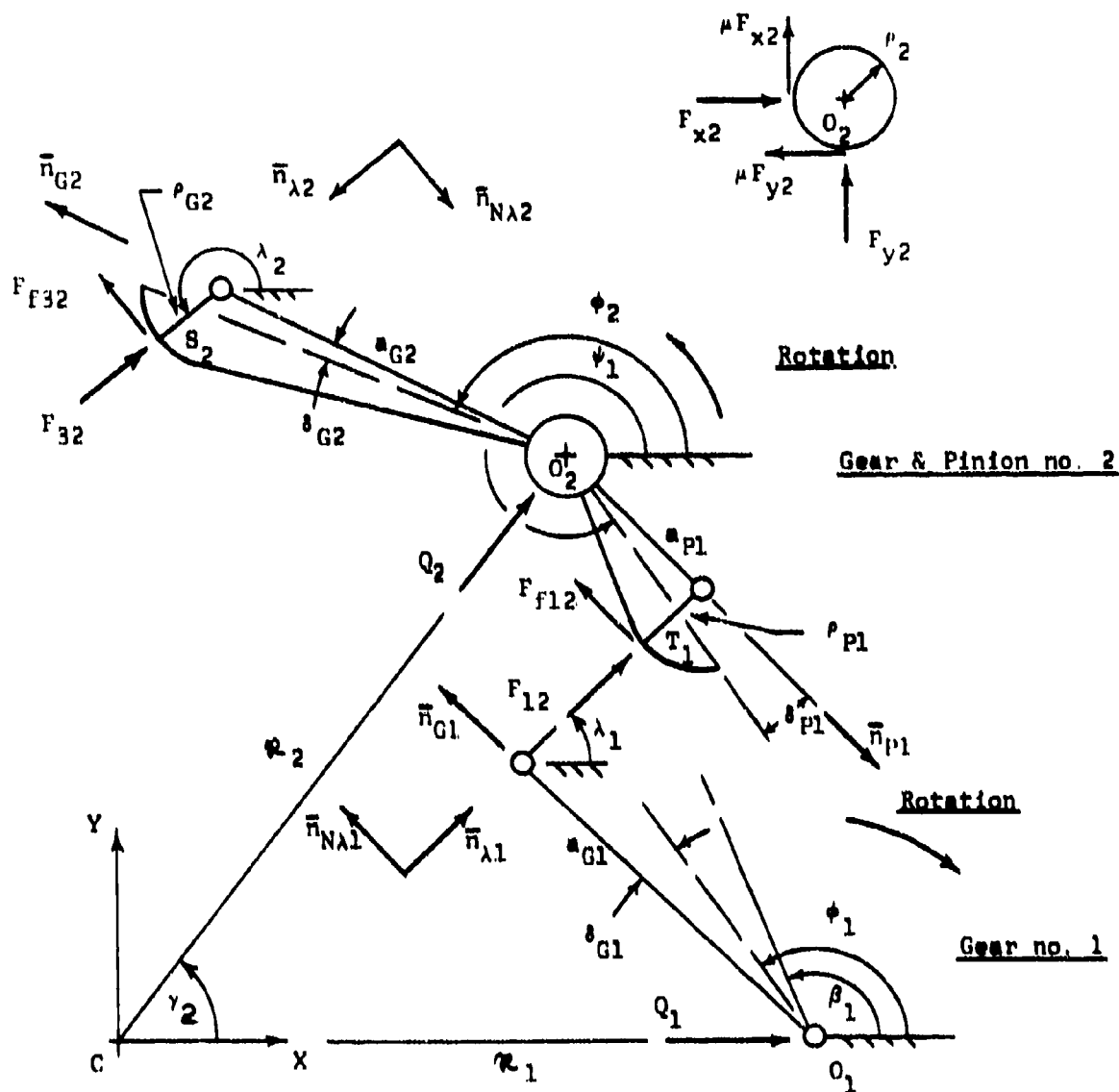
FIGURE H-3

FREE BODY DIAGRAM OF GEAR & PINION NO. 2
MESH NO. 2:   ROUND ON ROUND
MESH NO. 1:   ROUND ON ROUND

H-19

$$\bar{F}_{f12} = \mu s_{1R} F_{12} \bar{k}_{N\lambda 1} \qquad (H-44)$$

The pivot reactions, of the fuse body on the pivot shaft, together with the pivot friction forces, are shown in a separate diagram in Figure H-3. The centrifugal force, due to the mass of gear and pinion assembly no. 2, is given by

$$\bar{Q}_2 = Q_2(\cos\gamma_2\bar{I} + \sin\gamma_2\bar{J}) \qquad (H-45)$$

where

$$Q_2 = \mathcal{R}_2 m_2 \omega^2 \qquad (H-46)$$

Force equilibrium is assured by

$$-F_{23}\bar{n}_{\lambda 2} - \mu s_{2R} F_{23}\bar{n}_{N\lambda 2} + F_{12}\bar{n}_{\lambda 1} + \mu s_{1R} F_{12}\bar{n}_{N\lambda 1} + \bar{Q}_2$$

$$+ F_{x2}\bar{I} - \mu F_{y2}\bar{I} + F_{y2}\bar{J} + \mu F_{x2}\bar{J} = 0 \qquad (H-47)$$

H-20

Moment equilibrium must satisfy

$$-\mu \rho_2 (\tilde{F}_{x2} + \tilde{F}_{y2})\bar{k} + \left[ a_{G2}\bar{n}_{G2} + \rho_{G2}\bar{n}_{\lambda 2} \right] \times \left[ -F_{23}\bar{n}_{\lambda 2} - \mu a_{2R}F_{23}\bar{n}_{N\lambda 2} \right]$$

$$+ \left[ a_{P1}\bar{n}_{P1} - \rho_{P1}\bar{n}_{\lambda 1} \right] \times \left[ F_{12}\bar{n}_{\lambda 1} + \mu a_{1R}F_{12}\bar{n}_{N\lambda 1} \right] = 0 \qquad \text{(H-48)}$$

Equation (H-47) gives the following component expressions

$$-F_{23}\cos\lambda_2 + \mu a_{2R}F_{23}\sin\lambda_2 + F_{12}\cos\lambda_1 - \mu a_{1R}F_{12}\sin\lambda_1$$

$$+ Q_2\cos\gamma_2 + F_{x2} - \mu F_{y2} = 0 \qquad \text{(H-49)}$$

and

$$-F_{23}\sin\lambda_2 - \mu a_{2R}F_{23}\cos\lambda_2 + F_{12}\sin\lambda_1 + \mu a_{1R}F_{12}\cos\lambda_1$$

$$+ Q_2\sin\gamma_2 + F_{y2} + \mu F_{x2} = 0 \qquad \text{(H-50)}$$

The scalar form of the moment equation (H-48) becomes

$$-\mu \rho_2(\tilde{F}_{x2} + \tilde{F}_{y2}) + a_{G2}F_{23}\left[\sin(\phi_2 - \delta_{G2} - \lambda_2) - \mu s_{2R}\cos(\phi_2 - \delta_{G2} - \lambda_2)\right]$$

$$- \mu s_{2R}\rho_{G2}F_{23} + a_{P1}F_{12}\left[-\sin(\psi_1 + \delta_{P1} - \lambda_1) + \mu s_{1R}\cos(\psi_1 + \delta_{P1} - \lambda_1)\right]$$

$$- \mu s_{1R}\rho_{P1}F_{12} = 0 \qquad\qquad\qquad (H-51)$$

Simultaneous solution of equations (H-49) and (H-50) for $F_{x2}$ and $F_{y2}$ leads to

$$F_{x2} = \frac{1}{1 + \mu^2}\left\{F_{23}\left[(1 + \mu^2 s_{2R})\cos\lambda_2 - \mu(s_{2R} - 1)\sin\lambda_2\right]\right.$$

$$+ F_{12}\left[\mu(s_{1R} - 1)\sin\lambda_1 - (1 + \mu^2 s_{1R})\cos\lambda_1\right]$$

$$+ \left. Q_2\left[-\cos\gamma_2 - \mu\sin\gamma_2\right]\right\} \qquad\qquad (H-52)$$

and

$$F_{y2} = \frac{1}{1 + \mu^2}\left\{F_{23}\left[(1 + \mu^2 s_{2R})\sin\lambda_2 - \mu(1 - s_{2R})\cos\lambda_2\right]\right.$$

$$+ F_{12}\left[\mu(1 - s_{1R})\cos\lambda_1 - (1 + \mu^2 s_{1R})\sin\lambda_1\right]$$

$$+ \left. Q_2\left[\mu\cos\gamma_2 - \sin\gamma_2\right]\right\} \qquad\qquad (H-53)$$

The sum of $\tilde{F}_{x2}$ and $\tilde{F}_{y2}$ of equation (H-51) is now made up of equations (H-52) and (H-53) in the sense of equation (A-3b) of Appendix A

$$\tilde{F}_{x2} + \tilde{F}_{y2} = F_{23}A_{11} + F_{12}A_{12} + Q_2A_{13} + F_{23}A_{14} + F_{12}A_{15}$$

$$+ \; Q_2A_{16} \tag{H-54}$$

where

$$A_{11} = \left| \frac{(1 + \mu^2 s_{2R})\cos\lambda_2 - \mu(s_{2R} - 1)\sin\lambda_2}{1 + \mu^2} \right| \tag{H-55}$$

$$A_{12} = \left| \frac{\mu(s_{1R} - 1)\sin\lambda_1 - (1 + \mu^2 s_{1R})\cos\lambda_1}{1 + \mu^2} \right| \tag{H-56}$$

$$A_{13} = \left| \frac{- \cos\gamma_2 - \mu\sin\gamma_2}{1 + \mu^2} \right| \tag{H-57}$$

$$A_{14} = \left| \frac{(1 + \mu^2 s_{2R})\sin\lambda_2 - \mu(1 - s_{2R})\cos\lambda_2}{1 + \mu^2} \right| \tag{H-58}$$

$$A_{15} = \left| \frac{\mu(1 - s_{1R})\cos\lambda_1 - (1 + \mu^2 s_{1R})\sin\lambda_1}{1 + \mu^2} \right| \tag{H-59}$$

H-23

$$A_{16} = \left| \frac{\mu \cos \gamma_2 - \sin \gamma_2}{1 + \mu^2} \right| \qquad (H-60)$$

Equation (H-54) is now substituted into the moment equation (H-51). The resulting expression is then solved for the contact force $F_{12}$

$$F_{12} = \frac{-F_{23}C_6 + Q_2 C_7}{C_8} \qquad (H-61)$$

where

$$C_6 = a_{G2}\left[\sin(\phi_2 - \delta_{G2} - \lambda_2) - \mu s_{2R}\cos(\phi_2 - \delta_{G2} - \lambda_2)\right]$$
$$- \mu \rho_2 (A_{11} + A_{14}) - \mu s_{2R} \rho_{G2}$$

$$C_7 = \mu \rho_2 (A_{13} + A_{16})$$

$$C_8 = -\left\{ a_{P1}\left[\sin(\psi_1 + \delta_{P1} - \lambda_1) - \mu s_{1R}\cos(\psi_1 + \delta_{P1} - \lambda_1)\right] \right.$$
$$\left. + \mu \rho_2 (A_{12} + A_{15}) + \mu s_{1R} \rho_{P1} \right\}$$

## IV. FORCE AND MOMENT EQUILIBRIA OF INPUT GEAR NO. 1

Figure H-4 represents the free body diagram of the input gear no. 1 which has the input moment $M_{in}$ acting on it.

The forces of pinion no. 2 on gear no. 1 are given according to equations (H-43) and (H-44),

$$\bar{F}_{21} = - F_{12}\bar{n}_{\lambda 1} \tag{H-62}$$

and

$$\bar{F}_{f21} = - \mu s_{1R} F_{12}\bar{n}_{N\lambda 1} \tag{H-63}$$

The moments due to the friction forces on the pivot oppose rotation as indicated.

The centrifugal force, due to the mass of gear no. 1, is given by

$$\bar{Q}_1 = Q_1 \bar{i} \tag{H-64}$$

where

$$Q_1 = \mathcal{R}_1 m_1 \omega^2 \tag{H-65}$$
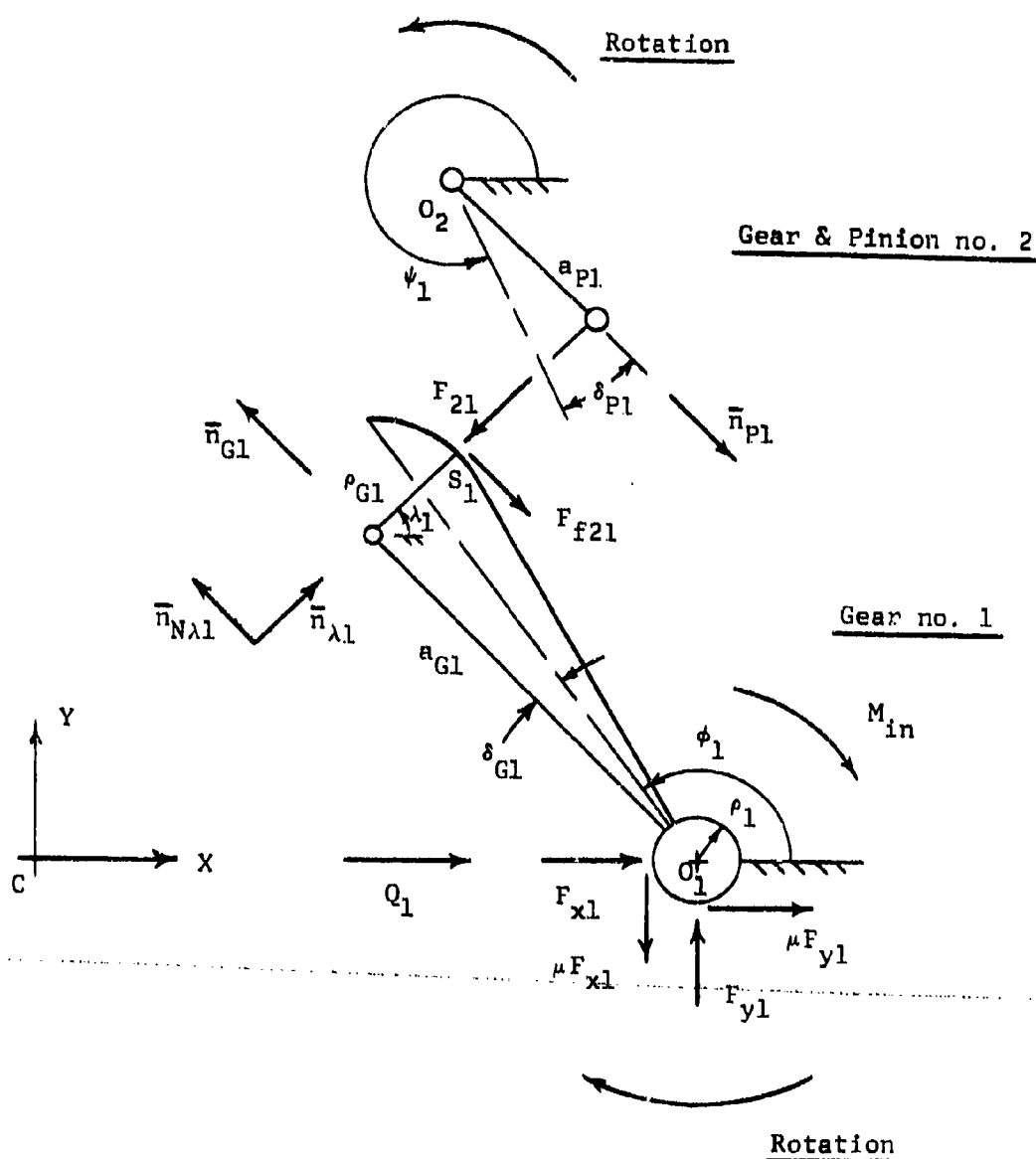
FIGURE H-4

FREE BODY DIAGRAM OF GEAR NO. 1
MESH NO. 1:  ROUND ON ROUND

Force equilibrium requires, that

$$-F_{12}\bar{n}_{\lambda 1} - \mu s_{1R}F_{12}\bar{n}_{N\lambda 1} + \bar{Q}_1 + F_{x1}\bar{I} + \mu F_{y1}\bar{I} + F_{y1}\bar{J}$$

$$- \mu F_{x1}\bar{J} = 0 \qquad (H-66)$$

Moment equilibrium is given by.

$$\mu \rho_1 (\widetilde{F}_{x1} + \widetilde{F}_{y1})\bar{k} - M_{1n}\bar{k} + \left[a_{G1}\bar{n}_{G1} + \rho_{G1}\bar{n}_{\lambda 1}\right] \times$$

$$\left[-F_{12}\bar{n}_{\lambda 1} - \mu s_{1R}F_{12}\bar{n}_{N\lambda 1}\right] = 0 \qquad (H-67)$$

Equation (H-66) furnishes the following component equations:

$$-F_{12}\cos\lambda_1 + \mu s_{1R}F_{12}\sin\lambda_1 + Q_1 + F_{x1} + \mu F_{y1} = 0 \qquad (H-68)$$

and

$$-F_{12}\sin\lambda_1 - \mu s_{1R}F_{12}\cos\lambda_1 + F_{y1} - \mu F_{x1} = 0 \qquad (H-69)$$

The scalar form of equation (H-67) becomes

$$\mu \rho_1 (\widetilde{F}_{x1} + \widetilde{F}_{y1}) - M_{1n} + a_{G1}F_{12}\left[\sin(\phi_1 + \delta_{G1} - \lambda_1)\right.$$

$$\left. - \mu s_{1R}\cos(\phi_1 + \delta_{G1} - \lambda_1)\right] - \mu s_{1R}\rho_{G1}F_{12} = 0 \qquad (H-70)$$

H-27

The simultaneous solution of equations (H-68) and (H-69) furnishes

$$F_{x1} = \frac{1}{1 + \mu^2} \left\{ F_{12} \left[ (1 - \mu^2 s_{1R})\cos\lambda_1 - \mu(1 + s_{1R})\sin\lambda_1 \right] - Q_1 \right\}$$

(H-71)

and

$$F_{y1} = \frac{1}{1 + \mu^2} \left\{ F_{12} \left[ (1 - \mu^2 s_{1R})\sin\lambda_1 + \mu(1 + s_{1R})\cos\lambda_1 \right] - \mu Q_1 \right\}$$

(H-72)

Then, with the same reasoning as before,

$$\widetilde{F}_{x1} + \widetilde{F}_{y1} = F_{12}A_{17} + Q_1 A_{18} + F_{12}A_{19} + Q_1 A_{20}$$

(H-73)

where

$$A_{17} = \left| \frac{(1 - \mu^2 s_{1R})\cos\lambda_1 - \mu(1 + s_{1R})\sin\lambda_1}{1 + \mu^2} \right|$$

(H-74)

$$A_{18} = \left| \frac{1}{1 + \mu^2} \right|$$

(H-75)

H-28

$$A_{19} = \left| \frac{(1 - \mu^2 s_{1R})\sin\lambda_1 + \mu(1 + s_{1R})\cos\lambda_1}{1 + \mu^2} \right| \quad\quad (H-76)$$

$$A_{20} = \left| \frac{\mu}{1 + \mu^2} \right| \quad\quad (H-77)$$

Equation (H-73) is now substituted into equation (H-70) and the result is solved for the contact force $F_{12}$

$$F_{12} = \frac{M_{1n} - Q_1 C_9}{C_{10}} \quad\quad (H-78)$$

where

$$C_9 = \mu \rho_1 (A_{18} + A_{20})$$

$$C_{10} = \mu \rho_1 (A_{17} + A_{19}) + a_{G1}\left[\sin(\phi_1 + \delta_{G1} - \lambda_1) - \mu s_{1R}\cos(\phi_1 + \delta_{G1} - \lambda_1)\right] - \mu s_{1R} \rho_{G1}$$

H-29

# V. MOMENT INPUT-OUTPUT RELATIONSHIP

Equations (H-61) and (H-78), which are both expressions in $F_{12}$, are now set equal to each other and the result is solved for $F_{23}$

$$F_{23} = \frac{-c_8}{c_6 c_{10}} (M_{1n} - Q_1 c_9) + \frac{c_7}{c_6} Q_2 \qquad (H-79)$$

The above is now equated to equation (H-40) and the result is solved for $F_{34}$

$$F_{34} = \frac{c_5 c_8 (M_{1n} - Q_1 c_9) - c_5 c_{10} c_7 Q_2 - c_4 c_6 c_{10} Q_3}{c_3 c_6 c_{10}} \qquad (H-80)$$

Finally equation (H-80) is equated to equation (H-19). This permits the determination of the equilibrant moment $M_{o41}$ (for case no. 1: RRR)

$$M_{o41} = M_{1n} \frac{c_2 c_5 c_8}{c_3 c_6 c_{10}} - Q_1 \frac{c_2 c_5 c_8 c_9}{c_3 c_6 c_{10}} - Q_2 \frac{c_2 c_5 c_7}{c_3 c_6} - Q_3 \frac{c_2 c_4}{c_3} - Q_4 c_1$$

$$(H-81)$$

**b. CASE NO. 2: RRF**

Since only mesh 1 is now assumed to be in the round on flat phase of motion, the forces $F_{34}$ and $F_{23}$ remain as given by equations (H-19) and (H-40), respectively.

**I. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 2**

Figure H-5 shows the free body diagram of the gear and pinion set with the necessary portions of mesh no. 1.

The forces of pinion no. 3 on gear no. 2 are given by equations (H-41) and (H-42), i.e.,

$$\bar{F}_{32} = -F_{23}\bar{n}_{\lambda 2} \qquad\qquad (H-82)$$

and

$$\bar{F}_{f32} = -\mu s_{2R} F_{23}\bar{n}_{N\lambda 2} \qquad\qquad (H-83)$$
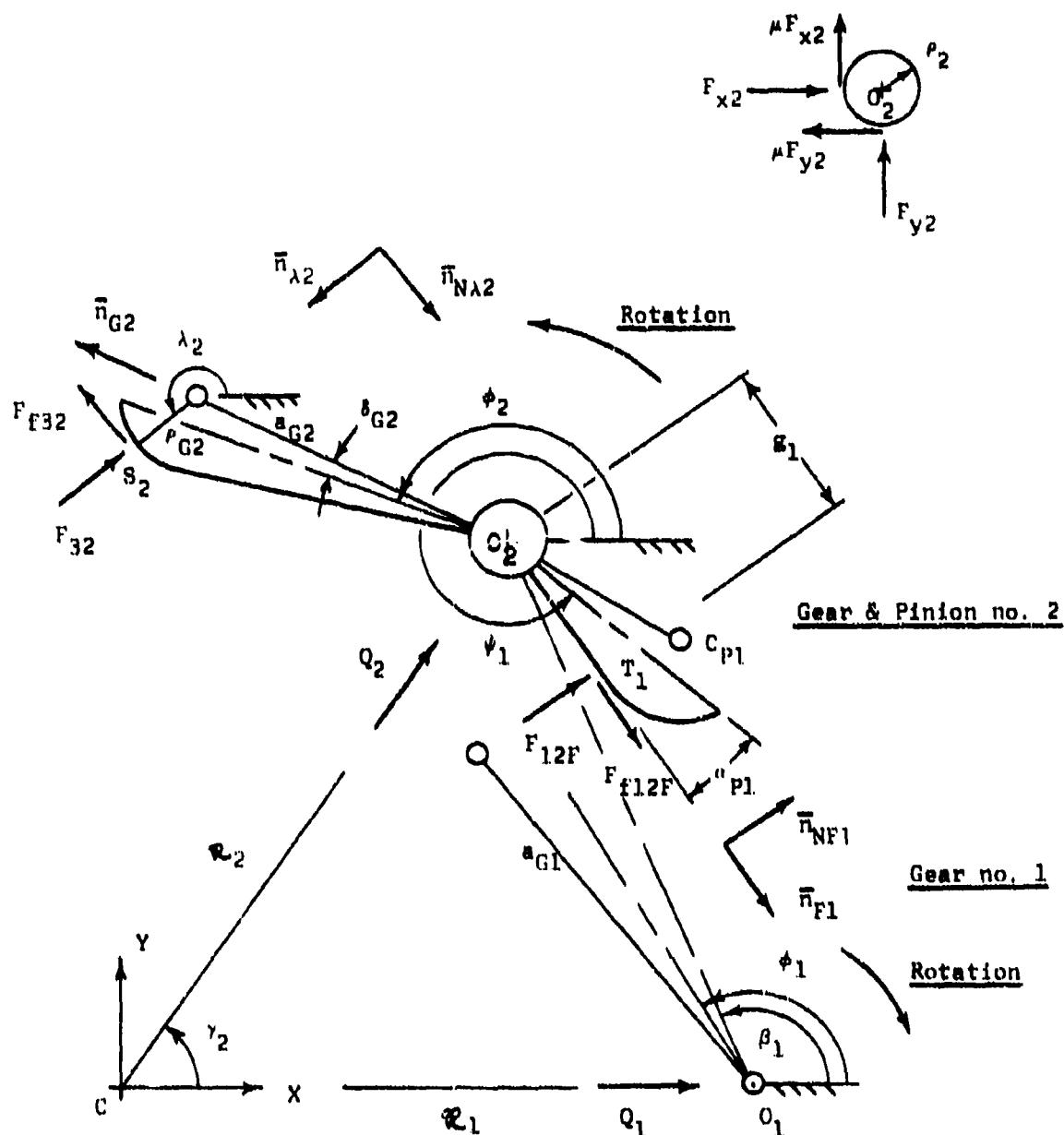
FIGURE H-5

FREE BODY DIAGRAM OF GEAR & PINION NO. 2
MESH NO. 2:  ROUND ON ROUND
MESH NO. 1:  ROUND ON FLAT

H-32

The contact force of gear no. 1 on pinion no. 2 is now given by

$$\bar{F}_{12F} = F_{12F}\bar{n}_{NF1} \qquad (H-84)$$

(Note that the additional subscript F is introduced to distinguish round on flat from round on round contact.)

The associated friction force is given by

$$\bar{F}_{f12F} = \mu s_{1F}F_{12F}\bar{n}_{F1} \qquad (H-85)$$

(See equation (H-2) for $s_{1F}$.)

The pivot reactions, together with the pivot friction forces, are shown in a separate diagram in Figure H-5. As before, the pivot friction moments oppose rotation.

The centrifugal force $\bar{Q}_2$ is again given by equations (H-45) and (H-46).

Force equilibrium is given by

$$-F_{23}\bar{n}_{\lambda 2} - \mu s_{2R}F_{23}\bar{n}_{N\lambda 2} + F_{12F}\bar{n}_{NF1} + \mu s_{1F}F_{12F}\bar{n}_{F1} + \bar{Q}_2$$

$$+ F_{x2}\bar{I} - \mu F_{y2}\bar{I} + F_{y2}\bar{J} + \mu F_{x2}\bar{J} = 0 \qquad (H-86)$$

H-33

Moment equilibrium about point $O_2$ requires

$$-\mu\rho_2(\tilde{F}_{x2} + \tilde{F}_{y2})\bar{k} + \left[a_{G2}\bar{n}_{G2} + \rho_{G2}\bar{n}_{\lambda 2}\right] \times \left[-F_{23}\bar{n}_{\lambda 2} - \mu s_{2R}F_{23}\bar{n}_{N\lambda 2}\right]$$

$$+ \quad s_1\bar{n}_{F1} \times F_{12F}\bar{n}_{NF1} \quad = \quad 0 \qquad\qquad (H-87)$$

Note that since the line of action of the friction force $F_{f12F}$ passes through point $O_2$, this friction force exerts no moment about point $O_2$.

Equation (H-86) furnishes the following component equations:

$$-F_{23}\cos\lambda_2 + \mu s_{2R}F_{23}\sin\lambda_2 - F_{12F}\sin(\psi_1 - \alpha_{P1}) + \mu s_{1F}F_{12F}\cos(\psi_1 - \alpha_{P1})$$

$$+ \quad F_{x2} - \mu F_{y2} + Q_2\cos\gamma_2 \quad = \quad 0 \qquad\qquad (H-88)$$

and

$$-F_{23}\sin\lambda_2 - \mu s_{2R}F_{23}\cos\lambda_2 + F_{12F}\cos(\psi_1 - \alpha_{P1}) + \mu s_{1F}F_{12F}\sin(\psi_1 - \alpha_{P1})$$

$$+ \quad F_{y2} + \mu F_{x2} + Q_2\sin\gamma_2 \quad = \quad 0 \qquad\qquad (H-89)$$

The scalar form of the moment equation (H-87) becomes

$$-\mu\rho_2(\tilde{F}_{x2} + \tilde{F}_{y2}) + a_{G2}F_{23}[\sin(\phi_2 - \delta_{G2} - \lambda_2) - \mu a_{2R}\cos(\phi_2 - \delta_{G2} - \lambda_2)]$$

$$- \mu a_{2R}a_{G2}F_{23} + F_{12F}\delta_1 = 0 \qquad (H-90)$$

Simultaneous solution of these component equations for $F_{x2}$ and $F_{y2}$ leads to

$$F_{x2} = \frac{1}{1 + \mu^2}\left\{F_{23}\left[\mu(1 - a_{2R})\sin\lambda_2 + (1 + \mu^2 a_{2R})\cos\lambda_2\right]\right.$$

$$+ F_{12F}\left[(1 - \mu^2 a_{1F})\sin(\psi_1 - \alpha_{P1}) - \mu(1 + a_{1F})\cos(\psi_1 - \alpha_{P1})\right]$$

$$\left. - Q_2\left[\mu\sin\gamma_2 + \cos\gamma_2\right]\right\} \qquad (H-91)$$

and

$$F_{y2} = \frac{1}{1 + \mu^2}\left\{F_{23}\left[(1 + \mu^2 a_{2R})\sin\lambda_2 - \mu(1 - a_{2R})\cos\lambda_2\right]\right.$$

$$+ F_{12F}\left[-\mu(1 + a_{1F})\sin(\psi_1 - \alpha_{P1}) + (\mu^2 a_{1F} - 1)\cos(\psi_1 - \alpha_{P1})\right]$$

$$\left. + Q_2\left[-\sin\gamma_2 + \mu\cos\gamma_2\right]\right\} \qquad (H-92)$$

H-35

The sum $(\tilde{F}_{x2} + \tilde{F}_{y2})$ of equation (H-90) is now made up of equations (H-91) and (H-92) in the sense of equation (A-3b) of Appendix A

$$\tilde{F}_{x2} + \tilde{F}_{y2} = F_{23}A_{21} + F_{12F}A_{22} + Q_2 A_{23} + F_{23}A_{24} + F_{12F}A_{25} + Q_2 A_{26}$$

(H-93)

where

$$A_{21} = \left| \frac{\mu(1 - s_{2R})\sin\lambda_2 + (1 + \mu^2 s_{2R})\cos\lambda_2}{1 + \mu^2} \right|$$

(H-94)

$$A_{22} = \left| \frac{(1 - \mu^2 s_{1F})\sin(\psi_1 - \alpha_{P1}) - \mu(1 + s_{1F})\cos(\psi_1 - \alpha_{P1})}{1 + \mu^2} \right|$$

(H-95)

$$A_{23} = \left| \frac{\mu\sin\gamma_2 + \cos\gamma_2}{1 + \mu^2} \right|$$

(H-96)

$$A_{24} = \left| \frac{(1 + \mu^2 s_{2R})\sin\lambda_2 - \mu(1 - s_{2R})\cos\lambda_2}{1 + \mu^2} \right|$$

(H-97)

$$A_{25} = \left| \frac{-\mu(1 + s_{1F})\sin(\psi_1 - \alpha_{P1}) + (\mu^2 s_{1F} - 1)\cos(\psi_1 - \alpha_{P1})}{1 + \mu^2} \right|$$

(H-98)

H-36

$$A_{26} = \left| \frac{-\sin\gamma_2 + \mu\cos\gamma_2}{1 + \mu^2} \right| \tag{H-99}$$

Equation (H-93) is now substituted into equation (H-90), and the resulting expression is solved for the contact force $F_{12F}$

$$F_{12F} = \frac{-F_{23}C_{11} + Q_2 C_{12}}{C_{13}} \tag{H-100}$$

where

$$C_{11} = a_{G2}\left[\sin(\phi_2 - \delta_{G2} - \lambda_2) - \mu a_{2R}\cos(\phi_2 - \delta_{G2} - \lambda_2)\right]$$
$$- \mu P_2(A_{21} + A_{24}) - \mu a_{2R}\rho_{G2}$$

$$C_{12} = \mu P_2(A_{23} + A_{26})$$

$$C_{13} = B_1 - \mu P_2(A_{22} + A_{25})$$

## II. FORCE AND MOMENT EQUILIBRIA OF INPUT GEAR NO. 1

Figure H-6 represents the free body diagram of input gear no. 1.

The forces of pinion no. 2 on this gear are given, according to equations (H-84) (H-85).

$$\bar{F}_{21F} = -F_{12F}\bar{n}_{NF1} \qquad (H-101)$$

and

$$\bar{F}_{f21F} = -\mu s_{1F}F_{12F}\bar{n}_{F1} \qquad (H-102)$$

The moments due to the pivot friction forces oppose the indicated rotation due to the moment $M_{in}$.

The centrifugal force $\bar{Q}_1$ has been defined by equations (H-64) and (H-65).

The force equilibrium equation is given by

$$-F_{12F}\bar{n}_{NF1} - \mu s_{1F}F_{12F}\bar{n}_{F1} + Q_1\bar{I} + F_{x1}\bar{I} + \mu F_{y1}\bar{I} + F_{y1}\bar{J} - \mu F_{x1}\bar{J}$$

$$= 0 \qquad (H-103)$$

H-38
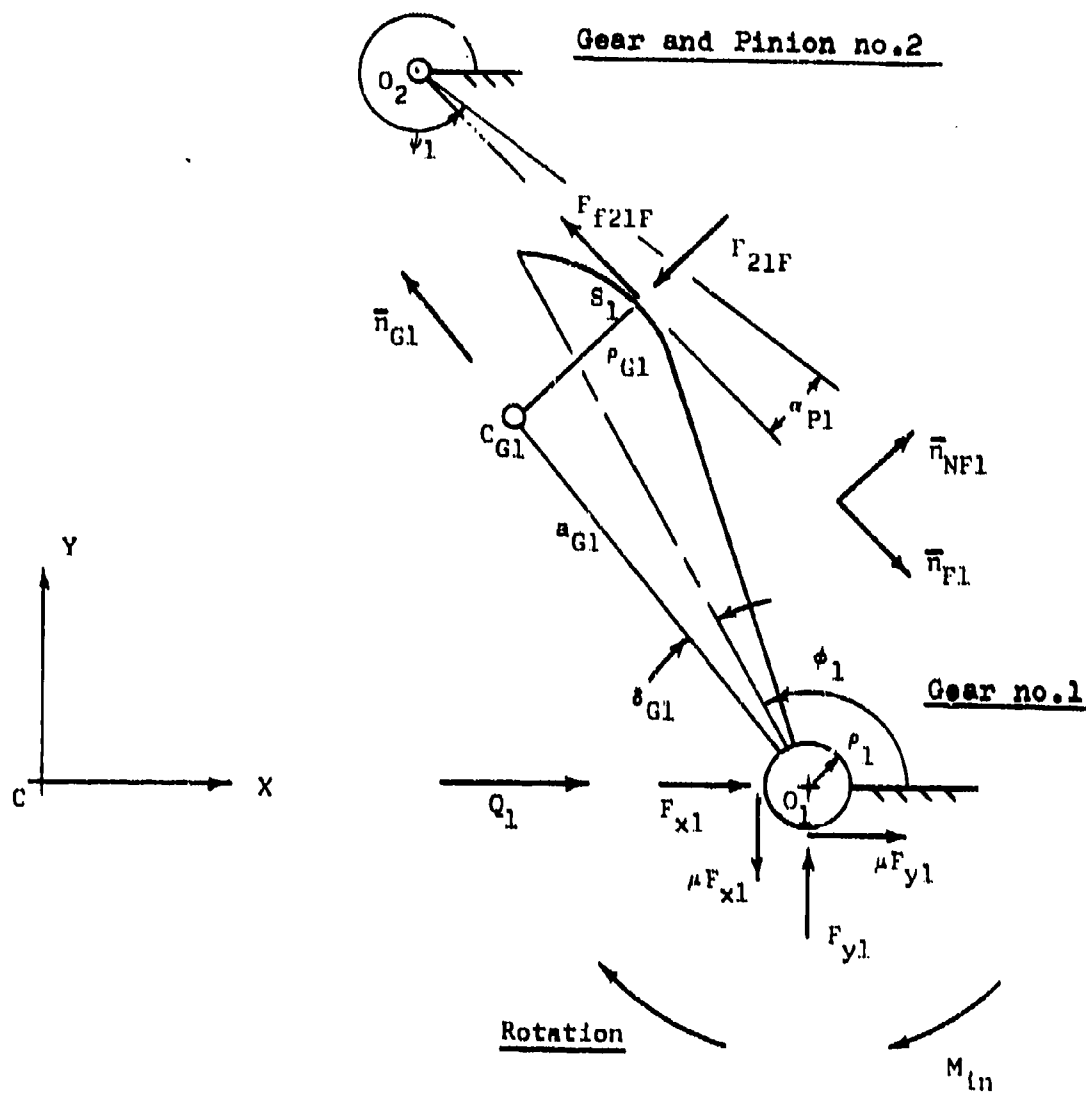
FIGURE H-6

FREE BODY DIAGRAM OF GEAR NO. 1
MECH NO. 1:   ROUND ON FLAT

H-39

The moment equilibrium equation becomes

$$-M_{1n}\bar{k} + \mu P_1(\tilde{F}_{x1} + \tilde{F}_{y1})\bar{k} + \left[a_{G1}\bar{n}_{G1} + \rho_{G1}\bar{n}_{NF1}\right] \times \left[-F_{12F}\bar{n}_{NF1} - \mu s_{1F}F_{12F}\bar{n}_{F1}\right]$$

$$= 0 \qquad\qquad (H-104)$$

Equation (H-103) furnishes the following component expressions

$$F_{12F}\sin(\psi_1 - \alpha_{P1}) - \mu s_{1F}F_{12F}\cos(\psi_1 - \alpha_{P1}) + Q_1 + F_{x1} + \mu F_{y1}$$

$$= 0 \qquad\qquad (H-105)$$

and

$$-F_{12F}\cos(\psi_1 - \alpha_{P1}) - \mu s_{1F}F_{12F}\sin(\psi_1 - \alpha_{P1}) + F_{y1} - \mu F_{x1}$$

$$= 0 \qquad\qquad (H-106)$$

The scalar form of the moment equation (H-104) becomes

$$-M_{1n} + \mu P_1(\tilde{F}_{x1} + \tilde{F}_{y1}) + \mu s_{1F}\rho_{G1}F_{12F} + a_{G1}F_{12F}\left[-\cos(\phi_1 + \delta_{G1} - \psi_1 + \alpha_{P1})\right.$$

$$\left. + \mu s_{1F}\sin(\phi_1 + \delta_{G1} - \psi_1 + \alpha_{P1})\right] = 0$$

$$(H-107)$$

Simultaneous solution of equations (H-105) and (H-106) for $F_{x1}$ and $F_{y1}$ furnishes

$$F_{x1} = \frac{F_{12F}\left[-(1 + \mu^2 s_{1F})\sin(\psi_1 - \alpha_{P1}) + \mu(s_{1F} - 1)\cos(\psi_1 - \alpha_{P1})\right] - Q_1}{1 + \mu^2}$$

$$(H-108)$$

H-40

and

$$F_{y1} = \frac{F_{12F}\left[(1 + \mu^2 s_{1F})\cos(\psi_1 - \alpha_{P1}) + \mu(s_{1F} - 1)\sin(\psi_1 - \alpha_{P1})\right] - \mu Q_1}{1 + \mu^2}$$

$$(H-109)$$

Now let

$$\widetilde{F}_{x1} + \widetilde{F}_{y1} = F_{12F}A_{27} + Q_1 A_{28} + F_{12F}A_{29} + Q_1 A_{30} \qquad (H-110)$$

where

$$A_{27} = \left| \frac{-(1 + \mu^2 s_{1F})\sin(\psi_1 - \alpha_{P1}) + \mu(s_{1F} - 1)\cos(\psi_1 - \alpha_{P1})}{1 + \mu^2} \right| \qquad (H-111)$$

$$A_{28} = \left| \frac{1}{1 + \mu^2} \right| \qquad (H-112)$$

$$A_{29} = \left| \frac{\mu(s_{1F} - 1)\sin(\psi_1 - \alpha_{P1}) + (1 + \mu^2 s_{1F})\cos(\psi_1 - \alpha_{P1})}{1 + \mu^2} \right| \qquad (H-113)$$

$$A_{30} = \left| \frac{\mu}{1 + \mu^2} \right| \qquad (H-114)$$

Equation (H-110) is now substituted into the moment equation (H-107).
This furnishes

$$F_{12F} = \frac{M_{1n} - Q_1 C_{14}}{C_{15}} \qquad (H-115)$$

where

$$C_{14} = \mu_{P1}(A_{28} + A_{30})$$

$$C_{15} = \mu_{P1}(A_{27} + A_{29}) + \mu_{s1F}\rho_{G1}$$

$$+ a_{G1}\left[\mu_{s1F}\sin(\phi_1 + \delta_{G1} - \psi_1 + \alpha_{P1}) - \cos(\phi_1 + \delta_{G1} - \psi_1 + \alpha_{P1})\right]$$

## III.  MOMENT INPUT-OUTPUT RELATIONSHIP

Equations (H-100) and (H-115), which are both expressions in $F_{12F}$, are now set equal to each other and the result is solved for $F_{23}$

$$F_{23} = \frac{-c_{13}}{c_{11}c_{15}} (M_{1n} - Q_1 c_{14}) + Q_2 \frac{c_{12}}{c_{11}} \qquad (H-116)$$

The above expression is now equated to equation (H-40) and solved for $F_{34}$

$$F_{34} = \frac{c_5 c_{13}}{c_3 c_{11} c_{15}} (M_{1n} - Q_1 c_{14}) - Q_2 \frac{c_5 c_{12}}{c_3 c_{11}} - Q_3 \frac{c_4}{c_3}$$

$$(H-117)$$

Finally, this expression is set equal to equation (H-19) and the result is solved for the equilibrant moment $M_{o42}$ (for case 2: RRF)

$$M_{o42} = M_{1n} \frac{c_2 c_5 c_{13}}{c_3 c_{11} c_{15}} - Q_1 \frac{c_2 c_5 c_{13} c_{14}}{c_3 c_{11} c_{15}} - Q_2 \frac{c_2 c_5 c_{12}}{c_3 c_{11}} - Q_3 \frac{c_2 c_4}{c_3} - Q_4 c_1$$

$$(H-118)$$

H-43

c.  CASE NO. 3:  RFF

   With both meshes no. 1 and no. 2 in the round on flat phase
of motion, only force $F_{34}$ of the round on round phase can be
incorporated for the present case.  The equilibrium equations
for gear and pinion set no. 3, gear and pinion set no. 2 and the
input gear no. 1 must be newly derived.


I.  FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 3

   Figure H-7 shows the free body diagram of gear and pinion
set no. 3, together with the necessary outline of mesh no. 2.

   The forces of pinion no. 4 on gear no. 3 are given by
equations (H-20) and (H-21)

$$\bar{F}_{43} = -F_{34}\bar{n}_{\lambda 3} \tag{H-119}$$

and

$$\bar{F}_{f43} = -\mu s_{3R}F_{34}\bar{n}_{N\lambda 3} \tag{H-120}$$

H-44
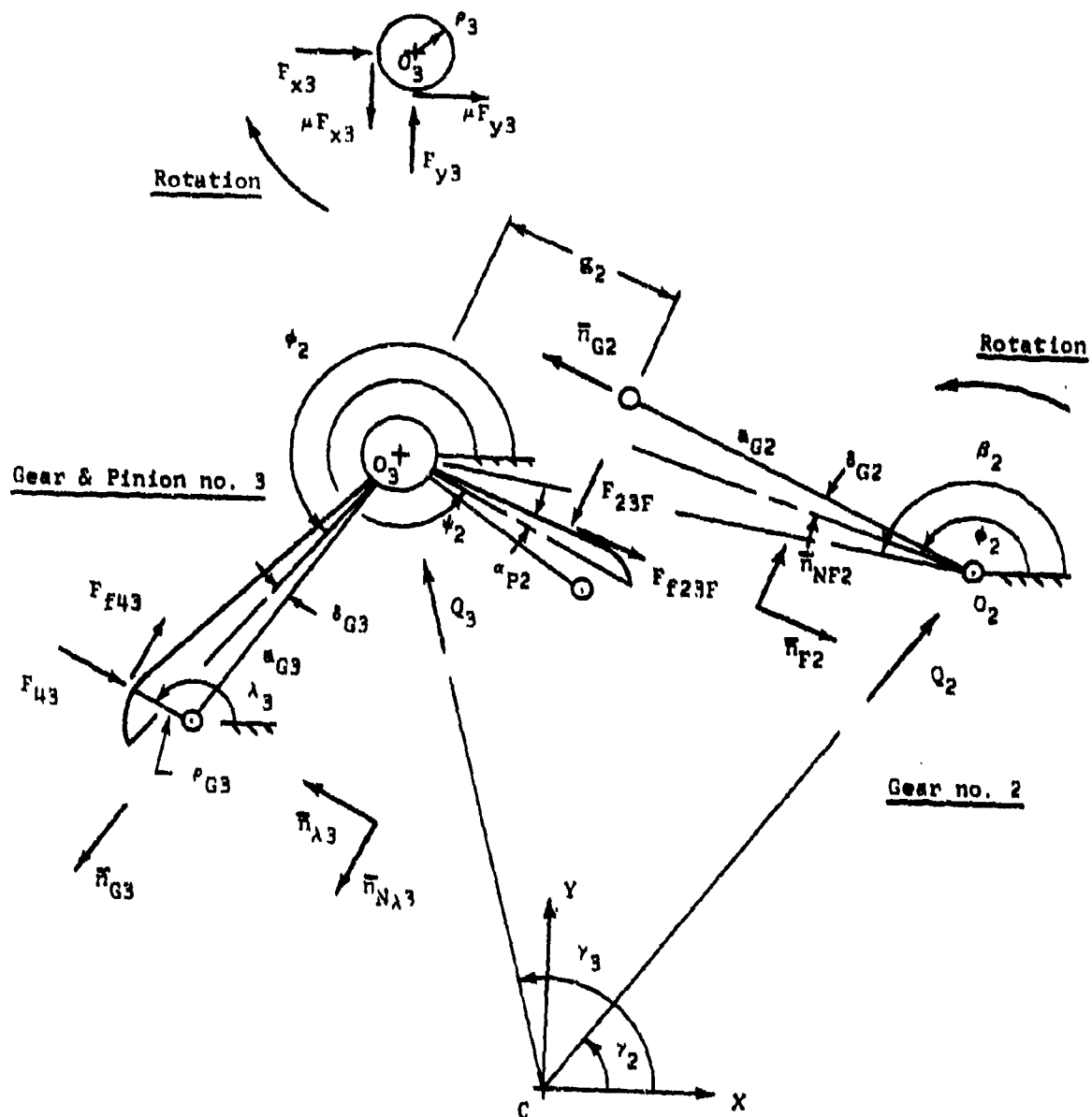
FIGURE H-7

FREE BODY DIAGRAM OF GEAR & PINION NO. 3
MESH NO. 3:   ROUND ON ROUND
MESH NO. 2:   ROUND ON FLAT

H-45

The normal contact force of gear no. 2 on pinion no. 3 is given by

$$\tilde{F}_{23F} = - F_{23F}\bar{n}_{NF2} \tag{H-121}$$

The associated friction force is given by

$$\bar{F}_{f23r} = \mu s_{2F}F_{23F}\bar{n}_{F2} \tag{H-122}$$

The pivot friction forces are chosen so that the resulting friction moments oppose the indicated rotation. The centrifugal force $\bar{Q}_3$ is that of equations (H-24) and (H-25).

Force equilibrium is given by

$$-F_{34}\bar{n}_{\lambda 3} - \mu s_{3R}F_{34}\bar{n}_{N\lambda 3} - F_{23F}\bar{n}_{NF2} + \mu s_{2F}F_{23F}\bar{n}_{F2} + \bar{Q}_3$$

$$+ F_{x3}\bar{i} + \mu F_{y3}\bar{i} + F_{y3}\bar{j} - \mu F_{x3}\bar{j} = 0 \tag{H-123}$$

Moment equilibrium about point $O_3$ is given by

$$\mu \rho_3 (\tilde{F}_{x3} + \tilde{F}_{y3})\bar{k} + \left[ s_{G3}\bar{n}_{G3} + \rho_{G3}\bar{n}_{\lambda 3} \right] \times \left[ -F_{34}\bar{n}_{\lambda 3} - \mu s_{3R}F_{34}\bar{n}_{N\lambda 3} \right]$$

$$+ s_2 \bar{n}_{F2} \times \bar{F}_{23F} = 0 \tag{H-124}$$

H-46

Note that the friction force $F_{f23F}$ exerts no moment about point $O_3$.

Equation (H-123) furnishes the following component equations:

$$-F_{34}\cos\lambda_3 + \mu s_{3R}F_{34}\sin\lambda_3 + Q_3\cos\gamma_3 + F_{x3} + \mu F_{y3} + F_{23F}\sin(\psi_2 + \alpha_{P2})$$

$$+ \mu s_{2F}F_{23F}\cos(\psi_2 + \alpha_{P2}) = 0 \qquad (\text{H-125})$$

and

$$-F_{34}\sin\lambda_3 - \mu s_{3R}F_{34}\cos\lambda_3 + Q_3\sin\gamma_3 + F_{y3} - \mu F_{x3} - F_{23F}\cos(\psi_2 + \alpha_{P2})$$

$$+ \mu s_{2F}F_{23F}\sin(\psi_2 + \alpha_{P2}) = 0 \qquad (\text{H-126})$$

The scalar form of the moment equation (H-124) becomes

$$\mu\rho_3(\widetilde{F}_{x3} + \widetilde{F}_{y3}) + a_{O3}F_{34}\left[\sin(\phi_3 + \delta_{G3} - \lambda_3) - \mu s_{3R}\cos(\phi_3 + \delta_{G3} - \lambda_3)\right]$$

$$- \mu s_{3R}\rho_{O3}F_{34} - \delta_2 F_{23F} = 0 \qquad (\text{H-127})$$

H-47

Simultaneous solution of equations (H-125) and (H-126) for $F_{x3}$ and $F_{y3}$ leads to

$$F_{x3} = \frac{1}{1 + \mu^2}\left\{ F_{34}\left[(1 - \mu^2 s_{3R})\cos\lambda_3 - \mu(1 + s_{3R})\sin\lambda_3\right] \right.$$

$$+ F_{23F}\left[-\mu(1 + s_{2F})\cos(\psi_2 + \alpha_{P2}) - (1 - \mu^2 s_{2F})\sin(\psi_2 + \alpha_{P2})\right]$$

$$\left. + Q_3\left[\mu\sin\gamma_3 - \cos\gamma_3\right]\right\} \qquad \text{(H-128)}$$

and

$$F_{y3} = \frac{1}{1 + \mu^2}\left\{ F_{34}\left[(1 - \mu^2 s_{3R})\sin\lambda_3 + \mu(1 + s_{3R})\cos\lambda_3\right] \right.$$

$$+ F_{23F}\left[(1 - \mu^2 s_{2F})\cos(\psi_2 + \alpha_{P2}) - \mu(1 + s_{2F})\sin(\psi_2 + \alpha_{P2})\right]$$

$$\left. - Q_3\left[\sin\gamma_3 + \mu\cos\gamma_3\right]\right\} \qquad \text{(H-129)}$$

The sum $\tilde{F}_{x3} + \tilde{F}_{y3}$ of equation (H-127) is now made up of equations (H-128) and (H-129) in the sense of equation (A-3b)

$$\tilde{F}_{x3} + \tilde{F}_{y3} = F_{34}A_{31} + F_{23F}A_{32} + Q_3A_{33} + F_{34}A_{34} + F_{23F}A_{35} + Q_3A_{36}$$

$$\text{(H-130)}$$

H-48

where

$$A_{31} = \left| \frac{(1 - \mu^2 s_{3R})\cos\lambda_3 - \mu(1 + s_{3R})\sin\lambda_3}{1 + \mu^2} \right| \tag{H-131}$$

$$A_{32} = \left| \frac{-\mu(1 + s_{2F})\cos(\psi_2 + \alpha_{P2}) - (1 - \mu^2 s_{2F})\sin(\psi_2 + \alpha_{P2})}{1 + \mu^2} \right| \tag{H-132}$$

$$A_{33} = \left| \frac{\mu\sin\gamma_3 - \cos\gamma_3}{1 + \mu^2} \right| \tag{H-133}$$

$$A_{34} = \left| \frac{(1 - \mu^2 s_{3R})\sin\lambda_3 + \mu(1 + s_{3R})\cos\lambda_3}{1 + \mu^2} \right| \tag{H-134}$$

$$A_{35} = \left| \frac{(1 - \mu^2 s_{2F})\cos(\psi_2 + \alpha_{P2}) - \mu(1 + s_{2F})\sin(\psi_2 + \alpha_{P2})}{1 + \mu^2} \right| \tag{H-135}$$

$$A_{36} = \left| \frac{\sin\gamma_3 + \mu\cos\gamma_3}{1 + \mu^2} \right| \tag{H-136}$$

Equation (H-130) is now substituted into equation (H-127) and the result is solved for $F_{23F}$

H-49

$$F_{23r} = \frac{-F_{34}C_{16} - Q_3 C_{17}}{C_{18}} \qquad (H-137)$$

where

$$C_{16} = \mu \rho_3 (A_{31} + A_{34}) + a_{G3}\left[\sin(\phi_3 + \delta_{G3} - \lambda_3)\right.$$

$$\left. - \mu a_{3R}\cos(\phi_3 + \delta_{G3} - \lambda_3)\right] - \mu a_{3R}\rho_{G3}$$

$$C_{17} = \mu \rho_3 (A_{33} + A_{36})$$

$$C_{18} = \mu \rho_3 (A_{32} + A_{35}) - B_2$$

## II.  FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 2

Figure H-8 shows the free body diagram of the gear and pinion set no. 2.

The forces of pinion no. 3 on gear no. 2 are equal and opposite to those given by equations (H-121) and (H-122), i.e.,

$$\bar{F}_{32F} = F_{23F}\bar{n}_{NF2} \qquad (H-138)$$

and

$$\bar{F}_{f32F} = -\mu s_{2F}F_{23F}\bar{n}_{F2} \qquad (H-139)$$

The forces of gear no. 1 on pinion no. 2 are those of equations (H-84) and (H-85).

The pivot friction is accounted for in the usual manner and the centrifugal force $\bar{Q}_2$ is given by equation (H-45).

Force equilibrium is given by

$$F_{23F}\bar{n}_{NF2} - \mu s_{2F}F_{23F}\bar{n}_{F2} + F_{12F}\bar{n}_{NF1} + \mu s_{1F}F_{12F}\bar{n}_{F1} + \bar{Q}_2 + F_{x2}\bar{i} - \mu F_{y2}\bar{i}$$

$$+ F_{y2}\bar{j} + \mu F_{x2}\bar{j} = 0 \qquad (H-140)$$

H-51
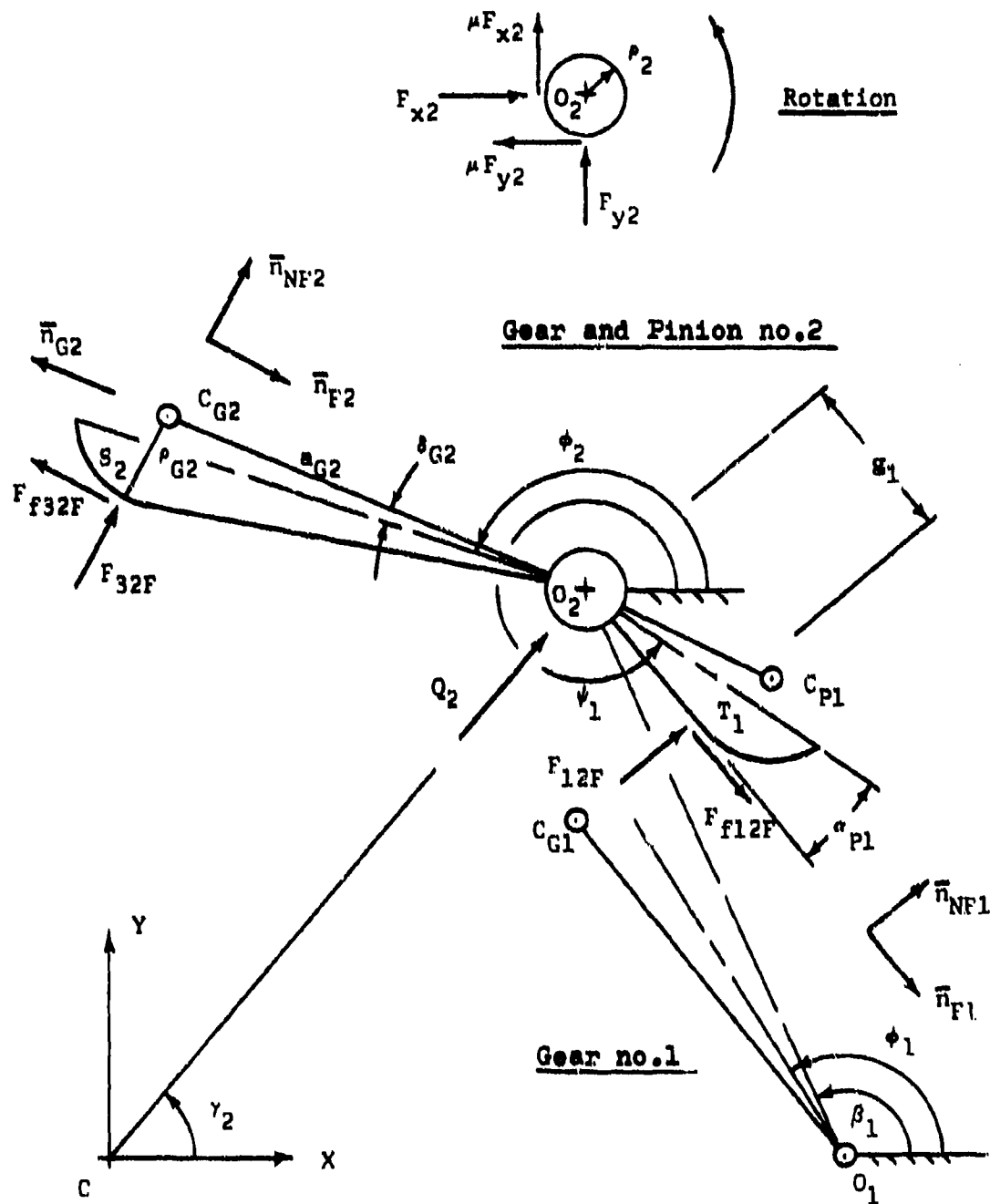
FIGURE H-8

FREE BODY DIAGRAM OF GEAR & PINION NO. 2
MESH NO. 2:   ROUND ON FLAT
MESH NO. 1:   ROUND ON FLAT

H-52

Moment equilibrium about point $O_2$ requires

$$-\mu\rho_2(\widetilde{F}_{x2} + \widetilde{F}_{y2})\bar{k} + [a_{G2}\bar{n}_{G2} - \rho_{G2}\bar{n}_{NF2}] \times [F_{23F}\bar{n}_{NF2} - \mu s_{2F}F_{23F}\bar{n}_{F2}]$$

$$+ g_1\bar{n}_{F1} \times F_{12F}\bar{n}_{NF1} = 0 \qquad (H-141)$$

Equation (H-140) gives the component equations

$$-F_{23F}\sin(\psi_2 + \alpha_{P2}) - \mu s_{2F}F_{23F}\cos(\psi_2 + \alpha_{P2}) + Q_2\cos\gamma_2 + F_{x2} - \mu F_{y2}$$

$$- F_{12F}\sin(\psi_1 - \alpha_{P1}) + \mu s_{1F}F_{12F}\cos(\psi_1 - \alpha_{P1}) = 0 \qquad (H-142)$$

and

$$F_{23F}\cos(\psi_2 + \alpha_{P2}) - \mu s_{2F}F_{23F}\sin(\psi_2 + \alpha_{P2}) + Q_2\sin\gamma_2 + F_{y2} + \mu F_{x2}$$

$$+ F_{12F}\cos(\psi_1 - \alpha_{P1}) + \mu s_{1F}F_{12F}\sin(\psi_1 - \alpha_{P1}) = 0 \qquad (H-143)$$

The scalar form of the moment equation (H-141) becomes

$$-\mu_{P2}(\widetilde{F}_{x2} + \widetilde{F}_{y2}) + a_{G2}F_{23F}\Big[\cos(\phi_2 - \delta_{G2} - \psi_2 - \alpha_{P2})$$

$$+ \mu s_{2F}\sin(\phi_2 - \delta_{G2} - \psi_2 - \alpha_{P2})\Big] - \mu s_{2F}\rho_{G2}F_{23F} + g_1 F_{12F} = 0$$

$$(H-144)$$

Simultaneous solution of the component equations (H-142) and (H-143) for $F_{x2}$ and $F_{y2}$ leads to

$$F_{x2} = \frac{1}{1 + \mu^2}\Big\{F_{23F}\Big[(1 + \mu^2 s_{2F})\sin(\psi_2 + \alpha_{P2}) + \mu(s_{2F} - 1)\cos(\psi_2 + \alpha_{P2})\Big]$$

$$+ F_{12F}\Big[(1 - \mu^2 s_{1F})\sin(\psi_1 - \alpha_{P1}) - \mu(1 + s_{1F})\cos(\psi_1 - \alpha_{P1})\Big]$$

$$- Q_2\Big[\mu\sin\gamma_2 + \cos\gamma_2\Big]\Big\} \qquad (H-145)$$

and

$$F_{y2} = \frac{1}{1 + \mu^2}\Big\{F_{23F}\Big[\mu(s_{2F} - 1)\sin(\psi_2 + \alpha_{P2}) - (1 + \mu^2 s_{2F})\cos(\psi_2 + \alpha_{P2})\Big]$$

$$- F_{12F}\Big[\mu(1 + s_{1F})\sin(\psi_1 - \alpha_{P1}) + (1 - \mu^2 s_{1F})\cos(\psi_1 - \alpha_{P1})\Big]$$

$$+ Q_2\Big[\mu\cos\gamma_2 - \sin\gamma_2\Big]\Big\} \qquad (H-146)$$

The sum $\widetilde{F}_{x2} + \widetilde{F}_{y2}$ of equation (H-144) is now made up of equations (H-145) and (H-146) in the sense of equation (A-3b)

$$\widetilde{F}_{x2} + \widetilde{F}_{y2} \;=\; F_{23F}A_{37} + F_{12F}A_{38} + Q_2 A_{39} + F_{23F}A_{40} + F_{12F}A_{41} + Q_2 A_{42}$$

$$(H-147)$$

where

$$A_{37} \;=\; \left| \frac{(1 + \mu^2 s_{2F})\sin(\psi_2 + \alpha_{P2}) + \mu(s_{2F} - 1)\cos(\psi_2 + \alpha_{P2})}{1 + \mu^2} \right| \qquad (H-148)$$

$$A_{38} \;=\; \left| \frac{(1 - \mu^2 s_{1F})\sin(\psi_1 - \alpha_{P1}) - \mu(1 + s_{1F})\cos(\psi_1 - \alpha_{P1})}{1 + \mu^2} \right| \qquad (H-149)$$

$$A_{39} \;=\; \left| \frac{\mu \sin\gamma_2 + \cos\gamma_2}{1 + \mu^2} \right| \qquad (H-150)$$

$$A_{40} \;=\; \left| \frac{\mu(s_{2F} - 1)\sin(\psi_2 + \alpha_{P2}) - (1 + \mu^2 s_{2F})\cos(\psi_2 + \alpha_{P2})}{1 + \mu^2} \right| \qquad (H-151)$$

$$A_{41} \;=\; \left| \frac{\mu(1 + s_{1F})\sin(\psi_1 - \alpha_{P1}) + (1 - \mu^2 s_{1F})\cos(\psi_1 - \alpha_{P1})}{1 + \mu^2} \right| \qquad (H-152)$$

$$A_{42} \;=\; \left| \frac{\mu\cos\gamma_2 - \sin\gamma_2}{1 + \mu^2} \right| \qquad (H-153)$$

Equation (H-147) is now substituted into equation (H-144) and the result

is solved for $F_{12F}$

$$F_{12F} = \frac{- F_{23F}C_{19} + Q_2 C_{20}}{C_{21}} \qquad (H-154)$$

where

$$C_{19} = -\mu\rho_2(A_{37} + A_{40}) + s_{G2}\left[\cos(\phi_2 - \delta_{G2} - \psi_2 - \alpha_{P2})\right.$$

$$\left. + \mu s_{2F}\sin(\phi_2 - \delta_{G2} - \psi_2 - \alpha_{P2})\right] - \mu s_{2F}\rho_{G2}$$

$$C_{20} = \mu\rho_2(A_{39} + A_{42})$$

$$C_{21} = -\mu\rho_2(A_{38} + A_{41}) + s_1$$

### III. FORCE AND MOMENT EQUILIBRIA OF INPUT GEAR NO. 1

While the numerical values of the force $F_{21F}$ and its associated friction force $F_{f12F}$, both acting on gear no. 1, are peculiar to the present combination of contact phases, its functional relationship to the input moment $M_{1n}$ and the centrifugal force $Q_1$ is identical to that derived in section 1b-II of this appendix. (See also Figure H-6.)

According to equation (H-115), one obtains for $F_{12F}$

$$F_{12F} = \frac{M_{1n} - Q_1 C_{14}}{C_{15}} \tag{H-155}$$

## IV. MOMENT INPUT-OUTPUT RELATIONSHIP

Equations (H-154) and (H-155), both in $F_{12F}$, are set equal to each other and the result is solved for $F_{23F}$

$$F_{23F} = \frac{-C_{21}}{C_{15}C_{19}} (M_{1n} - Q_1 C_{14}) + Q_2 \frac{C_{20}}{C_{19}} \qquad (H-156)$$

The above is now equated to equation (H-137) and the result is solved for $F_{34}$

$$F_{34} = \frac{C_{18}C_{21}}{C_{15}C_{16}C_{19}} (M_{1n} - Q_1 C_{14}) - Q_2 \frac{C_{18}C_{20}}{C_{16}C_{19}} - Q_3 \frac{C_{17}}{C_{16}} \qquad (H-157)$$

Finally, equation (H-157) is equated to equation (H-19), which corresponds to the round on round phase of mesh no. 3. The result is solved for the equilibrant moment $M_{o43}$ (for case 3: RFF)

$$M_{o43} = M_{1n} \frac{C_2 C_{18} C_{21}}{C_{15}C_{16}C_{19}} - Q_1 \frac{C_2 C_{14} C_{18} C_{21}}{C_{15}C_{16}C_{19}} - Q_2 \frac{C_2 C_{18} C_{20}}{C_{16}C_{19}} - Q_3 \frac{C_2 C_{17}}{C_{16}}$$

$$- Q_4 C_1 \qquad (H-158)$$

## d. CASE NO. 4: RFR

For this contact combination force $F_{34}$ may be taken from the results of case no. 1 [see equation (H-19)], since mesh no. 3 is in the round on round phase of motion. The force $F_{23F}$ of case no. 3, i.e., equation (H-137), also is incorporated.

The input-output relationship of the gear and pinion combination no. 2 must be newly derived, i.e., the force $F_{12}$ must be expressed in terms of the contact force $F_{32F}$ and the centrifugal force $Q_2$. Finally, the results of the equilibrium equations for the input gear no. 1 of case no. 1 are used. For this case of contact, the force $F_{12}$ is given by equation (H-78).

## I. FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 2

Figure H-9 shows the free body diagram of gear and pinion set no. 2 with the necessary portions of mesh no. 1.

The forces of pinion no. 3 on gear no. 2 were given by equations (H-138) and (H-139)
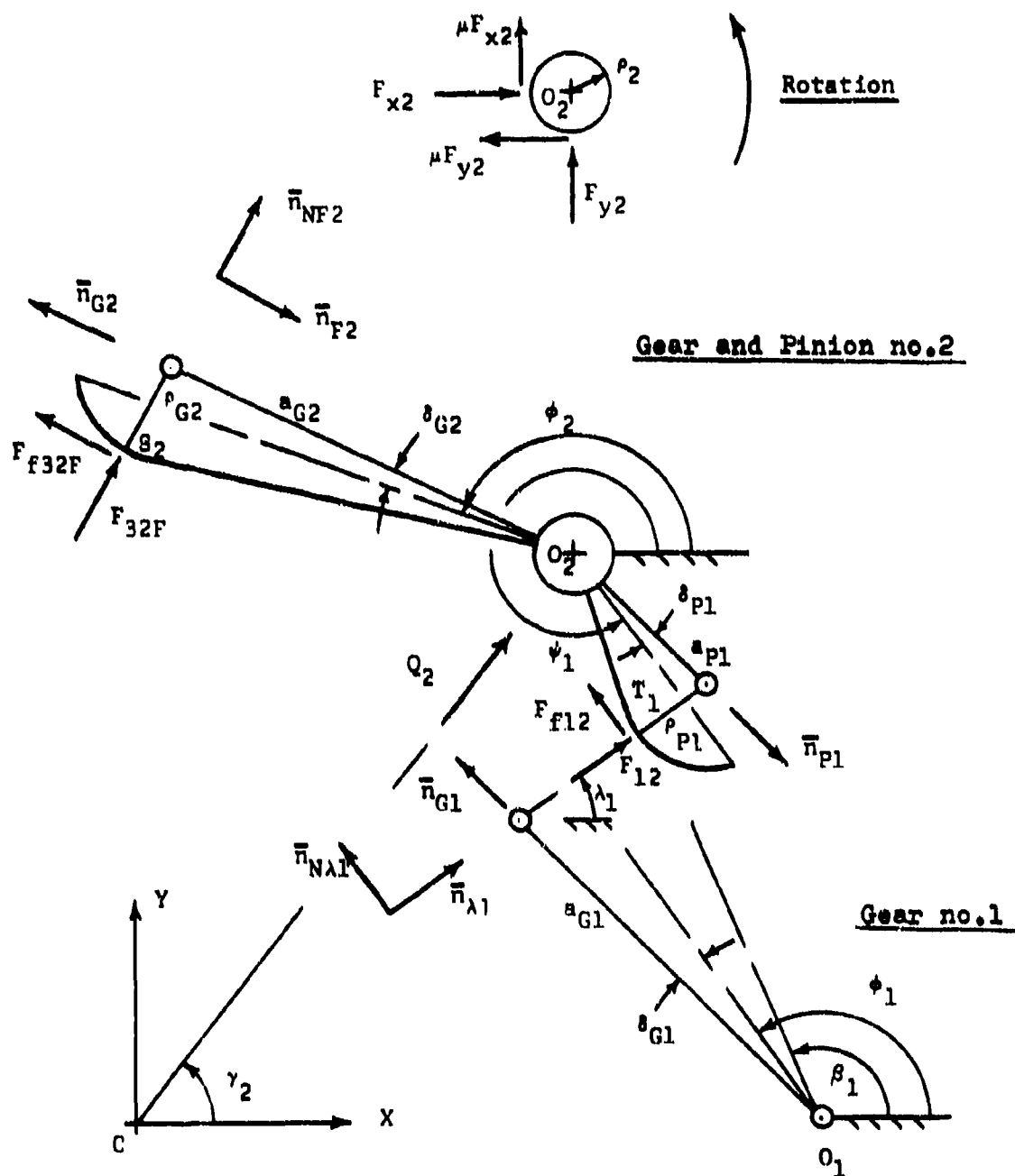
FIGURE H-9

FREE BODY DIAGRAM OF GEAR & PINION NO. 2
MESH NO. 2: ROUND ON FLAT
MESH NO. 1: ROUND ON ROUND

H-60

$$\bar{F}_{32F} = F_{23F}\bar{n}_{NF2} \tag{H-159}$$

and

$$\bar{F}_{f32F} = -\mu a_{2F}F_{23F}\bar{n}_{F2} \tag{H-160}$$

The forces of gear no. 1 on pinion no. 2 are given by equations (H-43) and (H-44)

$$\bar{F}_{12} = F_{12}\bar{n}_{\lambda 1} \tag{H-161}$$

and

$$\bar{F}_{f12} = \mu a_{1R}F_{12}\bar{n}_{N\lambda 1} \tag{H-162}$$

The centrifugal force $\bar{Q}_2$ is given by equation (H-45). The pivot reactions and friction forces are handled as before.

Force equilibrium is given by

$$F_{23F}\bar{n}_{NF2} - \mu a_{2F}F_{23F}\bar{n}_{F2} + F_{12}\bar{n}_{\lambda 1} + \mu a_{1R}F_{12}\bar{n}_{N\lambda 1} + \bar{Q}_2 + F_{x2}\bar{i} - \mu F_{y2}\bar{i}$$
$$+ F_{y2}\bar{j} + \mu F_{x2}\bar{j} = 0 \tag{H-163}$$

H-61

Moment equlibrium about point $O_2$ is given by

$$-\mu\rho_2(\tilde{F}_{x2} + \tilde{F}_{y2}) + \left[a_{G2}\bar{n}_{G2} - \rho_{G2}\bar{n}_{NF2}\right] \times \left[F_{23F}\bar{n}_{NF2} - \mu s_{2F}F_{23F}\bar{n}_{F2}\right]$$

$$+ \left[a_{P1}\bar{n}_{P1} - \rho_{P1}\bar{n}_{\lambda 1}\right] \times \left[F_{12}\bar{n}_{\lambda 1} + \mu s_{1R}F_{12}\bar{n}_{N\lambda 1}\right] = 0 \qquad (H-164)$$

Equation (H-163) gives the following component equations

$$-F_{23F}\sin(\psi_2 + \alpha_{P2}) - \mu s_{2F}F_{23F}\cos(\psi_2 + \alpha_{P2}) + F_{x2} - \mu F_{y2} + F_{12}\cos\lambda_1$$

$$- \mu s_{1R}F_{12}\sin\lambda_1 + Q_2\cos\gamma_2 = 0 \qquad (H-165)$$

and

$$F_{23F}\cos(\psi_2 + \alpha_{P2}) - \mu s_{2F}F_{23F}\sin(\psi_2 + \alpha_{P2}) + F_{y2} + \mu F_{x2} + F_{12}\sin\lambda_1$$

$$+ \mu s_{1R}F_{12}\cos\lambda_1 + Q_2\sin\gamma_2 = 0 \qquad (H-166)$$

H-62

The scalar form of the moment equation (H-164) becomes

$$- \rho_2(\widetilde{F}_{x2} + \widetilde{F}_{y2}) + a_{G2}F_{23F}\left[\cos(\phi_2 - \delta_{G2} - \psi_2 - \alpha_{P2})\right.$$

$$+ \mu a_{2F}\sin(\phi_2 - \delta_{G2} - \psi_2 - \alpha_{P2})\bigg] - \mu a_{2F}\rho_{G2}F_{23F} - \mu a_{1R}\rho_{P1}F_{12}$$

$$+ a_{P1}F_{12}\bigg[\mu a_{1R}\cos(\psi_1 + \delta_{P1} - \lambda_1) - \sin(\psi_1 + \delta_{P1} - \lambda_1)\bigg] = 0$$

$$(H-167)$$

Simultaneous solution of the force component equations (H-165)
and (H-166) for $F_{x2}$ and $F_{y2}$ results in

$$F_{x2} = \frac{1}{1 + \mu^2}\left\{F_{23F}\left[(1 + \mu^2 a_{2F})\sin(\psi_2 + \alpha_{P2}) - \mu(1 - a_{2F})\cos(\psi_2 + \alpha_{P2})\right]\right.$$

$$+ F_{12}\left[\mu(a_{1R} - 1)\sin\lambda_1 - (1 + \mu^2 a_{1R})\cos\lambda_1\right]$$

$$- Q_2\left[\mu\sin\gamma_2 + \cos\gamma_2\right]\Bigg\} \qquad (H-168)$$

and

$$F_{y2} = \frac{1}{1 + \mu^2}\left\{F_{23F}\left[\mu(a_{2F} - 1)\sin(\psi_2 + \alpha_{P2}) - (1 + \mu^2 a_{2F})\cos(\psi_2 + \alpha_{P2})\right]\right.$$

$$+ F_{12}\left[-(1 + \mu^2 a_{1R})\sin\lambda_1 + \mu(1 - a_{1R})\cos\lambda_1\right]$$

$$+ Q_2\left[-\sin\gamma_2 + \mu\cos\gamma_2\right]\Bigg\} \qquad (H-169)$$

H-63

The sum $\tilde{F}_{x2} + \tilde{F}_{y2}$ of equation(H-167) is now made up of equations (H-168) and (H-169) in the usual manner

$$\tilde{F}_{x2} + \tilde{F}_{y2} = F_{23F}A_{43} + F_{12}A_{44} + Q_2A_{45} + F_{23F}A_{46} + F_{12}A_{47} + Q_2A_{48}$$

$$(H-170)$$

where

$$A_{43} = \left| \frac{(1 + \mu^2 s_{2F})\sin(\psi_2 + \alpha_{P2}) - \mu(1 - s_{2F})\cos(\psi_2 + \alpha_{P2})}{1 + \mu^2} \right| \qquad (H-171)$$

$$A_{44} = \left| \frac{\mu(s_{1R} - 1)\sin\lambda_1 - (1 + \mu^2 s_{1R})\cos\lambda_1}{1 + \mu^2} \right| \qquad (H-172)$$

$$A_{45} = \left| \frac{\mu\sin\gamma_2 + \cos\gamma_2}{1 + \mu^2} \right| \qquad (H-173)$$

$$A_{46} = \left| \frac{\mu(s_{2F} - 1)\sin(\psi_2 + \alpha_{P2}) - (1 + \mu^2 s_{2F})\cos(\psi_2 + \alpha_{P2})}{1 + \mu^2} \right| \qquad (H-174)$$

$$A_{47} = \left| \frac{-(1 + \mu^2 s_{1R})\sin\lambda_1 + \mu(1 - s_{1R})\cos\lambda_1}{1 + \mu^2} \right| \qquad (H-175)$$

$$A_{48} = \left| \frac{-\sin\gamma_2 + \mu\cos\gamma_2}{1 + \mu^2} \right| \qquad (H-176)$$

Equation (H-170) is now substituted into the moment equation (H-167) and the result is solved for $F_{12}$

$$F_{12} = \frac{-F_{23F}C_{22} + Q_2 C_{23}}{C_{24}} \qquad (H-177)$$

where

$$C_{22} = -\mu\rho_2(A_{43} + A_{46}) + a_{G2}\left[\cos(\phi_2 - \delta_{G2} - \psi_2 - \alpha_{P2})\right.$$

$$\left. + \mu a_{2F}\sin(\phi_2 - \delta_{G2} - \psi_2 - \alpha_{P2})\right] - \mu a_{2F}\rho_{G2}$$

$$C_{23} = \mu\rho_2(A_{45} + A_{48})$$

$$C_{24} = -\mu\rho_2(A_{44} + A_{47}) + a_{P1}\left[\mu a_{1R}\cos(\psi_1 + \delta_{P1} - \lambda_1)\right.$$

$$\left. - \sin(\psi_1 + \delta_{P1} - \lambda_1)\right] - \mu a_{1R}\rho_{P1}$$

## II.  MOMENT INPUT-OUTPUT RELATIONSHIP

Equations (H-177) and (H-78) are now set equal to each other and the result is solved for $F_{23F}$

$$F_{23F} = \frac{-C_{24}}{C_{10}C_{22}}(M_{in} - Q_1 C_9) + Q_2 \frac{C_{23}}{C_{22}} \qquad (H-178)$$

The above is now equated to equation (H-137) to obtain $F_{34}$

$$F_{34} = \frac{C_{18}C_{24}}{C_{10}C_{16}C_{22}}(M_{in} - Q_1 C_9) - Q_2 \frac{C_{18}C_{23}}{C_{16}C_{22}} - Q_3 \frac{C_{17}}{C_{16}} \qquad (H-179)$$

Finally equations (H-179) and (H-19) are set equal to each other and the equilibrant moment $M_{o44}$ (For case 4: RFR) is determined

$$M_{o44} = M_{in}\frac{C_2 C_{18}C_{24}}{C_{10}C_{16}C_{22}} - Q_1\frac{C_2 C_9 C_{18}C_{24}}{C_{10}C_{16}C_{22}} - Q_2\frac{C_2 C_{18}C_{23}}{C_{16}C_{22}} - Q_3\frac{C_2 C_{17}}{C_{16}}$$

$$- Q_4 C_1 \qquad (H-180)$$

e. CASE NO. 5: FFF

For this contact combination it is necessary to determine
new expressions for the force $F_{34F}$ of gear no. 3 on pinion no. 4,
and for force $F_{23F}$ of gear no. 2 on pinion no. 3.

Equation (H-156), derived for case 3, and which relates
force $F_{23F}$ to the input moment $M_{in}$, may be used for the determination
of the final input-output relationship.

I. FORCE AND MOMENT EQUILIBRIA OF PINION NO. 4

Figure H-10 gives the free body diagram of pinion no. 4 in
the round on flat phase of motion with gear no. 3.

The equilibrant moment $M_{o4}$ acts in a clockwise direction
and opposes the counter-clockwise rotation of the pinion.

The normal contact force $\bar{F}_{34F}$ is given by

$$\bar{F}_{34F} = F_{34F}\bar{R}_{NF3} \qquad\qquad (H-181)$$
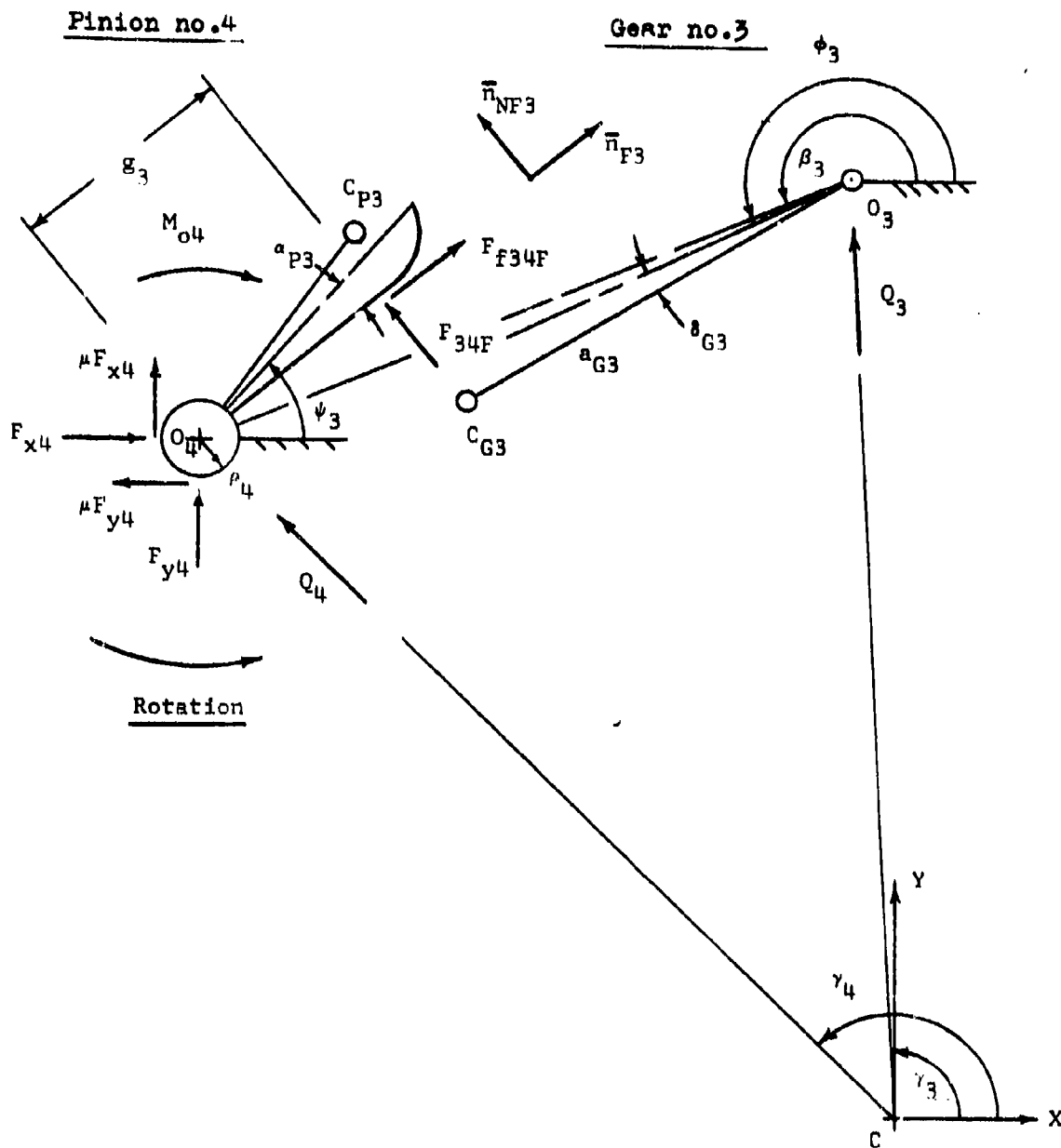
FIGURE H-10

FREE BODY DIAGRAM OF PINION NO. 4
MESH NO. 3: ROUND ON FLAT

H-68

The associated friction force becomes

$$\bar{F}_{f34F} = \mu s_{3F} F_{34F} \bar{n}_{F3} \tag{H-182}$$

The centrifugal force $\bar{Q}_4$ is given by equation (H-5) and the pivot friction forces are chosen such that they oppose rotation.

Force equilibrium is given by

$$F_{34F} \bar{n}_{NF3} + \mu s_{3F} F_{34F} \bar{n}_{F3} + F_{x4} \bar{i} - \mu F_{y4} \bar{i} + F_{y4} \bar{j} + \mu F_{x4} \bar{j}$$

$$+ Q_4 (\cos\gamma_4 \bar{i} + \sin\gamma_4 \bar{j}) = 0 \tag{H-183}$$

Moment equilibrium about point $O_4$ requires that

$$-M_{o4} \bar{k} - \mu \rho_4 (\widetilde{F}_{x4} + \widetilde{F}_{y4}) \bar{k} + s_3 \bar{n}_{F3} \times F_{34F} \bar{n}_{NF3} = 0 \tag{H-184}$$

Note that the friction force $F_{f34F}$ does not exert a moment about point $O_4$, since its line of action passes through it.

Equation (H-183) furnishes the following component equations

$$-F_{34F} \sin(\psi_3 - \alpha_{p3}) + \mu s_{3F} F_{34F} \cos(\psi_3 - \alpha_{p3}) + F_{x4} - \mu F_{y4}$$

$$+ Q_4 \cos\gamma_4 = 0 \tag{H-185}$$

H-69

and

$$F_{34F}\cos(\psi_3 - \alpha_{P3}) + \mu s_{3F}F_{34F}\sin(\psi_3 - \alpha_{P3}) + F_{y4} + \mu F_{x4}$$

$$+ Q_4 \sin\gamma_4 = 0 \qquad (H-186)$$

The scalar form of the moment equation (H-184) becomes

$$-M_{o4} - \mu\rho_4(\widetilde{F}_{x4} + \widetilde{F}_{y4}) + \varepsilon_3 F_{34F} = 0 \qquad (H-187)$$

Simultaneous solution of equations (H-185) and (H-186) for $F_{x4}$

and $F_{y4}$ results in

$$F_{x4} = \frac{1}{1 + \mu^2}\left\{F_{34F}\left[(1 - \mu^2 s_{3F})\sin(\psi_3 - \alpha_{P3}) - \mu(1 + s_{3F})\cos(\psi_3 - \alpha_{P3})\right]\right.$$

$$\left. + Q_4\left[-\mu\sin\gamma_4 - \cos\gamma_4\right]\right\} \qquad (H-188)$$

and

$$F_{y4} = \frac{1}{1 + \mu^2}\left\{F_{34F}\left[-\mu(1 + s_{3F})\sin(\psi_3 - \alpha_{P3}) - (1 - \mu^2 s_{3F})\cos(\psi_3 - \alpha_{P3})\right]\right.$$

$$\left. + Q_4\left[-\sin\gamma_4 + \mu\cos\gamma_4\right]\right\} \qquad (H-189)$$

H-70

The sum $\widetilde{F}_{x4} + \widetilde{F}_{y4}$ of equation (H-187) is now made up of equations (H-188) and (H-189) in the sense of equation (A-3b)

$$\widetilde{F}_{x4} + \widetilde{F}_{y4} = F_{34F}A_{49} + Q_4 A_{50} + F_{34F}A_{51} + Q_4 A_{52} \qquad (H-190)$$

where

$$A_{49} = \left| \frac{(1 - \mu^2 s_{3F})\sin(\psi_3 - \alpha_{P3}) - \mu(1 + s_{3F})\cos(\psi_3 - \alpha_{P3})}{1 + \mu^2} \right| \qquad (H-191)$$

$$A_{50} = \left| \frac{-\mu\sin\gamma_4 - \cos\gamma_4}{1 + \mu^2} \right| \qquad (H-192)$$

$$A_{51} = \left| \frac{-\mu(1 + s_{3F})\sin(\psi_3 - \alpha_{P3}) - (1 - \mu^2 s_{3F})\cos(\psi_3 - \alpha_{P3})}{1 + \mu^2} \right| \qquad (H-193)$$

$$A_{52} = \left| \frac{-\sin\gamma_4 + \mu\cos\gamma_4}{1 + \mu^2} \right| \qquad (H-194)$$

Equation (H-190) is now substituted into equation (H-187) and the result is solved for $F_{34F}$

$$F_{34F} = \frac{M_{o4} + Q_4 C_{25}}{C_{26}} \qquad (H-195)$$

where

$$C_{25} = \mu \rho_4 (A_{50} + A_{52})$$

$$C_{26} = B_3 - \mu \rho_4 (A_{49} + A_{51})$$

Figure H-11 gives the free body diagram of gear and pinion combination no. 3.  Both mesh 3 and mesh 2 are in their round on flat phase of contact.

The forces of pinion 4 on gear 3 are equal to, but opposite in direction to, those given by equations (H-181) and (H-182)

$$\bar{F}_{43F} = - F_{34F}\bar{n}_{NF3} \qquad \text{(H-196)}$$

and

$$\bar{F}_{f43F} = - \mu s_{3F}F_{34F}\bar{n}_{F3} \qquad \text{(H-197)}$$

The forces of gear 2 on pinion 3 are given by

$$\bar{F}_{23F} = - F_{23F}\bar{n}_{NF2} \qquad \text{(H-198)}$$

and

$$\bar{F}_{f23F} = \mu s_{2F}F_{23F}\bar{n}_{F2} \qquad \text{(H-199)}$$
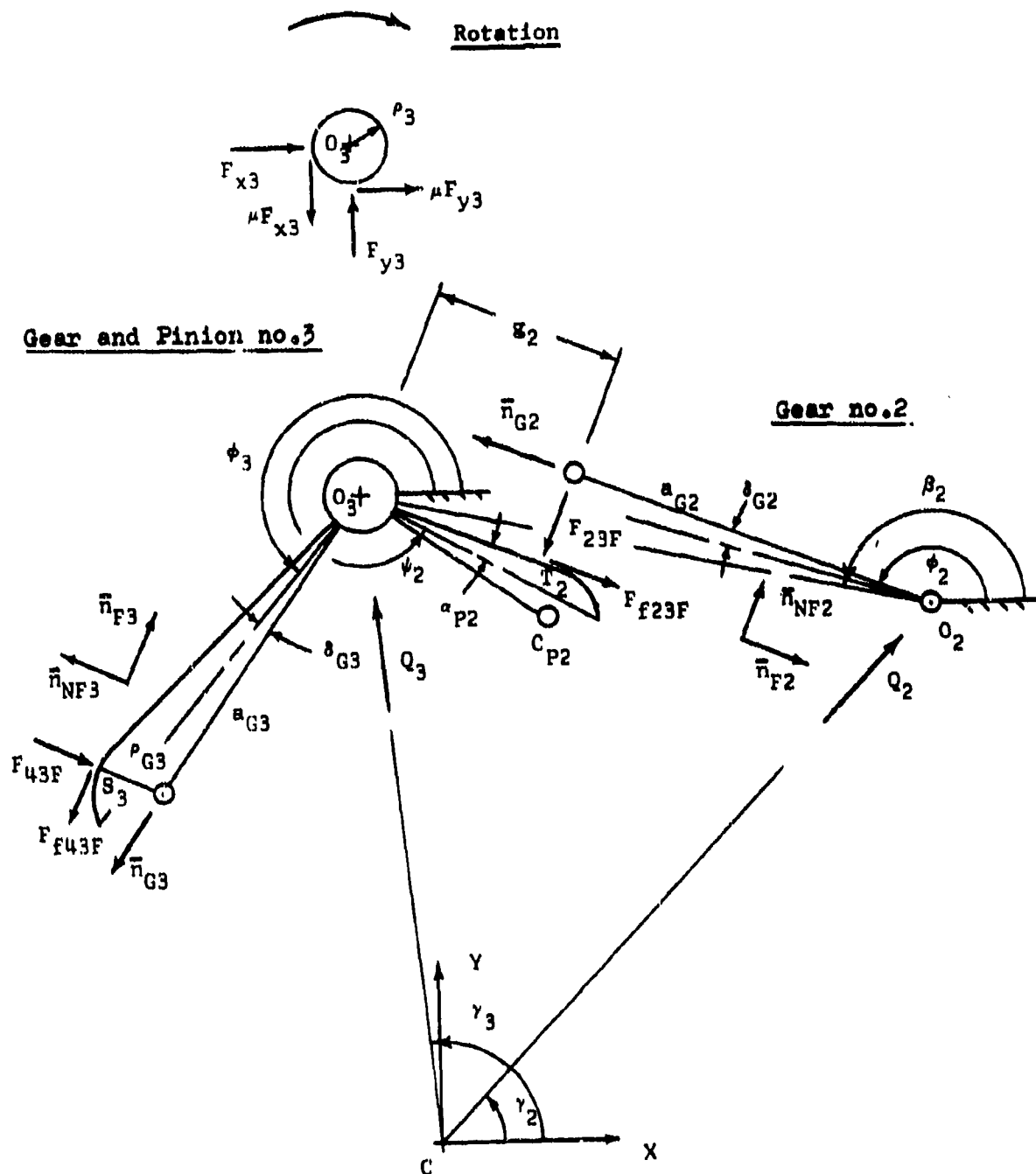
FIGURE H-11

FREE BODY DIAGRAM OF GEAR & PINION NO. 3
MESH NO. 3:  ROUND ON FLAT
MESH NO. 2:  ROUND ON FLAT

H-74

The pivot forces and moments are chosen in the usual manner and the centrifugal force $\bar{Q}_3$ is defined by equation (H-24).

Force equilibrium is given by

$$-F_{34F}\bar{n}_{NF3} - \mu s_{3F}F_{34F}\bar{n}_{F3} - F_{23F}\bar{n}_{NF2} + \mu s_{2F}F_{23F}\bar{n}_{F2} + \bar{Q}_3 + F_{x3}\bar{i}$$

$$+ \mu F_{y3}\bar{i} + F_{y3}\bar{j} - \mu F_{x3}\bar{j} = 0 \qquad (H-200)$$

Moment equilibrium about point $O_3$ requires that

$$\mu \rho_3(\tilde{F}_{x3} + \tilde{F}_{y3})\bar{k} + \left[ a_{G3}\bar{n}_{G3} + \rho_{G3}\bar{n}_{NF3} \right] \times \left[ -F_{34F}\bar{n}_{NF3} - \mu s_{3F}F_{34F}\bar{n}_{F3} \right]$$

$$+ s_2\bar{n}_{F2} \times (-)F_{23F}\bar{n}_{NF2} = 0 \qquad (H-201)$$

Note that the friction force $F_{f23F}$ does not exert a moment about point $O_3$.

Equation (H-200) furnishes, after all necessary substitutions, the following component equations:

$$F_{34F}\sin(\psi_3 - \alpha_{P3}) - \mu s_{3F}F_{34F}\cos(\psi_3 - \alpha_{P3}) + Q_3\cos\gamma_3 + F_{x3} + \mu F_{y3}$$

$$+ F_{23F}\sin(\psi_2 + \alpha_{P2}) + \mu s_{2F}F_{23F}\cos(\psi_2 + \alpha_{P2}) = 0 \qquad (H-202)$$

und

$$-F_{34F}\cos(\psi_3 - \alpha_{P3}) - \mu a_{3F}F_{34F}\sin(\psi_3 - \alpha_{P3}) + Q_3\sin\gamma_3 + F_{y3} - \mu F_{x3}$$

$$- F_{23F}\cos(\psi_2 + \alpha_{P2}) + \mu a_{2F}F_{23F}\sin(\psi_2 + \alpha_{P2}) = 0 \qquad (H\text{-}203)$$

The scalar form of the moment equation (H-201) becomes

$$\mu\rho_3(\widetilde{F}_{x3} + \widetilde{F}_{y3}) + a_{G3}F_{34F}\left[-\cos(\phi_3 + \delta_{G3} - \psi_3 + \alpha_{P3})\right.$$

$$+ \mu a_{3F}\sin(\phi_3 + \delta_{G3} - \psi_3 + \alpha_{P3})\Big] + \mu a_{3F}\rho_{G3}F_{34F} - \varepsilon_2 F_{23F} = 0$$

$$(H\text{-}204)$$

Simultaneous solution of the component equations (H-202) and (H-203) for $F_{x3}$ and $F_{y3}$ gives

$$F_{x3} = \frac{1}{1 + \mu^2}\left\{F_{34F}\left[-(1 + \mu^2 a_{3F})\sin(\psi_3 - \alpha_{P3}) + \mu(a_{3F} - 1)\cos(\psi_3 - \alpha_{P3})\right]\right.$$

$$+ F_{23F}\left[(\mu^2 a_{2F} - 1)\sin(\psi_2 + \alpha_{P2}) - \mu(1 + a_{2F})\cos(\psi_2 + \alpha_{P2})\right]$$

$$+ Q_3\left[\mu\sin\gamma_3 - \cos\gamma_3\right]\Big\} \qquad (H\text{-}205)$$

H-76

and

$$F_{y3} = \frac{1}{1 + \mu^2} \left\{ F_{34F}\left[\mu(s_{3F} - 1)\sin(\psi_3 - \alpha_{P3}) + (1 + \mu^2 s_{3F})\cos(\psi_3 - \alpha_{P3})\right] \right.$$

$$+ F_{23F}\left[-\mu(1 + s_{2F})\sin(\psi_2 + \alpha_{P2}) + (1 - \mu^2 s_{2F})\cos(\psi_2 + \alpha_{P2})\right]$$

$$\left. + Q_3\left[-\sin\gamma_3 - \mu\cos\gamma_3\right]\right\} \qquad (H\text{-}206)$$

The sum $\widetilde{F}_{x3} + \widetilde{F}_{y3}$ is now made up from equations (H-205) and (H-206)

$$\widetilde{F}_{x3} + \widetilde{F}_{y3} = F_{34F}A_{53} + F_{23F}A_{54} + Q_3 A_{55} + F_{34F}A_{56} + F_{23F}A_{57} + Q_3 A_{58}$$

$$(H\text{-}207)$$

where

$$A_{53} = \left| \frac{-(1 + \mu^2 s_{3F})\sin(\psi_3 - \alpha_{P3}) + \mu(s_{3F} - 1)\cos(\psi_3 - \alpha_{P3})}{1 + \mu^2} \right| \qquad (H\text{-}208)$$

$$A_{54} = \left| \frac{(\mu^2 s_{2F} - 1)\sin(\psi_2 + \alpha_{P2}) - \mu(1 + s_{2F})\cos(\psi_2 + \alpha_{P2})}{1 + \mu^2} \right| \qquad (H\text{-}209)$$

$$A_{55} = \left| \frac{\mu\sin\gamma_3 - \cos\gamma_3}{1 + \mu^2} \right| \qquad (H\text{-}210)$$

H-77

$$A_{56} = \left| \frac{\mu(s_{3F} - 1)\sin(\psi_3 - \alpha_{P3}) + (1 + \mu^2 s_{3F})\cos(\psi_3 - \alpha_{P3})}{1 + \mu^2} \right| \qquad \text{(H-211)}$$

$$A_{57} = \left| \frac{-\mu(1 + s_{2F})\sin(\psi_2 + \alpha_{P2}) + (1 - \mu^2 s_{2F})\cos(\psi_2 + \alpha_{P2})}{1 + \mu^2} \right| \qquad \text{(H-212)}$$

$$A_{58} = \left| \frac{\sin\gamma_3 + \mu\cos\gamma_3}{1 + \mu^2} \right| \qquad \text{(H-213)}$$

Equation (H-207) is now substituted into equation (H-204).
The result is solved for $F_{23F}$

$$F_{23F} = \frac{-F_{34F}C_{27} - Q_3 C_{28}}{C_{29}} \qquad \text{(H-214)}$$

where

$$C_{27} = \mu\rho_3(A_{53} + A_{56}) + a_{G3}\left[-\cos(\phi_3 + \delta_{G3} - \psi_3 + \alpha_{P3}) + \mu s_{3F}\sin(\phi_3 + \delta_{G3} - \psi_3 + \alpha_{P3})\right] + \mu s_{3F}\rho_{G3}$$

$$C_{28} = \mu\rho_3(A_{55} + A_{58})$$

$$C_{29} = \mu\rho_3(A_{54} + A_{57}) - \epsilon_2$$

H-78

# III. MOMENT INPUT-OUTPUT RELATIONSHIP

Equation (H-156) is an expression for $F_{23F}$ as a function of the moment $M_{1n}$ and the centrifugal forces $Q_1$ and $Q_2$, when both meshes no. 1 and no. 2 are in the round on flat phase of motion. This expression for $F_{23F}$ is now equated to equation (H-214). The result is solved for $F_{34F}$

$$F_{34F} = \frac{c_{21}c_{29}}{c_{15}c_{19}c_{27}} (M_{1n} - Q_1 c_{14}) - Q_2 \frac{c_{20}c_{29}}{c_{19}c_{27}} - Q_3 \frac{c_{28}}{c_{27}} \qquad \text{(H-215)}$$

The above is now equated to equation (H-195) and the resulting expression is used to determine the equilibrant moment $M_{o45}$ (for Case 5: FFF)

$$M_{o45} = M_{1n} \frac{c_{21}c_{26}c_{29}}{c_{15}c_{19}c_{27}} - Q_1 \frac{c_{14}c_{21}c_{26}c_{29}}{c_{15}c_{19}c_{27}} - Q_2 \frac{c_{20}c_{26}c_{29}}{c_{19}c_{27}}$$

$$- Q_3 \frac{c_{26}c_{28}}{c_{27}} - Q_4 c_{25} \qquad \text{(H-216)}$$

H-79

f. CASE NO. 6: FFR

MOMENT INPUT-OUTPUT RELATIONSHIP

The moment input-output relationship for this contact combination can be assembled entirely from previously derived component relationships. As for case no. 4, mesh no. 1 is in the round on round phase while mesh no. 2 is in the round on flat phase. Therefore, equation (H-178), which relates the force $F_{23F}$ to the input moment $M_{1n}$, may be used. The input-output relationship of the gear and pinion set no. 3, i.e. the relationship between the forces $F_{34F}$ and $F_{23F}$, is given by equation (H-214) of case no. 5. The force $F_{34F}$ may be obtained from equation (H-195). This expression was also derived for a round on flat contact in case no. 5.

Thus, equation (H-178) is first set equal to equation (H-214) and the result is solved for the force $F_{34F}$

$$F_{34F} = \frac{C_{24}C_{29}}{C_{10}C_{22}C_{27}} (M_{1n} - Q_1 C_9) - Q_2 \frac{C_{23}C_{29}}{C_{22}C_{27}} - Q_3 \frac{C_{28}}{C_{27}} \qquad (H-217)$$

The above expression is now set equal to equation (H-195). This then allows the determination of the equilibrant moment $M_{o46}$ for the present contact combination.

$$M_{o46} = M_{1n} \frac{c_{24} c_{26} c_{29}}{c_{10} c_{22} c_{27}} - Q_1 \frac{c_9 c_{24} c_{26} c_{29}}{c_{10} c_{22} c_{27}} - Q_2 \frac{c_{23} c_{26} c_{29}}{c_{22} c_{27}}$$

$$- Q_3 \frac{c_{26} c_{28}}{c_{27}} - Q_4 c_{25} \qquad (H-218)$$

g.  CASE NO. 7:  FRR

For the present contact combination the expression for
force $F_{34F}$ may be taken over from equation (H-195) of case no. 5.
The input-output relationship of gear and pinion set no. 3, which
relates the forces $F_{34F}$ and $F_{23}$, must be newly derived.  The
relationship between force $F_{23}$ and the input moment $M_{in}$ is
taken from case no. 1 in the form of equation (H-79).

## I.  FORCE AND MOMENT EQUILIBRIA OF GEAR AND PINION SET NO. 3

Figure H-12 gives the free body diagram of gear and pinion
set no. 3.  Mesh no. 3 is in the round on flat phase of contact,
while mesh no. 2 is in the round on round one.

The forces of pinion no. 4 on gear no. 3 are equal to, but
opposite in direction to those given by equations (H-181) and
(H-182)

$$\bar{\bar{F}}_{43F} = - F_{34F}\bar{\bar{n}}_{NF3} \qquad (H-219)$$

H-82
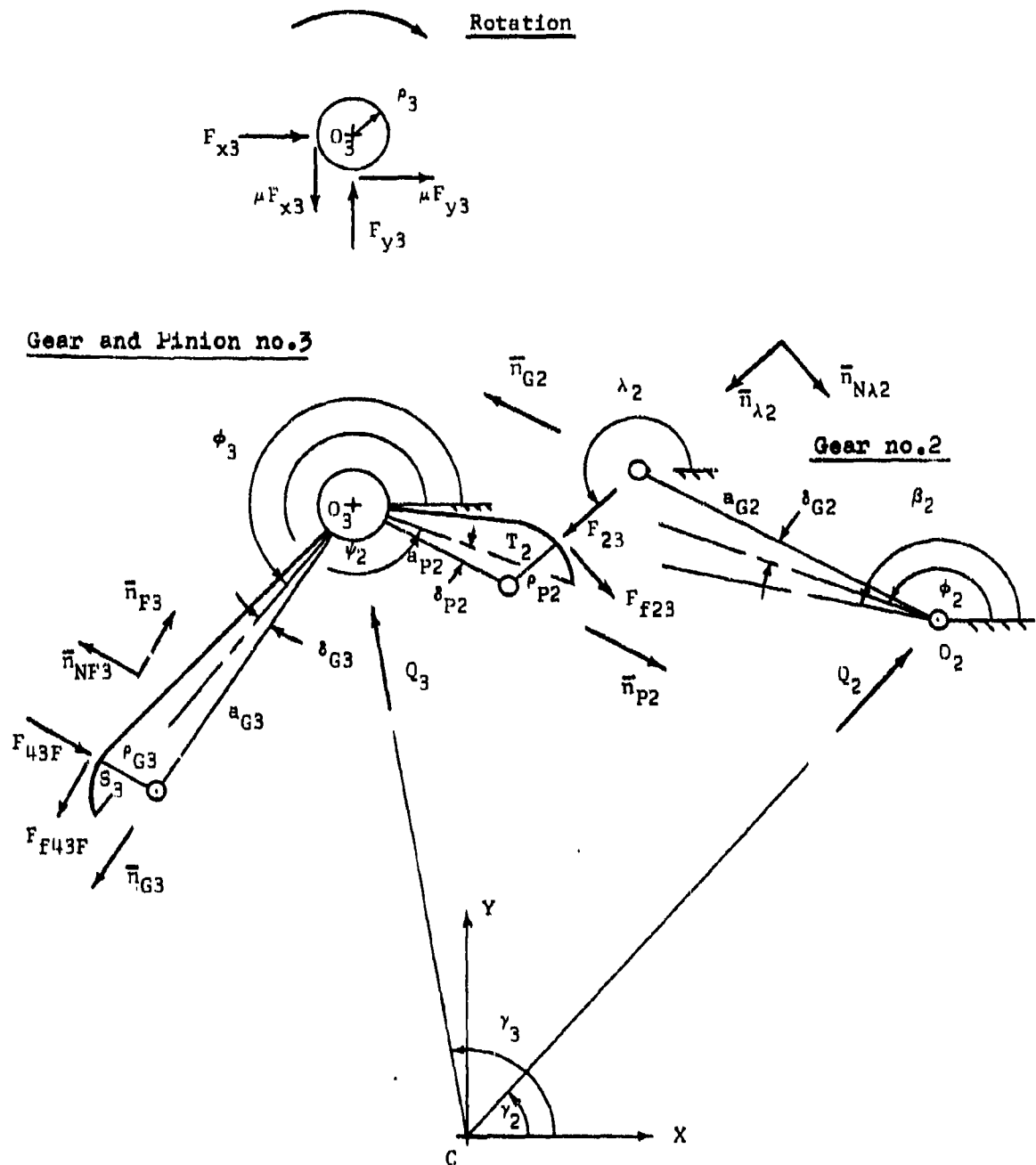
FIGURE H-12

FREE BODY DIAGRAM OF GEAR & PINION NO. 3

MESH NO. 3: ROUND ON FLAT

MESH NO. 2: ROUND ON ROUND

and

$$\bar{F}_{f43F} = -\mu s_{3F} F_{34F} \bar{n}_{F3} \tag{H-220}$$

The forces of gear no. 2 on pinion no. 3 are given by

$$\bar{F}_{23} = F_{23} \bar{n}_{\lambda 2} \tag{H-221}$$

and

$$\bar{F}_{f23} = \mu s_{2R} F_{23} \bar{n}_{N\lambda 2} \tag{H-222}$$

The pivot reactions are chosen in the same manner as before. The centrifugal force $\bar{Q}_3$ was defined by equation (H-24).

Force equilibrium of the gear set requires

$$-F_{34F} \bar{n}_{NF3} - \mu s_{3F} F_{34F} \bar{n}_{F3} + F_{23} \bar{n}_{\lambda 2} + \mu s_{2R} F_{23} \bar{n}_{N\lambda 2} + \bar{Q}_3 + F_{x3} \bar{I}$$

$$+ \mu F_{y3} \bar{I} + F_{y3} \bar{J} - \mu F_{x3} \bar{J} = 0 \tag{H-223}$$

Moment equilibrium about point $O_3$ is given by

$$\mu_{P3}(\widetilde{F}_{x3} + \widetilde{F}_{y3})\bar{k} + \left[a_{G3}\bar{n}_{G3} + \rho_{G3}\bar{n}_{NF3}\right] \times \left[-F_{34F}\bar{n}_{NF3} - \mu_{S3F}F_{34F}\bar{n}_{F3}\right]$$

$$+ \left[a_{P2}\bar{n}_{P2} - \rho_{P2}\bar{n}_{\lambda 2}\right] \times \left[F_{23}\bar{n}_{\lambda 2} + \mu_{S2R}F_{23}\bar{n}_{N\lambda 2}\right] = 0 \qquad (H\text{-}224)$$

Equation (H-223) gives the following component expressions

$$F_{34F}\sin(\psi_3 - \alpha_{P3}) - \mu_{S3F}F_{34F}\cos(\psi_3 - \alpha_{P3}) + Q_3\cos\gamma_3 + F_{x3} + \mu F_{y3}$$

$$+ F_{23}\cos\lambda_2 - \mu_{S2R}F_{23}\sin\lambda_2 = 0 \qquad (H\text{-}225)$$

and

$$-F_{34F}\cos(\psi_3 - \alpha_{P3}) - \mu_{S3F}F_{34F}\sin(\psi_3 - \alpha_{P3}) + Q_3\sin\gamma_3 + F_{y3} - \mu F_{x3}$$

$$+ F_{23}\sin\lambda_2 + \mu_{S2R}F_{23}\cos\lambda_2 = 0 \qquad (H\text{-}226)$$

H-85

The scalar form of the moment equation (H-224) becomes.

$$\mu\rho_3(\widetilde{F}_{x3} + \widetilde{F}_{y3}) + a_{G3}F_{34F}\left[-\cos(\phi_3 + \delta_{G3} - \psi_3 + \alpha_{P3})\right.$$

$$\left. + \mu a_{3F}\sin(\phi_3 + \delta_{G3} - \psi_3 + \alpha_{P3})\right] + \mu a_{3F}\rho_{G3}F_{34F}$$

$$+ a_{P2}F_{23}\left[-\sin(\psi_2 - \delta_{P2} - \lambda_2) + \mu a_{2R}\cos(\psi_2 - \delta_{P2} - \lambda_2)\right]$$

$$- \mu a_{2R}\rho_{P2}F_{23} = 0 \tag{H-227}$$

Simultaneous solution of the component equations (H-225) and (H-226) for $F_{x3}$ and $F_{y3}$ results in:

$$F_{x3} = \frac{1}{1 + \mu^2}\left\{F_{34F}\left[-(1 + \mu^2 a_{3F})\sin(\psi_3 - \alpha_{P3}) + \mu(a_{3F} - 1)\cos(\psi_3 - \alpha_{P3})\right]\right.$$

$$+ F_{23}\left[\mu(1 + a_{2R})\sin\lambda_2 + (\mu^2 a_{2R} - 1)\cos\lambda_2\right]$$

$$\left. + Q_3\left[\mu\sin\gamma_3 - \cos\gamma_3\right]\right\} \tag{H-228}$$

and

$$F_{y3} = \frac{1}{1 + \mu^2} \left\{ F_{34F} \left[ \mu(s_{3F} - 1)\sin(\psi_3 - \alpha_{P3}) + (1 + \mu^2 s_{3F})\cos(\psi_3 - \alpha_{P3}) \right] \right.$$

$$+ F_{23} \left[ (\mu^2 s_{2R} - 1)\sin\lambda_2 - \mu(1 + s_{2R})\cos\lambda_2 \right]$$

$$\left. + Q_3 \left[ -\sin\gamma_3 - \mu\cos\gamma_3 \right] \right\} \qquad \text{(H-229)}$$

The sum $\tilde{F}_{x3} + \tilde{F}_{y3}$ in equation (H-227) is now made up from equations (H-228) and (H-229) in the sense of equation (A-3b)

$$\tilde{F}_{x3} + \tilde{F}_{y3} = F_{34F}A_{59} + F_{23}A_{60} + Q_3 A_{61} + F_{34F}A_{62} + F_{23}A_{63} + Q_3 A_{64}$$

$$\text{(H-230)}$$

where

$$A_{59} = \left| \frac{-(1 + \mu^2 s_{3F})\sin(\psi_3 - \alpha_{P3}) + \mu(s_{3F} - 1)\cos(\psi_3 - \alpha_{P3})}{1 + \mu^2} \right| \qquad \text{(H-231)}$$

$$A_{60} = \left| \frac{\mu(1 + s_{2R})\sin\lambda_2 + (\mu^2 s_{2R} - 1)\cos\lambda_2}{1 + \mu^2} \right| \qquad \text{(H-232)}$$

$$A_{61} = \left| \frac{\mu\sin\gamma_3 - \cos\gamma_3}{1 + \mu^2} \right| \qquad \text{(H-233)}$$

H-87

$$A_{62} = \left| \frac{\mu(a_{3F} - 1)\sin(\psi_3 - \alpha_{P3}) + (1 + \mu^2 a_{3F})\cos(\psi_3 - \alpha_{P3})}{1 + \mu^2} \right| \qquad (H-234)$$

$$A_{63} = \left| \frac{(\mu^2 a_{2R} - 1)\sin\lambda_2 - \mu(1 + a_{2R})\cos\lambda_2}{1 + \mu^2} \right| \qquad (H-235)$$

$$A_{64} = \left| \frac{-\sin\gamma_3 - \mu\cos\gamma_3}{1 + \mu^2} \right| \qquad (H-236)$$

Equation (H-230) is now substituted into the moment equation (H-227) and the resulting expression is solved for $F_{23}$

$$F_{23} = \frac{-F_{34F}C_{30} - Q_3 C_{31}}{C_{32}} \qquad (H-237)$$

where

$$C_{30} = \mu_{P3}(A_{59} + A_{62}) - a_{G3}\left[\cos(\psi_3 + \delta_{G3} - \psi_3 + \alpha_{P3})\right.$$

$$\left. - \mu a_{3F}\sin(\psi_3 + \delta_{G3} - \psi_3 + \alpha_{P3})\right] + \mu a_{3F}\rho_{G3}$$

$$C_{31} = \mu_{P3}(A_{61} + A_{64})$$

$$C_{32} = \mu_{P3}(A_{60} + A_{63}) - a_{P2}\left[\sin(\psi_2 - \delta_{P2} - \lambda_2) - \mu s_{2R}\cos(\psi_2 - \delta_{P2} - \lambda_2)\right]$$

$$- \mu s_{2R}\rho_{P2}$$

## II.  MOMENT INPUT-OUTPUT RELATIONSHIP

Equation (H-79), which gives the force $F_{23}$ in terms of $M_{in}$, $Q_1$ and $Q_2$ for the appropriate contact combinations, is now set equal to equation (H-237).  Subsequently, one finds the following formulation for $F_{34F}$:

$$F_{34F} = \frac{C_8 C_{32}}{C_6 C_{10} C_{30}}(M_{in} - Q_1 C_9) - Q_2 \frac{C_7 C_{32}}{C_6 C_{30}} - Q_3 \frac{C_{31}}{C_{30}} \qquad \text{(H-238)}$$

The above expression is now set equal to equation (H-195) which gives $F_{34F}$ in terms of $M_{04}$ and $Q_4$.  The determination of the equilibrant moment $M_{047}$ (for case 7:  FRR) is now possible. Thus,

$$M_{o47} = M_{1n} \frac{c_8 c_{26} c_{32}}{c_6 c_{10} c_{30}} - Q_1 \frac{c_8 c_9 c_{26} c_{32}}{c_6 c_{10} c_{30}} - Q_2 \frac{c_7 c_{26} c_{32}}{c_6 c_{30}}$$

$$- Q_3 \frac{c_{26} c_{31}}{c_{30}} - Q_4 c_{25} \qquad\qquad (H\text{-}239)$$

## h. CASE NO. 8: FRF

### MOMENT INPUT-OUTPUT RELATIONSHIP

The input-output relationship for this contact combination can also be assembled entirely from existing expressions. With mesh no. 2 in the round on round phase of contact and mesh no. 1 in the round on flat one, the relationship between force $F_{23}$ and the moment $M_{in}$ is that of case no. 2. Equation (H-116) is applicable. The input-output relationship of gear and pinion set no. 3, which relates $F_{34F}$ to $F_{23}$, was derived for case no. 7 and is given by equation (H-237). Finally, with mesh no. 3 in the round on flat phase, one uses equation (H-195) for the relationship between force $F_{34F}$ and moment $M_{o4}$.
Thus, equation (H-116) is first set equal to equation (H-237) to obtain an expression for force $F_{34F}$

$$F_{34F} = \frac{c_{13}c_{32}}{c_{11}c_{15}c_{30}}(M_{in} - Q_1 c_{14}) - Q_2 \frac{c_{12}c_{32}}{c_{11}c_{30}} - Q_3 \frac{c_{31}}{c_{30}} \qquad \text{(H-240)}$$

H-91

The above expression is now set equal to equation (H-195).  This

allows the determination of $M_{o48}$ (for case 8:  FRF)

$$M_{o48} = M_{in} \frac{c_{13}c_{26}c_{32}}{c_{11}c_{15}c_{30}} - Q_1 \frac{c_{13}c_{14}c_{26}c_{32}}{c_{11}c_{15}c_{30}} - Q_2 \frac{c_{12}c_{26}c_{32}}{c_{11}c_{30}}$$

$$- Q_3 \frac{c_{26}c_{31}}{c_{30}} - Q_4 c_{25} \qquad \text{(H-241)}$$

## 2. INPUT-OUTPUT ANALYSIS OF TWO STEP-UP GEAR TRAIN

The following gives derivations for the moment input-output relationships associated with the four possible contact combinations of a two step-up gear train. (See Table H-2).

### a. CASE NO. 1: RR

For this contact combination, the relationship between the equilibrant moment $M_{o3}$ and force $F_{23}$, both acting on pinion no. 3, must first be newly obtained. The relationships between forces $F_{23}$ and $F_{12}$ of gear and pinion set no. 2, as well as between force $F_{12}$ and input moment $M_{in}$ of gear no. 1, may be taken from case no. 1 of the three step-up gear train analysis. Equations (H-61) and (H-78), respectively, are applicable.

### I. FORCE AND MOMENT EQUILIBRIA OF PINION NO. 3

Figure H-13 shows a schematic free body diagram of pinion no. 3 in the round on round phase of contact. The equilibrant

H-93
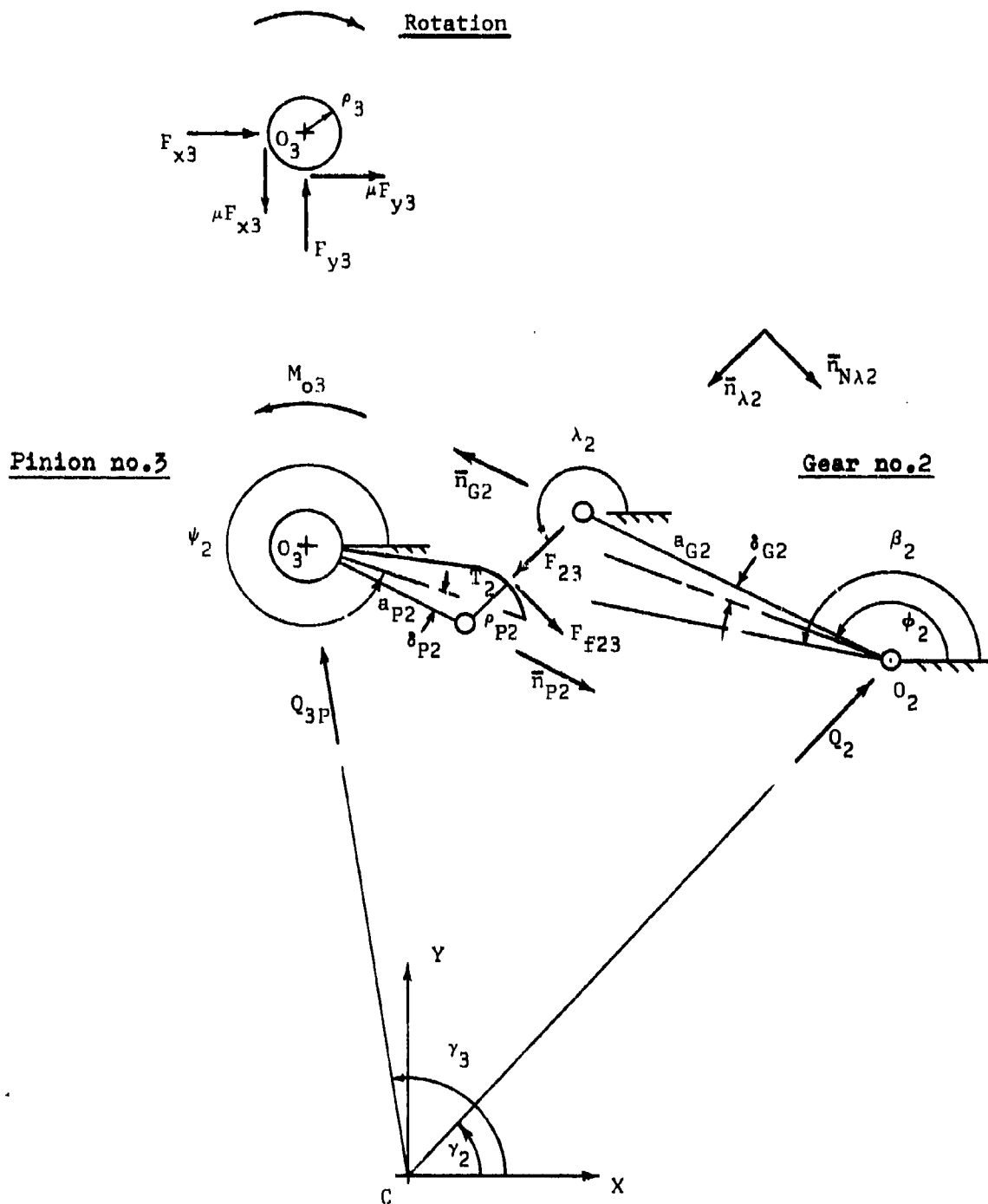
FIGURE H-13

FREE BODY DIAGRAM OF PINION NO. 3
MESH NO. 2: ROUND ON ROUND

moment $M_{o3}$ acts in the direction opposite to the clockwise rotation of the pinion. The normal contact force of gear no. 2 on pinion no. 3 is given by

$$\overline{F}_{23} = F_{23}\overline{n}_{\lambda 2} \tag{H-242}$$

The associated friction force becomes

$$\overline{F}_{f23} = \mu s_{2R} F_{23}\overline{n}_{N\lambda 2} \tag{H-243}$$

The pivot reactions are chosen in the usual manner.

The centrifugal force $\overline{Q}_{3P}$ is now due to the mass $m_{3P}$ of the pinion alone, i.e.,

$$\overline{Q}_{3P} = Q_{3P}(\cos\gamma_3\overline{I} + \sin\gamma_3\overline{J}) \tag{H-244}$$

where

$$Q_{3P} = m_{3P}\ell_3\omega^2 \tag{H-245}$$

Force equilibrium of the pinion is assured by

$$F_{23}\bar{n}_{\lambda 2} + \mu s_{2R}F_{23}\bar{n}_{N\lambda 2} + F_{x3}\bar{i} + \mu F_{y3}\bar{i} + F_{y3}\bar{j} - \mu F_{x3}\bar{j} + \bar{Q}_{3P} = 0$$

$$\text{(H-246)}$$

Moment equilibrium about point $O_3$ is given by

$$\mu \rho_3(\tilde{F}_{x3} + \tilde{F}_{y3})\bar{k} + M_{o3}\bar{k} + \left[a_{P2}\bar{n}_{P2} - \rho_{P2}\bar{n}_{\lambda 2}\right] \times \left[F_{23}\bar{n}_{\lambda 2} + \mu s_{2R}F_{23}\bar{n}_{N\lambda 2}\right]$$

$$= 0 \qquad \text{(H-247)}$$

Equation (H-246) gives the following component expressions

$$F_{23}\cos\lambda_2 - \mu s_{2R}F_{23}\sin\lambda_2 + F_{x3} + \mu F_{y3} + Q_{3P}\cos\gamma_3 = 0 \qquad \text{(H-248)}$$

and

$$F_{23}\sin\lambda_2 + \mu s_{2R}F_{23}\cos\lambda_2 + F_{y3} - \mu F_{x3} + Q_{3P}\sin\gamma_3 = 0 \qquad \text{(H-249)}$$

The scalar form of the moment equation (H-247) becomes

$$\mu \rho_3(\widetilde{F}_{x3} + \widetilde{F}_{y3}) + M_{o3} - \mu s_{2R} \rho_{P2} F_{23} + a_{P2} F_{23}\left[-\sin(\psi_2 - \delta_{P2} - \lambda_2)\right.$$

$$+ \left. \mu s_{2R}\cos(\psi_2 - \delta_{P2} - \lambda_2)\right] = 0 \qquad\qquad (H\text{-}250)$$

Simultaneous solution of equations (H-248) and (H-249) for the forces $F_{x3}$ and $F_{y3}$ gives

$$F_{x3} = \frac{1}{1 + \mu^2}\left\{F_{23}\left[\mu(1 + s_{2R})\sin\lambda_2 + (\mu^2 s_{2R} - 1)\cos\lambda_2\right]\right.$$

$$\left. + Q_{3P}\left[\mu\sin\gamma_3 - \cos\gamma_3\right]\right\} \qquad\qquad (H\text{-}251)$$

and

$$F_{y3} = \frac{1}{1 + \mu^2}\left\{F_{23}\left[(\mu^2 s_{2R} - 1)\sin\lambda_2 - \mu(1 + s_{2R})\cos\lambda_2\right]\right.$$

$$\left. + Q_{3P}\left[-\sin\gamma_3 - \mu\cos\gamma_3\right]\right\} \qquad\qquad (H\text{-}252)$$

The sum $\widetilde{F}_{x3} + \widetilde{F}_{y3}$ of equation (H-250) is now made up from equations (H-251) and (H-252) in the sense of equation (A-3b)

$$\widetilde{F}_{x3} + \widetilde{F}_{y3} = F_{23}A_{65} + Q_{3P}A_{66} + F_{23}A_{67} + Q_{3P}A_{68} \qquad \text{(H-253)}$$

where

$$A_{65} = \left| \frac{\mu(1 + s_{2R})\sin\lambda_2 + (\mu^2 s_{2R} - 1)\cos\lambda_2}{1 + \mu^2} \right| \qquad \text{(H-254)}$$

$$A_{66} = \left| \frac{\mu\sin\gamma_3 - \cos\gamma_3}{1 + \mu^2} \right| \qquad \text{(H-255)}$$

$$A_{67} = \left| \frac{(\mu^2 s_{2R} - 1)\sin\lambda_2 - \mu(1 + s_{2R})\cos\lambda_2}{1 + \mu^2} \right| \qquad \text{(H-256)}$$

$$A_{68} = \left| \frac{-\sin\gamma_3 - \mu\cos\gamma_3}{1 + \mu^2} \right| \qquad \text{(H-257)}$$

Equation (H-253) is now substituted into the moment equation (H-250) and the result is solved for $F_{23}$

$$F_{23} = \frac{-M_{o3} - Q_{3P}C_{33}}{C_{34}} \qquad \text{(H-258)}$$

H-98

where

$$C_{33} = \mu\rho_3(A_{66} + A_{68})$$

$$C_{34} = \mu\rho_3(A_{65} + A_{67}) - \mu s_{2R}\rho_{P2}$$

$$+ a_{P2}\left[\mu s_{2R}\cos(\psi_2 - \delta_{P2} - \lambda_2) - \sin(\psi_2 - \delta_{P2} - \lambda_2)\right]$$

## II.  MOMENT INPUT-OUTPUT RELATIONSHIP

In the derivation for case 1 of the three step-up gear train it was shown that if one sets equations (H-61) and (H-78) equal to each other, one obtains the following relationship [i.e., equation (H-79)] between force $F_{23}$ and the input moment $M_{1n}$:

$$F_{23} = \frac{-c_8}{c_6 c_{10}} (M_{1n} - Q_1 c_9) + Q_2 \frac{c_7}{c_6} \qquad (\text{H-259})$$

The above expression is now set equal to equation (H-258), and the result is solved for the equilibrant moment $M_{o31}$ (for case 1: RR)

$$M_{o31} = M_{1n} \frac{c_8 c_{34}}{c_6 c_{10}} - Q_1 \frac{c_8 c_9 c_{34}}{c_6 c_{10}} - Q_2 \frac{c_7 c_{34}}{c_6} - Q_{3P} c_{33} \qquad (\text{H-260})$$

b. CASE NO. 2: RF

MOMENT INPUT-OUTPUT RELATIONSHIP

The moment equation for the present case, in which mesh
no. 1 is in the round on flat phase and mesh no. 2 in the round
on round one, may be derived entirely from existing relationships.
Equation (H-116) gives an expression for the force $F_{23}$ in terms
of the input moment $M_{in}$ and the centrifugal forces $Q_1$ and $Q_2$
for the present combination of contacts in both meshes. When this
expression, from case no. 2 of the three step-up gear train, is
set equal to equation (H-259) of case no. 1 of the two step-up
gear train, one obtains for $M_{o32}$ (case no. 2: RF)

$$M_{o32} = M_{in} \frac{c_{13}c_{34}}{c_{11}c_{15}} - Q_1 \frac{c_{13}c_{14}c_{34}}{c_{11}c_{15}} - Q_2 \frac{c_{12}c_{34}}{c_{11}} - Q_{3p}c_{33}$$

(H-261)

H-101

## c. CASE NO. 3:  FF

For this contact combination, where both meshes no. 1 and no.2 are in the round on flat phase, the relationship between the equilibrant moment $M_{o3}$ and the normal contact force $F_{23F}$, both acting on pinion no. 3, must first be determined.  The resulting expression in $F_{23F}$ can then be set equal to the relationship between $F_{23F}$ and the input moment $M_{in}$, which is given by equation (H-156), and which was derived in conjunction with case no. 3 of the three step-up gear train.

## I.  FORCE AND MOMENT EQUILIBRIA OF PINION NO. 3

Figure H-14 shows the free body diagram of pinion no. 3 in the round on flat phase of contact.  Again, the equilibrant moment $M_{o3}$ acts in a counter-clockwise direction.  The normal force $F_{23F}$ of gear no. 2 on pinion no. 3 is given by

$$\bar{F}_{23F} = -F_{23F}\bar{n}_{NF2} \qquad (H\text{-}262)$$
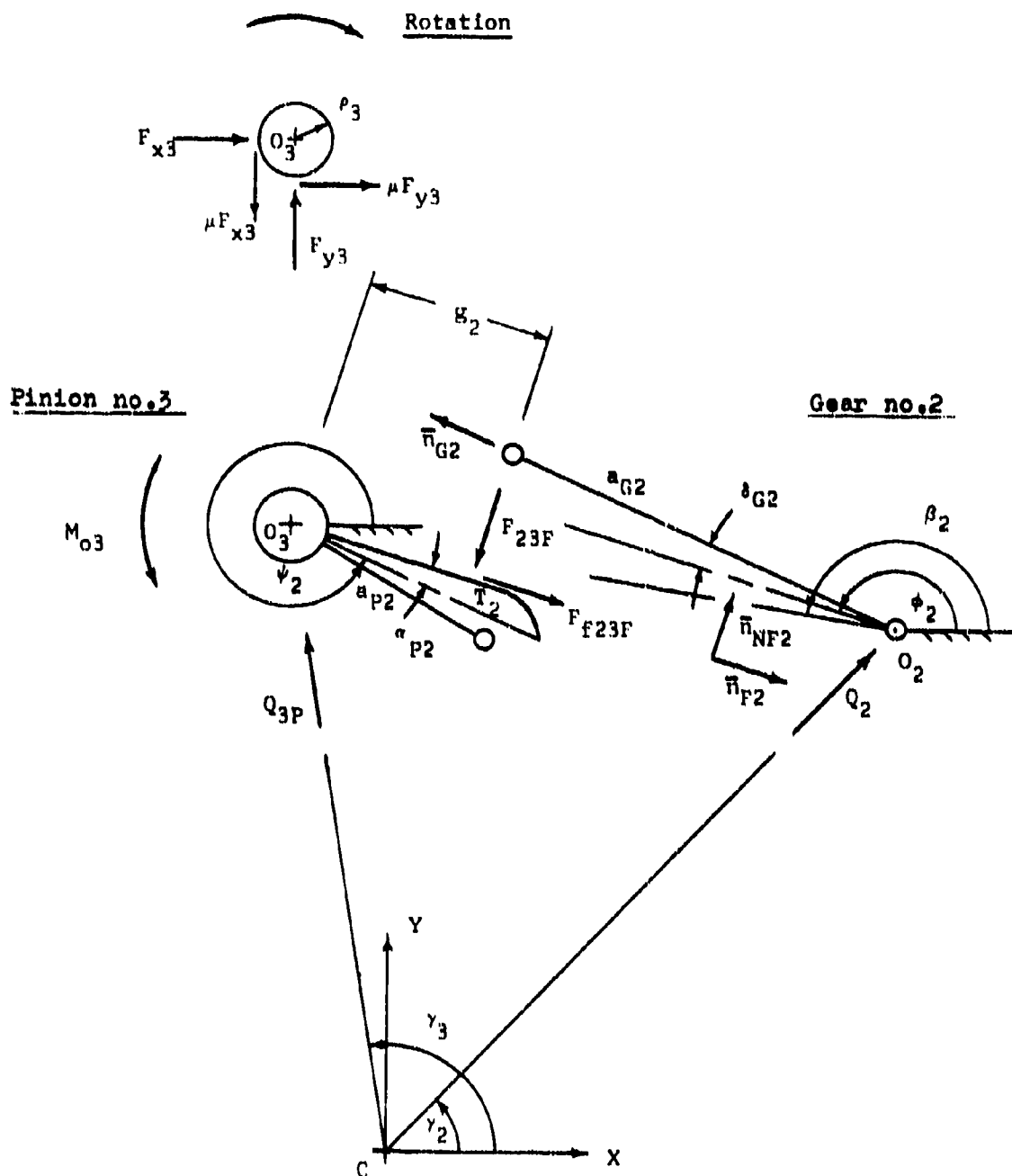
FIGURE H-14

FREE BODY DIAGRAM OF PINION NO. 3
MESH NO. 2:  ROUND ON FLAT

H-103

The associated friction force of gear no. 2 on pinion no. 3 becomes

$$\bar{F}_{f23F} = \mu s_{2F} F_{23F} \bar{n}_{F2} \tag{H-263}$$

The pivot reactions and pivot friction forces are chosen in the usual manner. The centrifugal force $\bar{Q}_{3P}$ was given previously by equation (H-244).

The force equilibrium expression becomes

$$-F_{23F}\bar{n}_{NF2} + \mu s_{2F} F_{23F}\bar{n}_{F2} + F_{x3}\bar{I} + \mu F_{y3}\bar{I} + F_{y3}\bar{J} - \mu F_{x3}\bar{J} + \bar{Q}_{3P}$$

$$= 0 \tag{H-264}$$

Moment equilibrium about point $O_3$ is assured by

$$\mu \rho_3(\tilde{F}_{x3} + \tilde{F}_{y3})\bar{k} + M_{o3}\bar{k} + \beta_2\bar{n}_{F2} \times (-)F_{23F}\bar{n}_{NF2} = 0 \tag{H-265}$$

As always before, the friction force $F_{f23F}$ exerts no moment about point $O_3$.

H-104

Equation (H-264) gives the following component expressions

$$F_{23F}\sin(\psi_2 + \alpha_{P2}) + \mu s_{2F}F_{23F}\cos(\psi_2 + \alpha_{P2}) + F_{x3} + \mu F_{y3} + Q_{3P}\cos\gamma_3$$

$$= \quad 0 \qquad\qquad (H-266)$$

and

$$-F_{23F}\cos(\psi_2 + \alpha_{P2}) + \mu s_{2F}F_{23F}\sin(\psi_2 + \alpha_{P2}) + F_{y3} - \mu F_{x3} + Q_{3P}\sin\gamma_3$$

$$= \quad 0 \qquad\qquad (H-267)$$

The scalar form of the moment equation (H-265) is given by

$$\mu P_3(\tilde{F}_{x3} + \tilde{F}_{y3}) + M_{o3} - s_2 F_{23F} = 0 \qquad\qquad (H-268)$$

Simultaneous solution of equations (H-266) and (H-267) for $F_{x3}$
and $F_{y3}$ leads to

$$F_{x3} = \frac{1}{1 + \mu^2}\left\{F_{23F}\left[(\mu^2 s_{2F} - 1)\sin(\psi_2 + \alpha_{P2}) - \mu(1 + s_{2F})\cos(\psi_2 + \alpha_{P2})\right]\right.$$

$$\left. + Q_{3P}\left[\mu\sin\gamma_3 - \cos\gamma_3\right]\right\} \qquad\qquad (H-269)$$

$$F_{y3} = \frac{1}{1 + \mu^2} \left\{ F_{23F} \left[ -\mu(1 + s_{2F})\sin(\psi_2 + \alpha_{P2}) + (1 - \mu^2 s_{2F})\cos(\psi_2 + \alpha_{P2}) \right] \right.$$

$$\left. + Q_{3P} \left[ -\sin\gamma_3 - \mu\cos\gamma_3 \right] \right\} \qquad \text{(H-270)}$$

The sum $\widetilde{F}_{x3} + \widetilde{F}_{y3}$ of equation (H-268) is now made up of equations (H-269) and (H-270) in the sense of equation (A-3b)

$$\widetilde{F}_{x3} + \widetilde{F}_{y3} = F_{23F}A_{69} + Q_{3P}A_{70} + F_{23F}A_{71} + Q_{3P}A_{72} \qquad \text{(H-271)}$$

where

$$A_{69} = \left| \frac{(\mu^2 s_{2F} - 1)\sin(\psi_2 + \alpha_{P2}) - \mu(1 + s_{2F})\cos(\psi_2 + \alpha_{P2})}{1 + \mu^2} \right| \qquad \text{(H-272)}$$

$$A_{70} = \left| \frac{\mu\sin\gamma_3 - \cos\gamma_3}{1 + \mu^2} \right| \qquad \text{(H-273)}$$

$$A_{71} = \left| \frac{-\mu(1 + s_{2F})\sin(\psi_2 + \alpha_{P2}) + (1 - \mu^2 s_{2F})\cos(\psi_2 + \alpha_{P2})}{1 + \mu^2} \right| \qquad \text{(H-274)}$$

$$A_{72} = \left| \frac{-\sin\gamma_3 - \mu\cos\gamma_3}{1 + \mu^2} \right| \qquad \text{(H-275)}$$

Equation (H-271) is now substituted into the moment equation (H-268), and the result is solved for $F_{23F}$

$$F_{23F} = \frac{-M_{o3} - Q_{3P}C_{35}}{C_{36}} \qquad \text{(H-276)}$$

where

$$C_{35} = \mu P_3(A_{70} + A_{72})$$

$$C_{36} = \mu P_3(A_{69} + A_{71}) - \epsilon_2$$

## II. MOMENT INPUT-OUTPUT RELATIONSHIP

The moment input-output relationship for the present case is obtained, as stated earlier, by setting equation (H-276) equal to equation (H-156) and solving the resulting expression for the equilibrant moment $M_{o33}$ (case no. 3: FF)

$$M_{o33} = M_{1n}\frac{C_{21}C_{36}}{C_{15}C_{19}} - Q_1\frac{C_{14}C_{21}C_{36}}{C_{15}C_{19}} - Q_2\frac{C_{20}C_{36}}{C_{19}} - Q_{3P}C_{35} \qquad \text{(H-277)}$$

## d. CASE NO. 4: FR

## MOMENT INPUT-OUTPUT RELATIONSHIP

The moment input-output relationship for this case, where mesh no. 1 is in round on round contact while mesh no. 2 is in the round on flat phase, may also be derived entirely by assembling existing relationships.

Equation (H-276), of the previous section, gives force $F_{23F}$ in terms of the equilibrant moment $M_{o3}$ when mesh no. 2 is in the round on flat phase. Equation (H-178), derived for case no. 4 (RFR) of the three step-up gear train, relates this force $F_{23F}$ to the input moment $M_{in}$.

Thus, one first sets equations (H-178) and (H-276) equal to each other and then solves the result for $M_{o34}$ (case 4: FR)

$$M_{o34} = M_{in} \frac{C_{24}C_{36}}{C_{10}C_{22}} - Q_1 \frac{C_9 C_{24} C_{36}}{C_{10}C_{22}} - Q_2 \frac{C_{23}C_{36}}{C_{22}} - Q_{3P}C_{35}$$

(H-278)

# APPENDIX I

## COMPUTER MODELS FOR THREE AND TWO STEP-UP GEAR TRAINS
## WITH CLOCK TEETH OPERATING IN A SPIN ENVIRONMENT

The following appendix contains descriptions, listings and sample outputs of the computer models relating to step-up gear trains containing clock (ogival) gear teeth:

1.  Program CLOCK 3:  Point and cycle efficiencies for three pass clock (ogival) step-up gear train in spin environment.

2.  Program CLOCK 4:  Point and cycle efficiencies for two pass clock (ogival) step-up gear train in spin environment.

The relevant background, the input parameters, the manner of the computations and the form of the output of each program are discussed in detail.  The program proper forms the last part of each section.

1. **Program CLOCK 3: Point and Cycle Efficiencies for Three**

   **Pass Clock (Ogival) Step-Up Gear Train**

   **in Spin Environment**

The kinematics of program CLOCK 3 is based on the work in Appendix G, while the moment input-output relationships are derived in section 1 of Appendix H. The nomenclature of the program is chosen to coincide as much as possible with the above appendices. It is to be noted that, even though the fuze related geometry produces different expressions for the various meshes, the kinematic computations of the individual meshes are very similar to those shown in CLOCK 1 in Appendix F for the single mesh in the standard position. It is also assumed that all three meshes will have been tested by program CLOCK 1 for their geometric suitability, i.e. whether there is enough room for tip radii.

a.  **Input Parameters** (see Program CLOCK 3, below)

The following parameters represent the input data for the program (for explanations and nomenclature, see sections 1 and 2 of Appendix F, as well as section 3 of Appendix C):

MU,  coefficient of friction, as before

RPM,  spin velocity

CAPRP1, CAPRP2, CAPRP3, RP2, RP3, RP4,  pitch radii of gears and pinions with nomenclature of Fig. G-1

RHOG1, RHOG2, RHOG3, RHOP1, RHOP2, RHOP3,  radii of curvature of circular arc portion of gear and pinion teeth

ACG1, ACG2, ACG3[*] = $a_{CG_i}$, distance from the center of rotation of the gear of the $i^{th}$ mesh to the center of curvature of the circular arc portion of the gear tooth. (Unless otherwise noted, this and all following numbering schemes refer to those associated with the mesh mechanics as given in the text of Appendices G and H.)

ACP1, ACP2, ACP3 = $a_{CP_i}$, distance from the center of rotation of the pinion of the $i^{th}$ mesh to the center of curvature of the circular arc portion of the pinion tooth

R1, R2, R3, R4 = $\mathcal{R}_i$ (nomenclature of Fig. G-1)

TG1, TG2, TG3, TP1, TP2, TP3, maximum thickness of gear and pinion teeth (mesh nomenclature)

NG1, NG2, NG3, NP2, NP3, NP4, numbers of teeth in various gears and pinions (nomenclature of Fig. G-1)

RHO1, RHO2, RHO3, RHO4, gear and/or pinion pivot radii (nomenclature of Fig. G-1)

M1, M2, M3, M4, masses of gear and/or pinion combinations

MD, see program INVOL 3

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[*]Since many parts of the computer program were written before the nomenclature for these distances was changed in the report from $a_{CG_i}$ and $a_{CP_i}$ to $a_{G_i}$ and $a_{P_i}$, there is a certain discrepancy between the program and the report.

K, range divisor

PHDOT1 = -1, all velocity computations are based on this value. The input motion in the fuze gearing model is negative (see Fig. (G-1)).

b. **Computations (see also COMMENT cards in program)**

I. **Computation of Gear Tooth Parameters**

The tooth parameters of the gears and pinions of all three meshes are first computed. These computations are essentially the same as those shown in program CLOCK 1 for a single mesh. Certain parameters are omitted because they have been checked separately by using CLOCK 1 and are not required for the kinematics of CLOCK 3.

In addition, the pivot to pivot distances B1, B2 and B3 are obtained.

II. **Computation of MIN, GAMMAS and BETAS**

To begin with, the program computes the input moment

$$\text{MIN} = M_{in} = md^2\omega^2 \qquad (I-1)$$

Subsequently, the angles $\gamma_2$, $\gamma_3$, $\gamma_4$ and $\beta_1$, $\beta_2$, $\beta_3$ are established according to the expressions of section 6b of Appendix A.

III. **Computation of Other Parameters**

The angles $\Delta\varphi_1$ and $\Delta\psi_1$ between the centerlines of adjacent gear and pinion teeth, respectively, are determined in this

section of the computations. In addition, the lengths $L_i$ are found (see eqs. (G-7), (G-53) and (G-88)). Finally, the centrifugal forces $Q_1$, $Q_2$, $Q_3$ and $Q_4$ are computed according to eqs. (H-65), (H-46), (H-25) and (H-6), respectively.

## IV. Proliminary Computations for Mesh 1

### A. Determination of Transition Angle

The primary consideration for determining the transition angles in the fuze related clock gear meshes is identical with that used in Appendix F. The transition angle $\psi_T$ is established as that angle for which, depending upon whether the input angle $\varphi$ has counterclockwise or clockwise motion, a small increase or decrease in $\varphi$, respectively, will cause the associated value of g to become smaller than its transition value $f_p$. Since the gear of mesh 1 turns in a clockwise direction, the above increment of $\varphi$ will be negative.

The program uses this criterion in the following manner:

(a) Transition angles $\psi_{1T1}$ and $\psi_{1T2}$ are computed according to eq. (G-39).

(b) The subroutine TRANS1 (which is valid for meshes in which the input gear has clockwise rotation, as is the case also for mesh 3) is called, and the angle $\varphi_{1T1}$ (PHIT), which is associated with $\psi_{1T1}$, is computed with the help of eqs. (G-40) and (G-41).

(c) The angle $\varphi$ is made slightly smaller than $\varphi_{1T1}$ to produce the angle PHINEXT, and eq. (G-29) is used to find the associated angle PSINEX. Since there are two such angles, the

subroutine selects the one which is closest in value to the transition angle $\psi_{1T1}$. Subsequently, the associated value of $g_{11}$ is computed according to eq. (G-27).

(d) Steps (b) and (c) are then repeated identically for the second transition angle $\psi_{1T2}$. This results in the determination of $g_{12}$.

(e) Control returns to the main program, and that value of $\psi_{1T}$ is chosen for which the associated value of $g_1$ is smaller than $f_{p1}$.

For checking, a subsidiary test, which is similar to the one shown in Appendix F, is added to the program. It is based on the idea that, for the correct transition angle $\psi_{1T}$, the line representing the flat portion of the pinion will make a smaller angle with the centerline $O_1O_2$ than will be the case for the incorrect one. TEST11 and TEST12 find these angles with the help of the expressions shown below. These expressions hold for all values of $\beta_1$ and make use of a new variable $\psi_{test}$, which had to be introduced since the tests require that the transition angles be expressed in a range between $-180^\circ$ and $+180^\circ$. Thus,

For $0^\circ < \psi_{test} < 180^\circ$

$$\text{TEST11} = |\pi - \beta_1 + \psi_{test} - \alpha_{P1}| \qquad (I-2)$$

For $-180^\circ < \psi_{test} < 0^\circ$

$$\text{TEST12} = |\pi + \beta_1 - (\psi_{test} + 2\pi - \alpha_{P1})| \qquad (I-3)$$

I-6

To determine the angle $\psi_{test}$, let

$$\psi_{test} = \psi_{1T} \quad \text{if} \quad -180° < \psi_{1T} < 180° \qquad (I-4)$$

$$\psi_{test} = \psi_{1T} + 2\pi \quad \text{if} \quad \psi_{1T} < -180° \qquad (I-5)$$

$$\psi_{test} = \psi_{1T} - 2\pi \quad \text{if} \quad \psi_{1T} > 180° \qquad (I-6)$$

### B. Determination of Correct Sign for Round on Flat Regime

The sign preceding the square root in eq. (G-29), for the round on flat regime, is determined with the help of $\varphi_{1T}$. The condition yielding that angle $\psi_{1F}$ which is closest to the angle $\psi_{1T}$ governs. The variable SIGN1F is used for the sign in question.

### C. Computation of Final and Initial Values of $\varphi_1$ and $\psi_1$

The final and initial values of the gear and pinion angles $\varphi_1$ and $\psi_1$, respectively, are found by continuously evaluating the round on flat regime eq. (G-29), using the previously determined value of SIGN1F, and simultaneously checking the contact condition for the subsequent set of teeth as given by eq. (G-46). This loop is initiated at the transition angle $\varphi_{1T}$, and it is terminated when the condition of eq. (G-46) is met. This allows the determination of the angles PHI1F and PSI1FF, at which the first set of teeth loses contact, as well as of the angles PHI1I and PSI1I at which the second set of teeth simultaneously comes into engagement. The initial engagement angles PHI1I and PSI1I are obtained by adding $\varphi_1$ to the "loss of contact" angle PHI1F

I-7

and by subtracting $\Delta\psi_1$ from the "loss of contact" angle PSI1FF.


D. <u>Determination of Correct Sign for Round on Round Regime</u>

Eq. (G-12) is used to determine the angle $\psi_1$, while the gear and pinion are in the round on round regime. The correct sign for this expression is obtained by comparing the value $\psi_1$, as computed with PHI1I, with the value for PSI1I. SIGN1R is the variable used for the desired sign.


V. <u>Preliminary Computations for Mesh 2</u>

A. <u>Determination of Transition Angle</u>

The primary criterion for determining the transition angle is again similar to that used in Appendix F and described earlier for mesh 1.

(a) Transition angles $\psi_{2T1}$ and $\psi_{2T2}$ are computed according to eq. (G-79).

(b) The subroutine TRANS2, which is valid for meshes in which the input gear has counterclockwise rotation, is called, and the angle $\varphi_{2T1}$, which is associated with $\psi_{2T1}$ is computed, with the help of eqs. (G-80) and (G-81).

(c) The angle $\varphi_2$ is made slightly larger than $\varphi_{2T1}$ to produce the angle PHINEXT, and eq. (G-71) is used to find the associated output angle PSINEX. Since there are two such angles, the subroutine selects the one which is closest to the transition value $\psi_{2T1}$. Subsequently, the associated value of $g_{21}$ is computed according to eq. (G-69).

(d)  Steps (b) and (c) are then repeated identically for the second transition angle $\psi_{2T2}$. This results in the determination of $g_{22}$.

(e)  Control returns to the main program, and that value of $\psi_{2T}$ is chosen for which the associated value of $g_2$ is smaller than $f_{p2}$.

The procedure for the associated subsidiary test for the transition angle is similar to that for mesh 1 and is given by

For  $0° < \psi_{test} < 180°$

$$TEST21 = |\pi - \beta_2 + \psi_{test} + \alpha_{P2}| \qquad (I-7)$$

For $-180° < \psi_{test} < 0°$

$$TEST22 = |\beta_2 + \pi - (\psi_{test} + 2\pi + \alpha_{P2})| \qquad (I-8)$$

To determine the angle $\psi_{test}$, let

$$\psi_{test} = \psi_{2T} \quad \text{if} \quad -180° < \psi_{2T} < 180° \qquad (I-9)$$

$$\psi_{test} = \psi_{2T} + 2\pi \quad \text{if} \quad \psi_{2T} < -180° \qquad (I-10)$$

$$\psi_{test} = \psi_{2T} - 2\pi \quad \text{if} \quad \psi_{2T} > 180° \qquad (I-11)$$

B.  <u>Determination of Correct Sign for Round on Flat Regime</u>

The sign preceding the square root in eq. (G-71), for the round on flat regime, is determined with the help of $\varphi_{2T}$. The condition yielding that angle $\psi_{2F}$ which is closest to the angle $\psi_{2T}$ governs. The variable SIGN2F is used for the sign in question.

### C. Computation of Final and Initial Values of $\varphi_2$ and $\psi_2$

The final and initial values of the gear and pinion angles $\varphi_2$ and $\psi_2$, respectively, are found by continuously evaluating the round on flat eq. (G-71), using the previously determined value of SIGN2F, and simultaneously checking the contact condition for the subsequent set of teeth, as given by eq. (G-86). This loop is initiated at the transition angle $\varphi_{2T}$ and is terminated when the condition of eq. (G-86) is met. (Recall that in meshes 1 and 3 the driving gear turns clockwise, while in mesh 2 it turns in a counterclockwise direction.) This allows the determination of the two angles PHI2F and PSI2FF at which the first set of teeth loses contact as well as of the angles PHI2I and PSI2I at which the second set of teeth simultaneously comes into contact. The initial engagement angles PHI2I and PSI2I are obtained by subtracting $\Delta\varphi_2$ from the "loss of contact" angle PHI2F and by adding $\Delta\psi_2$ to the "loss of contact" angle PSI2FF.

### D. Determination of Correct Sign for Round on Round Regime

Eq. (G-58) is used to determine the angle $\psi_2$ while the gear and pinion are in the round on round phase of motion. The correct sign for this expression is obtained by comparing the value of $\psi_2$, as computed with PHI2I, with the previously obtained value for PSI2I. SIGN2R is the variable used for the desired sign.

## VI. Preliminary Computations for Mesh 3

### A. Determination of Transition Angle

The determination of the transition angles for mesh 3 runs along parallel lines to the one shown for mesh 1 since the driving gear also rotates in a clockwise direction. In all cases, the parameters of section 3 of Appendix G are used.

(a) Transition angles $\psi_{3T1}$ and $\psi_{3T2}$ are computed with the help of eq. (G-99).

(b) The subroutine TRANS1 determines the angle $\varphi_{3T1}$, associated with $\psi_{3T1}$, according to eqs. (G-100) and (G-101).

(c) PHINEXT, which is now obtained by a decrease of the angle $\varphi_3$ from $\varphi_{3T1}$, serves as the input variable of eq. (G-94), and is used to determine PSINEX. Appropriate controls, as described before, determine the angle $\psi_{3T1}$. In addition, the associated value of $g_{31}$ is computed with the help of eq. (G-95).

(d) Steps (b) and (c) are again repeated for the second transition angle $\psi_{3T2}$ and $g_{22}$ is determined.

(e) After control is returned to the main program, that value of $\psi_{3T}$ is chosen for which the associated value of $g_3$ is smaller than $f_{p3}$.

The subsidiary test for the transition angles runs parallel to that described for mesh 1, i.e.,

For $0° < \psi_{test} < 180°$

$$TEST31 = |\pi - \beta_3 + \psi_{test} - \alpha_{p3}| \qquad (I-12)$$

For $-180° < \psi_{test} < 0°$

$$TEST32 = |\pi + \beta_3 - (\psi_{test} + 2\pi - \alpha_{P3})| \qquad (I-13)$$

To determine the angle $\psi_{test}$, let

$$\psi_{test} = \psi_{3T} \quad \text{if} \quad -180° < \psi_{3T} < 180° \qquad (I-14)$$

$$\psi_{test} = \psi_{3T} + 2\pi \quad \text{if} \quad \psi_{3T} < -180° \qquad (I-15)$$

$$\psi_{test} = \psi_{3T} - 2\pi \quad \text{if} \quad \psi_{3T} > 180° \qquad (I-16)$$

B. **Determination of Correct Sign for Round on Flat Regime**

The sign preceding the square root in eq. (G-94), for the round on flat regime, is determined with the help of the angle $\varphi_{3T}$. The condition yielding that angle $\psi_{3F}$ which is closest to the angle $\psi_{3T}$ will govern. The variable SIGN3F is used for the sign in question.

C. **Computation of Final and Initial Values of $\varphi_3$ and $\psi_3$**

The final and initial values of the gear and pinion angles $\varphi_3$ and $\psi_3$, respectively, are found by continuously evaluating the round on flat regime eq. (G-94), using the previously determined value of SIGN3F, and simultaneously checking the contact condition for the subsequent set of teeth, as given by eq. (G-102). This loop is initiated at the transition angle $\varphi_{3T}$, and it is terminated when the condition of eq. (G-102) is met. This allows the determination of the two angles PHI3F and PSI3FF at which the first set of teeth loses contact as well

I-12

as the angles PHI3I and PSI3I at which the second set of teeth simultaneously comes into contact. The initial engagement angles PHI3I and PSI3I are obtained by adding $\Delta\varphi_3$ to the "loss of contact" angle PHI3F and by subtracting $\Delta\psi_3$ from the "loss of contact" angle PSI3F.

### D. Determination of Correct Sign for Round on Round Regime

Eq. (G-87) is used to determine the angle $\psi_3$ while the gear and pinion are in the round on round phase of motion. The correct sign for this expression is obtained by comparing the value of $\psi_3$, as computed with PHI3I, with the previously obtained value for PSI3I. SIGN3R is the variable used for the desired sign.

### VII. Gear Train Motion Model: Kinematics, Point and Cycle Efficiency

The simulation of the gear train model, which is necessary for the computation of both POINTEF and CYCLEFF, is found in a loop, starting with statement label no. 29 (card no. 458) and ending with card no. 812. The motions of the individual driving gears are initialized at their respective angles PHI1I, PHI2I and PHI3I. (This again is arbitrary.) The meshes will be in round on round contact until they reach their respective transition angles PHI1T, PHI2T and PHI3T. Once the transition angles are passed, the meshes will be in round on flat contact. These regimes continue until the final angles PHI1F, PHI2F and PHI3F are reached.

The increment DDPHI1 of the angle PHI1 of the input gear 1

is obtained from an adaptation of eqs. (A-211) and (A-213), in which tooth numbers, rather than base circle radii, are used. The increment DDPHI2 of gear 2 is related to the increment of the pinion angle PSI1. Similarly, the increment DDPHI3 is obtained with the help of the pinion angle PSI2.

While the motion of gear 1 is terminated when the angle PHI1 reaches the angle PHI1F (or rather PHI1F + DDPHI1 for moment summation purposes), both gears 2 and 3 must be reset to their respective starting angles whenever their final angles PHI2F and PHI3F are reached.

The appropriate choice of moment equation depends upon which of the eight possible combinations of contact conditions, as indicated by Table H-1, is applicable.

The following discusses the kinematics of the individual meshes as well as the determination of the point and cycle efficiencies in greater detail.

## A. Kinematics

### (1) Mesh 1

Depending on whether PHI1 is larger or smaller than PHI1T, the parameters of the round on round or the round on flat regime are computed. (Recall that gear 1 turns in a clockwise direction.)

For the round on round phase, the following calculations are made:

$\psi_1$, according to eq. (G-11), and with the help of the previously determined SIGN1R

$\lambda_1$, according to eqs. (G-13) and (G-14)

$\dot{\psi}_1$, according to eq. (G-15)

$V_{S1/T1_R}$, according to eq. (G-20)

$s_{1R}$, according to eq. (H-1) as adapted to mesh 1

For the round on flat phase, the following calculations are made:

$\psi_1$, according to eq. (G-29), and with the help of the previously determined SIGN1F

$g_1$, according to eq. (G-27)

$\dot{\psi}_1$, according to eq. (G-30)

$V_{S1/T1_F}$, according to eq. (G-33)

$s_{1F}$, according to eq. (H-2) as adapted to mesh 1

### (2) Mesh 2

The increment DDPHI2 for each round of computation is obtained with the help of the change in the angle $\psi_1$ between the present and the previous computation, i.e., as shown at statement label no. 31

$$DDPHI2 = PSI1 - PSI1P \qquad (I-17)$$

For the first round of computations, the "previous" $\psi_1$, i.e., PSI1P, is equal to PSI1I.

It must be recalled that gear 2 rotates in a positive direction, and therefore, the angle $\phi_2$ increases with continued

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motion. The angle PHI2 is re-indexed to PHI2I once it becomes larger than PHI2F.

As for mesh 1, comparison with the transition angle decides whether the mesh is in the round on round or in the round on flat regime.

The following round on round parameters are calculated:

$\psi_2$, according to eq. (G-58), and with the help of the previously determined SIGN2R

$\lambda_2$, according to eqs. (G-59) and (G-60)

Note that the "input angular velocity" for mesh 2, i.e., $\dot{\varphi}_2$, equals the momentary value of $\dot{\psi}_1$.

$\dot{\psi}_2$, according to eq. (G-61)

$V_{S2/T2_R}$, according to eq. (G-63)

$s_{2R}$, according to eq. (H-1) as adapted to mesh 2

For the round on flat phase, the following calculations are made:

$\psi_2$, according to eq. (G-71), and with the help of the previously determined SIGN2F

$g_2$, according to eq. (G-69)

Again, $\dot{\varphi}_2$ equals the momentary value of $\dot{\psi}_1$

$\dot{\psi}_2$, according to eq. (G-72)

$V_{S2/T2_F}$, according to eq. (G-74)

$s_{2F}$, according to eq. (H-2) as adapted to mesh 2

### (3) Mesh 3

The increment DDPHI3, for each round of computation, is obtained with the help of the change in the angle $\psi_2$ between the present and the previous computation, i.e., as shown at statement label no. 33:

$$DDPHI3 = PSI2 - PSI2P \qquad (I-18)$$

For the first round of computations, the "previous" $\psi_2$, i.e., PSI2P, is equal to PSI2I.

Gear 3 rotates in a negative (clockwise) direction, and therefore, the angle $\varphi_3$ decreases with continued rotation. The angle PHI3, which represents this angle, is re-indexed to PHI3I once it becomes smaller than PHI3F.

As for meshes 1 and 2, comparison with the applicable transition angle decides whether the mesh is in the round on round or in the round on flat regime.

The following round on round parameters are calculated:

$\psi_3$, according to eq. (G-87), and with the help of the previously determined SIGN3R

$\lambda_3$, according to eqs. (G-89) and (G-90)

Note that the "input angular velocity" for mesh 3, i.e., $\dot{\varphi}_3$, equals the momentary value of $\dot{\psi}_2$.

$\dot{\psi}_3$, according to eq. (G-91)

$V_{S3/T3_R}$, according to eq. (G-92)

$s_{3R}$, according to eq. (H-1) as adapted to mesh 3

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For the round on flat phase, the following calculations are made:

$\psi_3$, according to eq. (G-94), and with the help of the previously determined SIGN3F

$g_3$, according to eq. (G-95)

Again, $\dot{\varphi}_3$ is equal to the momentary value of $\dot{\psi}_2$.

$\dot{\psi}_3$, according to eq. (G-96)

$V_{S3/T3F}$, according to eq. (G-97)

$s_{3F}$, according to eq. (H-2) as adapted to mesh 3

### B.  Moment Computations, Point and Cycle Efficiencies

Regardless of the combination of contact conditions, the point efficiency is computed according to eq. (3), i.e.,

$$'_p = \text{POINTEF} = K_{ratio} \frac{M_{o41}}{M_{in}} \tag{I-19}$$

where, with $\dot{\varphi}_1 = -1$,

$$K_{ratio} = |\dot{\psi}_3| \tag{I-20}$$

The cycle efficiency determination is based on eq. (C-10) in Appendix C, which represents an adaptation of eq. (4):

$$'_C = \frac{\Delta \varphi_1 \Sigma \, '_p}{\varphi_{1FIN} - \varphi_{1IN}} \tag{I-21}$$

(See page C-18.)  The associated expression in the program, at statement label no. 45 becomes

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$$\text{CYCLEFF} = -\text{MTOT*DDPHI1}/(\text{PHI1F-PHI1I}) \qquad (I-22)$$

where

$$\text{MTOT} = \text{MTOT} + \text{POINTEF} \qquad (I-23)$$

The moment computations begin with the statement label no. 35 , and initially consist of the determination of the variables A1 to A64 and C1 to C32 of section 1 of Appendix H. The governing contact combination (see also Table H-1) is determined with the help of the 8 moment control statements, which start with card no. 737 . Once the appropriate combination is established, the program is directed to one of the 8 associated moment expressions. These expressions for $M_{o41}$ coincide in nomenclature with those given by eqs. (H-81), (H-118), (H-158), (H-180), (H-216), (H-218), (H-239) and (H-241). They are listed in the above order, beginning with statement label no. 36 and ending with statement label no. 43 .

In devising the control statements, the manner of rotation of the individual mesh input gears had to be taken into account. Thus:

For mesh 3:

    Round on round (R) corresponds to

        PHI3I > PHI3 > PHI3T

    Round on flat (F) corresponds to

        PHI3T > PHI3 > PHI3F

For mesh 2:

Round on round (R) corresponds to

$$PHI2I < PHI2 < PHI2T$$

Round on flat (F) corresponds to

$$PHI2T < PHI2 < PHI2F$$

For mesh 1:

Round on round (R) corresponds to

$$PHI1I > PHI1 > PHI1T$$

Round on flat (F) corresponds to

$$PHI1T > PHI1 > PHI1F$$

c.  <u>Output (see Program CLOCK 3, below)</u>

The output of the program is best explained with the help of the sample problem at the end of the program.

I.  <u>Input Parameters</u>

<u>Mesh 1</u>

CAPRP1 $=$ $R_{P1}$ $=$ .47725 in. (1.212 cm)

RP2 $=$ $r_{P2}$ $=$ .09085 in.   (0.231 cm)

ACG1 $=$ $a_{G1}$ $=$ .47725 in.   (1.212 cm)

ACP1 $=$ $a_{P1}$ $=$ .09085 in.   (0.231 cm)

RHOG1 $=$ $\rho_{G1}$ $=$ .03870 in.   (0.098 cm)

RHOP1 $=$ $\rho_{P1}$ $=$ .01740 in.   (0.044 cm)

TG1 $=$ $t_{G1}$ $=$ .03480 in.   (0.088 cm)

TP1 $=$ $t_{P1}$ $=$ .02800 in.   (0.071 cm)

NG1 $=$ $n_{G1}$ $=$ 42

NP2 $=$ $n_{P2}$ $=$ 8

## Mesh 2

CAPRP2 $= R_{P2} = $ .20670 in.   (0.525 cm)

RP3 $= r_{P3} = $ .06890 in.   (0.175 cm)

ACG2 $= a_{G2} = $ .20670 in.   (0.525 cm)

ACP2 $= a_{P2} = $ .06890 in.   (0.175 cm)

RHOG2 $= \rho_{G2} = $ .02070 in.   (0.053 cm)

RHOP2 $= \rho_{P2} = $ .01040 in.   (0.026 cm)

TG2 $= t_{G2} = $ .02520 in.   (0.064 cm)

TP2 $= t_{P2} = $ .02080 in.   (0.053 cm)

NG2 $= n_{G2} = $ 27

NP3 $= n_{P3} = $ 9

## Mesh 3

CAPRP3 $= R_{P3} = $ .17560 in.   (0.446 cm)

RP4 $= r_{P4} = $ .05905 in.   (0.150 cm)

ACG3 $= a_{G3} = $ .17560 in.   (0.446 cm)

ACP3 $= a_{P3} = $ .05905 in.   (0.150 cm)

RHOG3 $= \rho_{G3} = $ .01910 in.   (0.049 cm)

RHOP3 $= \rho_{P3} = $ .00875 in.   (0.022 cm)

TG3 $= t_{G3} = $ .02170 in.   (0.055 cm)

TP3 $= t_{P3} = $ .01750 in.   (0.044 cm)

NG3 $= n_{G3} = $ 27

NP4 $= n_{P4} = $ 9

## In addition

MU $= \mu = $ .2

RPM $= $ 1000

M1 $= m_1 = $ .69515 x $10^{-4}$ lb-sec$^2$/in.   (12.171 g)

$M2 = m_2 = .97028 \times 10^{-5}$ lb-sec$^2$/in. (1.699 g)

$M3 = m_3 = .70027 \times 10^{-5}$ lb-sec$^2$/in. (1.226 g)

$M4 = m_4 = .79188 \times 10^{-6}$ lb-sec$^2$/in. (0.139 g)

$R1 = \mathcal{R}_1 = .750$ in. (1.905 cm)

$R2 = \mathcal{R}_2 = .750$ in. (1.905 cm)

$R3 = \mathcal{R}_3 = .750$ in. (1.905 cm)

$R4 = \mathcal{R}_4 = .750$ in. (1.905 cm)

$RHO1 = \rho_1 = .060$ in.(0.152 cm)

$RHO2 = \rho_2 = .030$ in.(0.076 cm)

$RHO3 = \rho_3 = .025$ in.(0.064 cm)

$RHO4 = \rho_4 = .020$ in.(0.051 cm)

$MD = md^2 = .15 \times 10^{-4}$ lb-sec$^2$ in. (16.944 g - cm$^2$)

$K = 25$

## II. Computed Values

At the beginning of the output, one finds MIN = $M_{in}$.
Subsequently, the following are listed for each mesh:

$f_{P1}$,   the length of the pinion flats

$\beta_1$,   the fuze body pivot to pivot line angles

$\psi_{T1}$ and $\varphi_{T1}$,   the transition angles as well as the associated
      subsidiary tests

$\varphi_{IN1}$ and $\psi_{IN1}$,   the initial angles

$\varphi_{FIN1}$ and $\psi_{FIN1}$,   the final angles

Finally, for the full range of the input angle $\varphi_1$, the
point efficiency POINTEF is listed, in addition to other parameters
which are useful for checking purposes.  Note that DPSI1, DPSI2

and DPSI3 represent $\dot{\psi}_1$, $\dot{\psi}_2$ and $\dot{\psi}_3$, respectively. The cycle efficiency CYCLEFF is found at the end of the output.

```
C
      PROGRAM CLOCK3(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)             A  1
C
C POINT AND CYCLE EFFICIENCIES FOR THREE PASS CLOCK (OGIVAL) STEP UP   A  3
C GEAR TRAIN IN SPIN ENVIRONMENT                                       A  4

      REAL MU,LAMDA1,LAMDA2,LAMDA3,MD4,MD41,MD42,MD43,MD44,MD45,MD46,MD4  A  6
     17,MC48,LK1,LY1,L11,LK2,LY2,L12,LK3,LY3,L13,NP2,NP3,NP4,NG1,NG2,NG3,NI A 7
     2N,R1,R2,R3,R4,MTOT,MD,L11,LL2,LL3,K                             A  8
    1 READ (5,61) MU,RPM,CAPRP1,CAPRP2,CAPRP3,RP2,RP3,RP4,ACG1,ACG2,ACG3 A  9
     1,ACP1,ACP2,ACP3,ISTOP                                           A 10
      READ (5,82) R1,R2,R3,R4                                         A 11
      READ (5,62) RHOG1,RHOG2,RHOG3,RHOP1,RHOP2,RHOP3                 A 12
      READ (5,63) TG1,TG2,TG3,TP1,TP2,TP3                            A 13
      READ (5,64) NG1,NG2,NG3,NP2,NP3,NP4                            A 14
      READ (5,85) R1,R2,R3,R4                                        A 15
      READ (5,65) RHO1,RHO2,RHO3,RHO4,MD,K                           A 16
      PI=3.14159                                                     A 17
      Z=PI/180.                                                      A 18
      OMEGA=RPM*2.*PI/6G.                                            A 19
      OM2=OMEGA*OMEGA                                                A 20
      PHDOT1=-1.                                                     A 21

C
C COMPUTATION OF GEAR TOOTH PARAMETERS                                A 23

      CKG1=RHOG1-TG1/2.                                              A 25
      DELG1=ASIN(CKG1/CAPRP1)                                        A 26
      CXP1=RHOP1-TP1/2.                                              A 27
      DELP1=ASIN(CXP1/RP2)                                           A 28
      GAMEP1=ASIN(RHOP1/RP2)                                         A 29
      ALPHP1=GAMEP1-DELP1                                            A 30
      FP1=ZCP1-COS(GAMEP1)                                           A 31
      B1=LAPRP1-RP2                                                  A 32
      CKG2=RHOG2-TG2/2.                                              A 33
      DELG2=ASIN(CKG2/CAPRP2)                                        A 34
      CXP2=RHOP2-TP2/2.                                              A 35
      DELP2=ASIN(CXP2/RP3)                                           A 36
      GAMEP2=ASIN(RHOP2/RP3)                                         A 37
      ALPHP2=GAMEP2-DELP2                                            A 38
      FP2=ZCP2-COS(GAMEP2)                                           A 39
      B2=CAPRP2-RP3                                                  A 40
      CKG3=RHOG3-TG3/2.                                              A 41
      DELG3=ASIN(CKG3/CAPRP3)                                        A 42
      CXP3=RHOP3-TP3/2.                                              A 43
      DELP3=ASIN(CXP3/RP4)                                           A 44
      GAMEP3=ASIN(RHOP3/RP4)                                         A 45
      ALPHP3=GAMEP3-DELP3                                            A 46
      FP3=4CP3=COS(GAMEP3)                                           A 47
      B3=CAPRP3-RP4                                                  A 48

C
C COMPUTATION OF BIN, GAMMAS AND BETAS                                A 50

      BIN=MD-OM2                                                     A 51
      DELTA2=ACOS((((CAPRP1+RP2)=(CAPRP1+RP2)+R1=R1-R2=R2)/(2.=R1=(CAPRP1 A 53
```

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```
                                    1+RP2)))
      DELTA3=ACOS(((CAPRP2+RP3)*(CAPRP2+RP3)+R2*R2-R3*R3)/(2.*R2*(CAPRP2    A  54
     1+RP3)))                                                               A  55
                                                                           A  56
      DELTA4=ACOS(((CAPRP3+RP4)*(CAPRP3+RP4)+R3*R3-R4*R4)/(2.*R3*(CAPRP3    A  57
     1+RP4)))                                                               A  58
      GAMMA2=ACOS((R1*R1+R2*R2-(CAPRP1+RP2)*(CAPRP1+RP2))/(2.*R1*R2))       A  59
      GAMMA3P=ACOS((R2*R2+R3*R3-(CAPRP2+RP3)*(CAPRP2+RP3))/(2.*R2*R3))      A  60
      GAM13=GAMMA2+GAMMA3P                                                  A  61
      GAMMA4P=ACOS((R3*R3+R4*R4-(CAPRP3+RP4)*(CAPRP3+RP4))/(2.*R3*R4))      A  62
      GAMMA4=GAMMA3+GAMMA4P                                                 A  63
      BETA1=PI-DELTA2                                                       A  64
      BETA2=GAMMA2+PI-DELTA3                                                A  65
      BETA3=GAMMA3+PI-DELTA4                                                A  66
      BETA1D=BETA1/Z                                                        A  67
      BETA2D=BETA2/Z                                                        A  68
      BETA3D=BETA3/Z                                                        A  69
      WRITE (6,66) BIN,MU,RPM,CAPRP1,CAPRP2,CAPRP3,RP2,RP3,RP4,ACG1,ACG2    A  70
     1,ACG3,ACP1,ACP2,ACP3                                                  A  71
      WRITE (6,83) R1,R2,R3,R4                                              A  72
      WRITE (6,67) RHOG1,RHOG2,RHOG3,RHOP1,RHOP2,RHOP3                      A  73
      WRITE (6,68) TG1,TG2,TG3,TP1,TP2,TP3                                  A  74
      WRITE (6,69) MG1,MG2,MG3,MP2,MP3,MP4                                  A  75
      WRITE (6,66) M1,M2,M3,M4                                              A  76
      WRITE (6,70) RHO1,RHO2,RHO3,RHO4,MD,K,PHOOT1                          A  77
      WRITE (6,84) FP1,FP2,FP3                                              A  78
      WRITE (6,71) BETA1D,BETA2D,BETA3D                                     A  79
C                                                                          A  80
C     COMPUTATION OF OTHER PARAMETERS                                      A  81
C                                                                          A  82
      DPHI1=360./MG1*Z                                                     A  83
      DPSI1=360./MP2*Z                                                     A  84
      DPHI2=360./MG2*Z                                                     A  85
      DPSI2=360./MP3*Z                                                     A  86
      DPHI3=360./MG3*Z                                                     A  87
      DPSI3=360./MP4*Z                                                     A  88
      L1=RHOG1+RHOP1                                                       A  89
      L2=RHOG2+RHOP2                                                       A  90
      L3=RHOG3+RHOP2                                                       A  91
      Q1=M1*R1*DM2                                                         A  92
      Q2=M2*R2*DM2                                                         A  93
      Q3=M3*R3*DM2                                                         A  94
      Q4=M4*R4*DM2                                                         A  95
C                                                                          A  96
C     PRELIMINARY COMPUTATIONS FOR MESH 1                                  A  97
C                                                                          A  98
C     DETERMINATION OF TRANSITION ANGLE OF MESH 1                          A  99
      A1T=RHOG1*COS(BETA1+ALPHP1)+FP1*SIN(BETA1+ALPHP1)                    A 100
      B1T=-RHOG1*SIN(BETA1+ALPHP1)+FP1*COS(BETA1+ALPHP1)                   A 101
      C1T=(ACG1-RHOG1+RHOG1-B1-FP1*FP1)/(2.*B1)                            A 102
      ROOT1T=A1T*A1T+B1T-C1T*C1T                                           A 103
      Y1T1=A1T+SQRT(ROOT1T)                                                A 104
      Y1T2=A1T-SQRT(ROOT1T)                                                A 105
```

```
      X1T=B1T+C1T                                                      A 107
      PSI1T1=2.*ATAN2(Y1T1,X1T)                                        A 108
      PSI1T2=2.*ATAN2(Y1T2,X1T)                                        A 109
      PSITE11=PSI1T1                                                   A 110
      PSITE12=PSI1T2                                                   A 111
      IF (PSI1T1.GT.PI) PSITE11=PSI1T1-2.*PI                           A 112
      IF (PSI1T1.LT.-PI) PSITE11=PSI1T1+2.*PI                          A 113
      IF (PSI1T2.GT.PI) PSITE12=PSI1T2-2.*PI                           A 114
      IF (PSI1T2.LT.-PI) PSITE12=PSI1T2+2.*PI                          A 115
      IF (PSITE11.GE.-3.) TEST11=ABS(PI-BETA1+PSITE11-ALPHP1)/Z        A 116
      IF (PSITE12.GE.0.) TEST12=ABS(PI-BETA1+PSITE12-ALPHP1)/Z         A 117
      IF (PSITE11.LT.0.) TEST11=ABS(PI+BETA1-(PSITE11+2.*PI-ALPHP1))/Z A 118
      IF (PSITE12.LT.0.) TEST12=ABS(PI+BETA1-(PSITE12+2.*PI-ALPHP1))/Z A 119
      IF (PSI1T1.LT.0.) PSI1T1=PSI1T1+2.*PI                            A 120
      IF (PSI1T2.LT.0.) PSI1T2=PSI1T2+2.*PI                            A 121
      PSI1T1D=PSI1T1/Z                                                 A 122
      PSI1T2D=PSI1T2/Z                                                 A 123
      WRITE (6,46) PSI1T1D,TEST11                                      A 124
      WRITE (6,47) PSI1T2D,TEST12                                      A 125
      CALL TRANS1 (RHOG1,ALPHP1,BETA1,FP1,ACG1,B1,DELG1,Z,PSI1T1,PHI1T1, A 126
     1G11)                                                             A 127
      IF (G11.GT.FP1) GO TO 2                                          A 128
      PHI1T=PHI1T1                                                     A 129
      PSI1T=PSI1T1                                                     A 130
      GO TO 4                                                          A 131
    2 CALL TRANS1 (RHOG1,ALPHP1,BETA1,FP1,ACG1,B1,DELG1,Z,PSI1T2,PHI1T2, A 132
     1G12)                                                             A 133
      IF (G12.LT.FP1) GO TO 3                                          A 134
      WRITE (6,72)                                                     A 135
      STOP                                                             A 136
    3 PHI1T=PHI1T2                                                     A 137
      PSI1T=PSI1T2                                                     A 138
    4 IF (PHI1T.LT.0.) PHI1T=PHI1T+2.*PI                               A 139
      IF (PSI1T.LT.0.) PSI1T=PSI1T+2.*PI                               A 140
      PHI1TD=PHI1T/Z                                                   A 141
      PSI1TD=PSI1T/Z                                                   A 142
      WRITE (6,73) PHI1TD,PSI1TD                                       A 143
C                                                                      A 144
C   DETERMINATION OF CORRECT SIGN FOR ROUND ON FLAT REGIME OF MESH 1   A 145
C                                                                      A 146
      A1F=ACG1*COS(PHI1T+DELG1+ALPHP1)-B1*COS(BETA1+ALPHP1)            A 147
      B1F=-ACG1*SIN(PHI1T+DELG1+ALPHP1)+B1*SIN(BETA1+ALPHP1)           A 148
      C1F=RHOG1                                                        A 149
      ROD1TF=A1F-A1F+B1F-B1F-C1F*C1F                                   A 150
      Y1F1=A1F+SQRT(ROD1TF)                                            A 151
      Y1F2=A1F-SQRT(ROD1TF)                                            A 152
      X1F=B1F+C1F                                                      A 153
      PSI1F1=2.*ATAN2(Y1F1,X1F)                                        A 154
      PSI1F2=2.*ATAN2(Y1F2,X1F)                                        A 155
      IF (PSI1F1.LT.0.) PSI1F1=PSI1F1+2.*PI                            A 156
      IF (PSI1F2.LT.0.) PSI1F2=PSI1F2+2.*PI                            A 157
      IF (ABS(PSI1F1-PSI1T).LT.ABS(PSI1F2-PSI1T)) GO TO 5             A 158
      SIGN1F=-1.                                                       A 159
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```
        GO TO 6                                                          A 160
    5   SIGN1F=1,                                                        A 161
C                                                                        A 162
C       COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI FOR MESH 1 A 163
C                                                                        A 164
    6   DO 7 I=1,200                                                     A 165
        PHID1=PHI1FD-(I-1.)/100.                                         A 166
        PHI1=PHID1*Z                                                     A 167
        A1F=ACG1*COS(PHI1+DELG1+ALPHP1)-B1*COS(BETA1+ALPHP1)            A 168
        B1F=-ACG1*SIN(PHI1+DELG1+ALPHP1)+B1*SIN(BETA1+ALPHP1)          A 169
        C1F=RHOG1                                                        A 170
        ROOT1F=A1F*A1F+B1F*B1F-C1F*C1F                                   A 171
        Y1F=A1F+SIGN1F*SQRT(ROOT1F)                                      A 172
        X1F=B1F+C1F                                                      A 173
        PSI1F=2.*ATAN2(Y1F,X1F)                                          A 174
        IF (PSI1F.LT.0.) PSI1F=PSI1F+2.*PI                               A 175
        LX1=B1*COS(BETA1)+ACP1*COS(PSI1F-DPSI1+DELP1)-ACG1*COS(PHI1+DPHI1+ A 176
       1DELG1)                                                           A 177
        LY1=B1*SIN(BETA1)+ACP1*SIN(PSI1F-DPSI1+DELP1)-ACG1*SIN(PHI1+DPHI1+ A 178
       1DELG1)                                                           A 179
        LL1=SQRT(LX1*LX1+LY1*LY1)                                        A 180
        DELEL1=LL1-L1                                                    A 181
        IF (DELEL1.LE.0.) GO TO 8                                        A 182
    7   CONTINUE                                                         A 183
    8   PHI1F=PHI1                                                       A 184
        PSI1FF=PSI1F                                                     A 185
        PHI1I=PHI1F+DPHI1                                                A 186
        PSI1I=PSI1FF-DPSI1                                               A 187
        IF (PSI1I.LT.0.) PSI1I=PSI1I+2.*PI                               A 188
        PHI1ID=PHI1I/Z                                                   A 189
        PSI1ID=PSI1I/Z                                                   A 190
        PHI1FD=PHI1F/Z                                                   A 191
        PSI1FD=PSI1FF/Z                                                  A 192
        WRITE (6,74) PHI1ID,PSI1ID,PHI1FD,PSI1FD                         A 193
C                                                                        A 194
C       DETERMINATION OF CORRECT SIGN FOR ROUND ON ROUND REGIME OF MESH 1 A 195
C                                                                        A 196
        A1R=ACG1*SIN(PHI1I+DELG1-DELP1)-B1*SIN(BETA1-DELP1)             A 197
        B1R=ACG1*COS(PHI1I+DELG1-DELP1)-B1*COS(BETA1-DELP1)             A 198
        C1R=(ACP1-ACP1+ACG1*ACG1+B1*B1-L1*L1-2.*ACG1*B1*COS(PHI1I+DELG1-BE A 199
       1TA1))/(2.*ACP1)                                                  A 200
        ROOT1R=A1R*A1R+B1R*B1R-C1R*C1R                                   A 201
        Y1R1=A1R+SQRT(ROOT1R)                                           A 202
        Y1R2=A1R-SQRT(ROOT1R)                                           A 203
        X1R=B1R+C1R                                                      A 204
        PSI1R1=2.*ATAN2(Y1R1,X1R)                                        A 205
        PSI1R2=2.*ATAN2(Y1R2,X1R)                                        A 205
        IF (PSI1R1.LT.0.) PSI1R1=PSI1R1+2.*PI                            A 207
        IF (PSI1R2.LT.0.) PSI1R2=PSI1R2+2.*PI                            A 208
        IF (ABS(PSI1I-PSI1R1).LT.ABS(PSI1I-PSI1R2)) GO TO 9             A 209
        SIGN1R=-1.                                                       A 210
        GO TO 10                                                         A 211
    9   SIGN1R=1.                                                        A 212
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C                                                                        A 213
C      PRELIMINARY COMPUTATIONS FOR MESH 2                               A 214
C                                                                        A 215
C      DETERMINATION OF TRANSITION ANGLE OF MESH 2                       A 216
C                                                                        A 217
 10    A2T=-RHOG2*COS(BETA2-ALPHP2)+FP2*SIN(BETA2-ALPHP2)                A 218
       B2T=RHOG2*SIN(BETA2-ALPHP2)+FP2*COS(BETA2-ALPHP2)                 A 219
       C2T=(ACG2*ACG2-RHOG2*RHOG2-B2*B2-FP2*FP2)/(2.*B2)                 A 220
       ROOT2T=A2T*A2T+B2T*B2T-C2T*C2T                                    A 221
       Y2T1=A2T+SQRT(ROOT2T)                                             A 222
       Y2T2=A2T-SQRT(ROOT2T)                                             A 223
       X2T=B2T+C2T                                                       A 224
       PSI2T1=2.*ATAN2(Y2T1,X2T)                                        A 225
       PSI2T2=2.*ATAN2(Y2T2,X2T)                                        A 226
       PSITE21=PSI2T1                                                    A 227
       PSITE22=PSI2T2                                                    A 228
       IF (PSI2T1.GT.PI) PSITE21=PSI2T1-2.*PI                           A 229
       IF (PSI2T1.LT.PI) PSITE21=PSI2T1+2.*PI                           A 230
       IF (PSI2T2.GT.PI) PSITE22=PSI2T2-2.*PI                           A 231
       IF (PSI2T2.LT.PI) PSITE22=PSI2T2+2.*PI                           A 232
       IF (PSITE21.GE.0.) TEST21=ABS(PI-BETA2+PSITE21+ALPHP2)/Z         A 233
       IF (PSITE21.LT.0.) TEST21=ABS(BETA2+PI-(PSITE21+2.*PI+ALPHP2))/Z A 234
       IF (PSITE22.GE.0.) TEST22=ABS(PI-BETA2+PSITE22+ALPHP2)/Z         A 235
       IF (PSITE22.LT.0.) TEST22=ABS(BETA2+PI-(PSITE22+2.*PI+ALPHP2))/Z A 236
       IF (PSI2T1.LT.0.) PSI2T1=PSI2T1+2.*PI                            A 237
       IF (PSI2T2.LT.0.) PSI2T2=PSI2T2+2.*PI                            A 238
       PSI2T1D=PSI2T1/Z                                                  A 239
       PSI2T2D=PSI2T2/Z                                                  A 240
       WRITE (6,48) PSI2T1D,TEST21                                       A 241
       WRITE (6,49) PSI2T2D,TEST22                                       A 242
       CALL TRANS2 (RHOG2,ALPHP2,BETA2,FP2,ACG2,B2,DELG2,Z,PSI2T1,PHI2T1, A 243
      1G21)                                                              A 244
       IF (G21.GT.FP2) GO TO 11                                          A 245
       PHI2T=PHI2T1                                                      A 246
       PSI2T=PSI2T1                                                      A 247
       GO TO 13                                                          A 248
 11    CALL TRANS2 (RHOG2,ALPHP2,BETA2,FP2,ACG2,B2,DELG2,Z,PSI2T2,PHI2T2, A 249
      1G22)                                                              A 250
       IF (G22.LT.FP2) GO TO 12                                          A 251
       WRITE (6,75)                                                      A 252
       STOP                                                              A 253
 12    PHI2T=PHI2T2                                                      A 254
       PSI2T=PSI2T2                                                      A 255
 13    IF (PHI2T.LT.0.) PHI2T=PHI2T+2.*PI                               A 256
       IF (PSI2T.LT.0.) PSI2T=PSI2T+2.*PI                               A 257
       PHI2TD=PHI2T/Z                                                    A 258
       PSI2TD=PSI2T/Z                                                    A 259
       WRITE (6,76) PHI2TD,PSI2TD                                        A 260
C                                                                        A 261
C      DETERMINATION OF CORRECT SIGN FOR ROUND OR FLAT REGIME OF MESH 2  A 262
C                                                                        A 263
       A2F=ACG2*COS(PHI2T-DELG2-ALPHP2)-B2*COS(BETA2-ALPHP2)            A 264
       B2F=-ACG2*SIN(PHI2T-DELG2-ALPHP2)+B2*SIN(BETA2-ALPHP2)           A 265
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            C2F=-RHOG2                                                                A 265
            ROOT2F=A2F*A2F+B2F*B2F-C2F*C2F                                            A 267
            Y2F1=A2F+SQRT(ROOT2F)                                                     A 268
            Y2F2=A2F-SQRT(ROOT2F)                                                     A 269
            X2F=B2F+C2F                                                               A 270
            PSI2F1=2.*ATAN2(Y2F1,X2F)                                                 A 271
            PSI2F2=2.*ATAN2(Y2F2,X2F)                                                 A 272
            IF (PSI2F1.LT.0.) PSI2F1=PSI2F1+2.*PI                                     A 273
            IF (PSI2F2.LT.0.) PSI2F2=PSI2F2+2.*PI                                     A 274
            IF (ABS(PSI2F1-PSI2T).LT.ABS(PSI2F2-PSI2T)) GO TO 14                      A 275
            PSI2F1D=PSI2F1/Z                                                          A 276
            PSI2F2D=PSI2F2/Z                                                          A 277
            SIGN2F=-1.                                                                A 278
            GO TO 15                                                                  A 279
         14 SIGN2F=1.                                                                 A 280
      C                                                                               A 281
      C     COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI FOR MESH 2         A 282
      C                                                                               A 283
         15 DO 16 I=1,1000                                                            A 284
            PHID2=PHI2TD+(I-1.)/100.                                                  A 285
            PHI2=PHID2*Z                                                              A 286
            A2F=ACG2*COS(PHI2-DELG2-ALPHP2)-B2*COS(BETA2-ALPHP2)                      A 287
            B2F=-ACG2*SIN(PHI2-DELG2-ALPHP2)+B2*SIN(BETA2-ALPHP/)                     A 288
            C2F=-RHOG2                                                                A 289
            ROOT2F=A2F*A2F+B2F*B2F-C2F*C2F                                            A 290
            Y2F=A2F+SIGN2F*SQRT(ROOT2F)                                               A 291
            X2F=B2F+C2F                                                               A 292
            PSI2F=2.*ATAN2(Y2F,X2F)                                                   A 293
            IF (PSI2F.LT.0.) PSI2F=PSI2F+2.*PI                                        A 294
            LX2=B2*COS(BETA2)+ACP2*COS(PSI2F+DPSI2-DELP2)-ACG2*COS(PHI2-DPHI2-        A 295
           1DELG2)                                                                    A 296
            LY2=B2*SIN(BETA2)+ACP2*SIN(PSI2F+DPSI2-DELP2)-ACG2*SIN(PHI2-DPHI2-        A 297
           1DELG2)                                                                    A 298
            LL2=SQRT(LX2*LX2+LY2*LY2)                                                 A 299
            DELEL2=LL2-L2                                                             A 300
            IF (DELEL2.LE.0.) GO TO 17                                                A 301
         16 CONTINUE                                                                  A 302
         17 PHI2F=PHI2                                                                A 303
            PSI2FF=PSI2F                                                              A 304
            PHI2I=PHI2F-DPHI2                                                         A 305
            PSI2I=PSI2FF+DPSI2                                                        A 306
            IF (PSI2I.GT.2.*PI) PSI2I=PSI2I-2.*PI                                     A 307
            PHI2ID=PHI2I/Z                                                            A 308
            PSI2ID=PSI2I/Z                                                            A 309
            PHI2FD=PHI2F/Z                                                            A 310
            PSI2FD=PSI2FF/Z                                                           A 311
            WRITE (6,77) PHI2ID,PSI2ID,PHI2FD,PSI2FD                                  A 312
      C                                                                               A 313
      C     DETERMINATION OF CORRECT SIGN FOR ROUND ON ROUND REGIME OF MESH 2         A 314
      C                                                                               A 315
            A2R=B2*SIN(BETA2+DELP2)-ACG2*SIN(PHI2I-DELG2+DELP2)                       A 316
            B2R=B2*COS(BETA2+DELP2)-ACG2*COS(PHI2I-DELG2+DELP2)                       A 317
            C2R=(L2*L2-B2-B2-ACG2*ACG2*ACP2*ACP2+2.*ACG2*B2*COS(PHI2I-DELG2-BE        A 318
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                    1TA2))/(2.*ACP2)                                                                    A 319
                    ROOT2R=A2R*A2R+B2R*B2R-C2R*C2R                                                      A 320
                    Y2R1=A2R+SQRT(ROOT2R)                                                               A 321
                    Y2R2=A2R-SQRT(ROOT2R)                                                               A 322
                    X2R=B2R+C2R                                                                         A 323
                    PSI2R1=2.*ATAN2(Y2R1,X2R)                                                           A 324
              325   PSI2R2=2.*ATAN2(Y2R2,X2R)                                                           A 325
                    IF (PSI2R1.LT.0.) PSI2R1=PSI2R1+2.*PI                                                A 326
                    IF (PSI2R2.LT.0.) PSI2R2=PSI2R2+2.*PI                                                A 327
                    IF (ABS(PSI2R1-PSI2R1).LT.ABS(PSI2R1-PSI2R2)) GO TO 18                               A 328
                    SIGN2R=-1.                                                                          A 329
              330   GO TO 19                                                                            A 330
                18  SIGN2R=1.                                                                           A 331
                                                                                                        A 332
                 C  PRELIMINARY COMPUTATIONS FOR MESH 3                                                 A 333
                 C                                                                                      A 334
              335 C DETERMINATION OF TRANSITION ANGLE OF MESH 3                                         A 335
                 C                                                                                      A 336
                19  A3T=RHOG3-COS(BETA3+ALPHP3)+FP3-SIN(BETA3+ALPHP3)                                    A 337
                    B3T=-RHOG3-SIN(BETA3+ALPHP3)+FP3*COS(BETA3+ALPHP3)                                   A 338
                    C3T=(ACG3*ACG3-RHOG3*RHOG3-B3-FP3*FP3)/(2.*R3)                                       A 339
              340   ROOT3T=A3T+A3T+B3T*B3T-C3T*C3T                                                      A 340
                    Y3T1=A3T+SQRT(ROOT3T)                                                               A 341
                    Y3T2=A3T-SQRT(ROOT3T)                                                               A 342
                    X3T=B3T+C3T                                                                         A 343
                    PSI3T1=2.*ATAN2(Y3T1,X3T)                                                           A 344
              345   PSI3T2=2.*ATAN2(Y3T2,X3T)                                                           A 345
                    PSITE31=PSI3T1                                                                      A 346
                    PSITE32=PSI3T2                                                                      A 347
                    IF (PSI3T1.GT.PI) PSITE31=PSI3T1-2.*PI                                              A 348
                    IF (PSI3T1.LT.PI) PSITE31=PSI3T1+2.*PI                                              A 349
              350   IF (PSI3T2.GT.PI) PSITE32=PSI3T2-2.*PI                                              A 350
                    IF (PSI3T2.LT.PI) PSITE32=PSI3T2+2.*PI                                              A 351
                    IF (PSITE31.GE.0.) TEST31=ABS(PI-BETA3+PSITE31-ALPHP3)/Z                            A 352
                    IF (PSITE31.LT.0.) TEST31=ABS(PI-BETA3-(PSITE31+2.*PI-ALPHP3))/Z                    A 353
                    IF (PSITE32.GE.0.) TEST32=ABS(PI-BETA3+PSITE32-ALPHP3)/Z                            A 354
              355   IF (PSITE32.LT.0.) TEST32=ABS(PI-BETA3-(PSITE32+2.*PI-ALPHP3))/Z                    A 355
                    IF (PSI3T1.LT.0.) PSI3T1=PSI3T1+2.*PI                                               A 356
                    IF (PSI3T2.LT.0.) PSI3T2=PSI3T2+2.*PI                                               A 357
                    PSI3T1D=PSI3T1/Z                                                                    A 358
                    PSI3T2D=PSI3T2/Z                                                                    A 359
              360   WRITE (6.50) PSI3T1D,TEST31                                                         A 360
                    WRITE (6.51) PSI3T2D,TEST32                                                         A 361
                    CALL TRANS1 (RHOG3,ALPHP3.BETA3,FP3,ACG3.B3,DELG3,Z,PSI3T1,PHI3T1,                  A 362
                1G31)                                                                                   A 363
                    PHI3T=PHI3T1                                                                        A 364
              365   PSI3T=PSI3T1                                                                        A 365
                    GO TO 22                                                                            A 366
                20  CALL TRANS1 (RHOG3,ALPHP3,BETA3,FP3,ACG3,B3,DELG3,Z,PSI3T2,PHI3T2,                  A 367
                1G32)                                                                                   A 368
                    IF (G32.LT.FP3) GO TO 21                                                            A 369
              370   WRITE (5.78)                                                                        A 370
                                                                                                        A 371
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          STOP                                                      A 372
       21 PHI3T=PHI3T2                                              A 373
          PSI3T=PSI3T2                                              A 374
       22 IF (PHI3T.LT.0.) PHI3T=PHI3T+2.*PI                        A 375
          IF (PSI3T.LT.0.) PSI3T=PSI3T+2.*PI                        A 376
          PHI3TD=PHI3T/Z                                            A 377
          PSI3TD=PSI3T/Z                                            A 378
          WRITE (6,79) PHI3TD,PSI3TD                                A 379
C                                                                   A 380
C         DETERMINATION OF CORRECT SIGN FOR ROUND ON FLAT REGIME OF MESH 3   A 381
C                                                                   A 382
          A3F=ACG3*COS(PHI3T+DELG3+ALPHP3)-B3*COS(BETA3+ALPHP3)     A 383
          B3F=-ACG3*SIN(PHI3T+DELG3+ALPHP3)+B3*SIN(BETA3+ALPHP3)    A 384
          C3F=RHOG3                                                 A 385
          ROOT3F=A3F*A3F+B3F*B3F-C3F*C3F                            A 386
          Y3F1=A3F+SQRT(ROOT3F)                                     A 387
          Y3F2=A3F-SQRT(ROOT3F)                                     A 388
          X3F=B3F+C3F                                               A 389
          PSI3F1=2.*ATAN2(Y3F1,X3F)                                 A 390
          PSI3F2=2.*ATAN2(Y3F2,X3F)                                 A 391
          IF (PSI3F1.LT.0.) PSI3F1=PSI3F1+2.*PI                     A 392
          IF (PSI3F2.LT.0.) PSI3F2=PSI3F2+2.*PI                     A 393
          IF (ABS(PSI3F-PSI3T).LT.ABS(PSI3F2-PSI3T)) GO TO 23       A 394
          SIGN3F=-1.                                                A 395
          GO TO 24                                                  A 396
       23 SIGN3F=1.                                                 A 397
C                                                                   A 398
C         COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI FOR MESH 3   A 399
C                                                                   A 400
       24 DO 25 I=1,200G                                            A 401
          PHID3=PHI3TD-(I-1.)/100.                                  A 402
          PHI3=PHID3*Z                                              A 403
          A3F=ACG3*COS(PHI3+DELG3+ALPHP3)-B3*COS(BETA3+ALPHP3)      A 404
          B3F=-ACG3*SIN(PHI3+DELG3+ALPHP3)+B3*SIN(BETA3+ALPHP3)     A 405
          C3F=RHOG3                                                 A 406
          ROOT3F=A3F*A3F+B3F*B3F-C3F*C3F                            A 407
          Y3F=A3F+SIGN3F*SQRT(ROOT3F)                               A 408
          X3F=B3F+C3F                                               A 409
          PSI3F=2.*ATAN2(Y3F,X3F)                                   A 410
          IF (PSI3F.LT.0.) PSI3F=PSI3F+2.*PI                        A 411
          LX3=B3*COS(BETA3)+ACP3*COS(PSI3F-DPSI3+DELP3)-ACG3*COS(PHI3+DPHI3+   A 412
         1DELG3)                                                    A 413
          LY3=B3*SIN(BETA3)+ACP3*SIN(PSI3F-DPSI3+DELP3)-ACG3*SIN(PHI3+DPHI3+   A 414
         1DELG3)                                                    A 415
          LL3=SQRT(LX3*LX3+LY3*LY3)                                 A 416
          DELEL3=LL3-L3                                             A 417
          IF (DELEL3.LE.0.) GO TO 26                                A 418
       25 CONTINUE                                                  A 419
       26 PHI3FF=PHI3F                                              A 420
          PSI3FF=PSI3F                                              A 421
          PHI3I=PHI3F+DPHI3                                         A 422
          PSI3I=PSI3FF-DPSI3                                        A 423
          IF (PSI3I.LT.0.) PSI3I=PSI3I+2.*PI                        A 424
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425        IF (PSI3I.LT.0.) PSI3I=PSI3I+2.*PI                                    A 425
           PHI3ID=PSI3I/Z                                                       A 426
           PSI3ID=PSI3I/Z                                                       A 427
           PHI3FD=PHI3F/Z                                                       A 428
           PSI3FD=PSI3FF/Z                                                      A 429
430        WRITE (6.80) PHI3ID,PSI3ID,PHI3FD,PSI3FD                             A 430
      C                                                                         A 431
      C    DETERMINATION OF CORRECT SIGN OF ROUND ON ROUND REGIME FOR MESH 3    A 432
      C                                                                         A 433
435        A3R=ACG3*SIN(PHI3I+DELG3-DELP3)-B3*SIN(BETA3-DELP3)                  A 434
           B3R=ACG3*COS(PHI3I+DELG3-DELP3)-B3*COS(BETA3-DELP3)                  A 435
           C3R=(ACP3*ACP3+ACG3*ACG3+B3*B3-L3*L3-2.*ACG3*B3*COS(PHI3I+DELG3-BE   A 436
          1TA3))/(2.*ACP3)                                                      A 437
           ROOT3R=A3R*A3R+B3R*B3R-C3R*C3R                                       A 438
440        Y3R1=A3R+SQRT(ROOT3R)                                                A 439
           Y3R2=A3R-SQRT(ROOT3R)                                               A 440
           X3R=B3R+C3R                                                          A 441
           PSI3R1=2.*ATAN2(Y3R1,X3R)                                            A 442
           PSI3R2=2.*ATAN2(Y3R2,X3R)                                            A 443
           IF (PSI3R1.LT.0.) PSI3R1=PSI3R1+2.*PI                                A 444
445        IF (PSI3R2.LT.0.) PSI3R2=PSI3R2+2.*PI                                A 445
           IF (ABS(PSI3I-PSI3R1).LT.ABS(PSI3I-PSI3R2)) GO TO 27                 A 446
           SIGN3R=-1.                                                           A 447
           GO TO 28                                                            A 448
      27   SIGN3R=1.                                                            A 449
450   C                                                                         A 450
      C    GEAR TRAIN MOTION MODEL, KINEMATICS                                  A 451
      C                                                                         A 452
      28   DOPHI1=NP2*NP3-(PHI3I-PHI3F)/(K*MG1*MG2)                             A 453
           PHI1I=PHI1I+DOPHI1                                                   A 454
455        WRITE (6.52)                                                         A 455
      29   PHI1=PHI1-DOPHI1                                                     A 456
           PHI1D=PHI1/Z                                                        A 457
           IF (PHI1.LE.PHI1F+DOPHI1) GO TO 45                                   A 458
      C                                                                         A 459
460   C    MESH 1                                                               A 460
      C                                                                         A 461
           IF (PHI1.LE.PHI1T) GO TO 30                                          A 462
           A1R=ACG1*SIN(PHI1+DELG1-DELP1)-B1*SIN(BETA1-DELP1)                   A 463
           B1R=ACG1*COS(PHI1+DELG1-DELP1)-B1*COS(BETA1-DELP1)                   A 464
465        C1R=(ACP1*ACP1+ACG1*ACG1+B1*B1-L1*L1-2.*ACG1*B1*COS(PHI1+DELG1-BET   A 465
          1A1))/(2.*ACP1)                                                       A 466
           ROOT1R=A1R*A1R+B1R*B1R-C1R*C1R                                       A 467
           Y1R=A1R+SIGN1R*SQRT(ROOT1R)                                          A 468
           X1R=B1R+C1R                                                          A 469
470        PSI1=2.*ATAN2(Y1R,X1R)                                               A 470
           IF (PSI1.LT.0.) PSI1=PSI1+2.*PI                                      A 471
           PSI1D=PSI1/Z                                                         A 472
           IF (ABS(PHI1-PHI1I).LT.0.0001) PSI1I=PSI1                            A 473
           IF(ABS(PHI1-PHI1I).LT..0001)PSI1P=PSI1I                              A 474
475        SLAM1=(B1*SIN(BETA1)+ACP1*SIN(PSI1+DELP1)-ACG1*SIN(PHI1+DELG1))/L1   A 475
           CLAM1=(B1*COS(BETA1)+ACP1*COS(PSI1+DELP1)-ACG1*COS(PHI1+DELG1))/L1   A 476
           LAMDA1=ATAN2(SLAM1,CLAM1)                                            A 477
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      IF (LAMDA1.LT.0.) LAMDA1=LAMDA1+2.*PI                            A 478
      PSDOT1=PHDOT1+ACG1*(B1/ACP1*SIN(PHI1+DELG1-BETA1)+SIN(PHI1-PSI1+DE A 479
     1LG1-DELP1))/((A1R-COS(PSI1)-B1R*SIN(PSI1))                       A 480
      VST1R=PHDOT1*(ACG1*COS(PHI1+DELG1-LAMDA1)+RHOG1)-PSDOT1*(ACP1*COS( A 481
     1PSI1+DELP1-LAMDA1)-RHOP1)                                        A 482
      S1R=VST1R/ABS(VST1P)                                             A 483
      GO TO 31                                                         A 484
   30 A1F=ACG1*COS(PHI1+DELG1+ALPHP1)-B1*COS(BETA1+ALPHP1)             A 485
      B1F=-ACG1*SIN(PHI1+DELG1+ALPHP1)+B1*SIN(BETA1+ALPHP1)            A 486
      C1F=RHOG1                                                        A 487
      ROOT1F=A1F*A1F+B1F*B1F-C1F*C1F                                   A 488
      Y1F=A1F+SIGM1F*SQRT(ROOT1F)                                      A 489
      X1F=B1F+C1F                                                      A 490
      PSI1=2.*ATAN2(Y1F,X1F)                                          A 491
      IF (PSI1.LT.0.) PSI1=PSI1+2.*PI                                  A 492
      PSI1D=PSI1/Z                                                     A 493
      G1=(ACG1*SIN(PHI1+DELG1)+RHOG1*COS(PSI1-ALPHP1)-B1*SIN(BETA1))/SIN A 494
     1(PSI1-ALPHP1)                                                    A 495
      PSDCT1=PHDOT1*(ACG1*COS(PHI1-PSI1+DELG1+ALPHP1))/(A1F*COS(PSI1))-B1 A 496
     1*SIN(PSI1))                                                      A 497
      VST1F=PHDOT1*(ACG1*SIN(PSI1-ALPHP1-PHI1-DELG1)-RHOG1)            A 498
      S1F=VST1F/ABS(VST1F)                                            A 499
C                                                                     A 500
C   MESH 2                                                            A 501
C                                                                     A 502
   31 DDPHI2=PSI1-PSI1P                                                A 503
      IF (ABS(PHI1-PHI1I).LT.0.0001) PHI2=PHI2I                        A 504
      PHI2=PHI2+DDPHI2                                                 A 505
      PSI1P=PSI1                                                       A 506
      IF (PHI2.GT.,PHI2F) PHI2=PHI2I                                   A 507
      PHI2D=PHI2/Z                                                     A 508
      IF (PHI2.GE.PHI2I) GO TO 32                                      A 509
      A2R=B2*SIN(BETA2+DELP2)-ACG2*SIN(PHI2-DELG2+DELP2)               A 510
      B2R=B2*COS(BETA2+DELP2)-ACG2*COS(PHI2-DELG2+DELP2)               A 511
      C2R=(L2*L2-B2-B2-ACG2*ACG2-ACP2*ACP2+2.*ACG2*B2*COS(PHI2-DELG2-BET A 512
     1A2))/(2.*ACP2)                                                   A 513
      ROOT2R=A2R*A2R+B2R*B2R-C2R*C2R                                   A 514
      Y2R=A2R+SIGM2R*SQRT(ROOT2R)                                      A 515
      X2R=B2R+C2R                                                      A 516
      PSI2=2.*ATAN2(Y2R,X2R)                                          A 517
      IF (PSI2.LT.0.) PSI2=PSI2+2.*PI                                  A 518
      PSI2D=PSI2/Z                                                     A 519
      IF (ABS(PHI2-PHI2I).LT.0.0001) PSI2I=PSI2                        A 520
      IF(ABS(PHI2-PHI2I).LT.0.0001)PSI2P=PSI2I                         A 521
      SLAM2=(B2-SIN(BETA2)-ACG2-SIN(PHI2-DELG2)-ACG2-SIN(PHI2-DELG2))/L2 A 522
      CLAM2=(B2*COS(BETA2)+ACP2*COS(PSI2-DELP2)-ACG2*COS(PHI2-DELG2))/L2 A 523
      LAMDA2=ATAN2(SLAM2,CLAM2)                                        A 524
      IF (LAMDA2.LT.0.) LAMDA2=LAMDA2+2.*PI                            A 525
      PHDOT2=PSDOT1                                                    A 526
      PSDOT2=PHDOT2*ACG2*(-SIN(PHI2-PSI2-DELG2+DELP2)-B2/ACP2*SIN(PHI2-B A 527
     1ELG2-BETA2))/(A2R*COS(PSI2)-B2R*SIN(PSI2))                       A 528
      VST2R=PHDOT2*(ACG2*COS(PHI2-DELG2-LAMDA2)+RHOG2)-PSDOT2*(ACP2*COS( A 529
     1PSI2-DELP2-LAMDA2)-RHOP2)                                        A 530
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        S2R=VST2R/ABS(VST2R)                                              A 531
        GO TO 33                                                          A 532
32      A2F=*CG2*COS(PHI2-DELG2-ALPHP2)-B2*COS(BETA2-ALPHP2)             A 533
        B2F=-ACG2*SIN(PHI2-DELG2-ALPHP2)+B2*SIN(BETA2-ALPHP2)            A 534
        C2F=-BHOG2                                                        A 535
        ROGT2F=A2F*A2F+B2F*B2F-C2F*C2F                                   A 536
        Y2F=*2F+SIGN2F*SQRT(ROOT2F)                                      A 537
        X2F=B2F+C2F                                                       A 538
        PSI2=2.*ATAN2(Y2F,X2F)                                            A 539
        IF (PSI2.LT.0.) PSI2=PSI2+2.*PI                                  A 540
        PSI2D=PSI2/Z                                                      A 541
        G2=(ACG2*SIN(PHI2-DELG2)-BHOG2*COS(PSI2+ALPHP2)-B2*SIN(BETA2))/SIN A 542
       1(PSI2+ALPHP2)                                                     A 543
        PHDOT2=PSDOT!                                                     A 544
        PSDOT2=(PHDOT2*ACG2*COS(PHI2-DELG2-ALPHP2-PSI2))/(A2F*COS(PSI2)-B2 A 545
       1F*SIN(PSI2))                                                      A 546
        VST2F=PHDOT2*(ACG2*SIN(PSI2+ALPHP2-PHI2+DELG2)+BHOG2)            A 547
        S2F=VST2F/ABS(VST2F)                                              A 548

C       RESH 3                                                           A 549
C
C
33      DDPHI3=PSI2-PSI2P                                                 A 550
        IF (ABS(PHI1-PHI11).LT.0.C001) PHI3=PHI3I                        A 551
        PHI3=PHI3+DOPHI3                                                  A 552
        PSI2P=PSI2                                                        A 553
        IF (PHI3.LT.PHI3F) PHI3=PHI3I                                    A 554
        PHI30=PHI3/Z                                                      A 555
        IF (PHI3.LE.PHI3T) GO TO 34                                      A 556
        A3F=ACG3*SIN(PHI3+DELG3-DELP3)-B3*SIN(BETA3-DELP3)               A 557
        B3F=ACG3*COS(PHI3+DELG3-DELP3)-B3*COS(BETA3-DELP3)               A 558
        C3F=(ACP3*ACP3+ACG3*ACG3+B3*B3-L3*L3-2.*ACG3*B3*COS(PHI3+DELG3-BET A 559
       1A3))/(2.-*ACP3)                                                   A 560
        ROOT3R=A3R*A3R+B3F*B3R-C3R*C3R                                   A 561
        Y3R=A3R*SIGN3R*SQRT(ROOT3R)                                      A 562
        X3R=B3R+C3R                                                       A 563
        PSI3=2.*ATAN2(Y3R,X3R)                                            A 564
        IF (PSI3.LT.0.) PSI3=PSI3+2.*PI                                  A 565
        IF (ABS(PHI3-PHI3I).LT.0.000I) PSI3=PSI3I                        A 566
        PSI30=PSI3/Z                                                      A 567
        SLAP3=(B3*SIN(BETA3)+ACP3*SIN(PSI3+DELP3)-ACG3*SIN(PHI3+DELG3))/L3 A 568
        CLAP3=(B3*COS(BETA3)+ACP3*COS(PSI3+DELP3)-ACG3*COS(PHI3+DELG3))/L3 A 569
        LAMDA3=ATAN2(SLAM3,CLAM3)                                        A 570
        IF (LAMDA3.LT.0.0) LAMDA3=LAMDA3+2.*PI                           A 571
        PHDOT3=PSDOT2                                                     A 572
        PSDOT3=PHDOT3*ACG3*(B3/ACP3*SIN(PHI3+DELG3-BETA3)+SIN(PHI3-PSI3-DE A 573
       1LG3-DELP3))/(A3*COS(PSI3)-B3*SIN(PSI3))                          A 574
        VST3R=PHDOT3*(ACG3*COS(PHI3+DELG3-LAMDA3)+BHOG3)-PSDOT3*(ACP3*COS( A 575
       1PSI3+DELP3-LAMDA3)-BHOG3)                                        A 576
        S3R=VST3R/ABS(VST3R)                                              A 577
        GO TO 35                                                         A 578
34      A3F=ACG3*COS(PHI3+DELG3+ALPHP3)-B3*COS(BETA3+A.PHP3)             A 579
        B3F=-ACG3*SIN(PHI3+DELG3+ALPHP3)+B3*SIN(BETA3+ALPHP3)            A 580
        C3F=BHOG3                                                         A 581
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        ROOT3F=A3F*A3F+B3F*B3F-C3F+C3F                                          A 584
        Y3F=A3F+SI(GA3F*SQRT(ROOT3F)                                            A 585
        X3F=B3F+C3F                                                             A 586
        PSI3=2.*ATAN2(Y3F,X3F)                                                  A 587
        IF (PSI3.LT.0.) PSI3=PSI3+2.*PI                                         A 588
        PSI3G=PSI3/2                                                            A 589
        G3=(ACG3-SIN(PHI3+DELG3)+BMOG3-COS(PSI3-ALPHP3)-B3-SIN(BETA3))/SIN      A 590
       1(PSI3-ALPHP3)                                                          A 591
        PHDOT3=PSOOT2                                                           A 592
        PSOGT3=PHDOT3-ACG3-COS(PHI3-PSI3-DELG3+ALPHP3)/(A3F-COS(PSI3)-B3F+     A 593
       1SIN(PSI3))                                                             A 594
        VST3F=PHDOT3=(ACG3-SIN(PSI3-ALPHP3-PHI3-DELG3)-BMOG3)                  A 595
        S3F=VST3F/ABS(VST3F)                                                    A 596
                                                                               A 597
  C                                                                            A 598
  C     MOMENT COMPUTATIONS                                                    A 599
  C                                                                            A 600
   35   DM=1.+BU-BU                                                            A 601
        A1=ABS((BU-SIN(GAMMA4)+COS(GAMMA4))/DM)                                A 602
        A2=ABS((BU-(S3R-1.)-SIN(LAMDA3)-(1.+BU-BU-S3R)-COS(LAMDA3))/DM)        A 603
        A3=ABS((SIN(GAMMA4)-BU-COS(GAMMA4))/DM)                                A 604
        A4=ABS((1.+BU-BU-S3R)-SIN(LAMDA3)+BU-(S3R-1.)-COS(LAMDA3))/DM)         A 605
        A5=ABS((1.-BU-BU-S3R)-COS(LAMDA3)-BU-(1.+S3R)-SIN(LAMDA3))/DM)         A 606
        A6=ABS((BU-(1.+S2R)-SIN(LAMDA2)-COS(GAMMA3))/DM)                       A 607
        A7=ABS((BU-SIN(GAMMA3)-COS(GAMMA3))/DM)                                A 608
        A8=ABS((1.-BU-BU-S3R)-SIN(LAMDA3)-BU-(1.+S3R)-COS(LAMDA3))/DM)         A 609
        A9=ABS((BU-BU-S2R-1.)-SIN(LAMDA2)-BU-(1.+S2R)-COS(LAMDA2))/DM)         A 610
        A10=ABS((SIN(GAMMA3)-BU-COS(GAMMA3))/DM)                               A 611
        A11=ABS((1.+BU-BU-S2R)-COS(LAMDA2)-BU-(S2R-1.)-SIN(LAMDA2))/DM)        A 612
        A12=ABS((BU-(S1R-1.)-SIN(LAMDA1)-(1.-BU-BU-S1R)-COS(LAMDA1))/DM)       A 613
        A13=ABS((-CGS(GAMMA2)-BU-SIN(GAMMA2))/DM)                              A 614
        A14=ABS((1.+BU-BU-S2R)-SIN(LAMDA2)-BU-(1.-S2R)-COS(LAMDA2))/DM)        A 615
        A15=ABS((BU-(1.-S1R)-COS(LAMDA1)-(1.+BU-BU-S1R)-SIN(LAMDA1))/DM)       A 616
        A16=ABS((BU-COS(GAMMA2)-SIN(GAMMA2))/DM)                               A 617
        A17=ABS((1.+S1R)-SIN(LAMDA1)-BU-(1.+S1R)-SIN(LAMDA1))/DM)              A 618
        A18=ABS(1./DM)                                                         A 619
        A19=ABS((1.-BU-BU-S1R)-SIN(LAMDA1)+BU-(1.+S1R)-COS(LAMDA1))/DM)        A 620
        A20=ABS(BU/DM)                                                         A 621
        A21=ABS((BU-(1.-S2R)-SIN(LAMDA2)-(1.+BU-BU-S2R)-COS(LAMDA2))/DM)       A 622
        A22=ABS((1.-S1F)-SIN(PSI1-ALPHP1)-BU-(1.+S1F)-COS(PSI1-ALPH           A 623
       1P1))/DM)                                                              A 624
        A23=ABS((BU-SIN(GAMMA2)+COS(GAMMA2))/DM)                               A 624
        A24=ABS((1.+BU-BU-S2R)-SIN(LAMDA2)-BU-(1.-S2R)-COS(LAMDA2))/DM)        A 625
        A25=ABS((-BU-(1.+S1F)-SIN(PSI1-ALPHP1)-(BU-BU-S1F-1.)-COS(PSI1-ALP     A 626
       1HP1))/DM)                                                             A 627
        A26=ABS((-SIN(GAMMA2)+BU-COS(GAMMA2))/DM)                              A 628
        A27=ABS((-(1.+BU-BU-S1F)-SIN(PSI1-ALPHP1)+BU-(S1F-1.)-COS(PSI1-ALP     A 629
       1HP1))/DM)                                                             A 630
        A28=ABS(1./DM)                                                         A 631
        A29=ABS((BU-(S1F-1.)-SIN(PSI1-ALPHP1)+(1.+BU-BU-S1F)-COS(PSI1-ALPH     A 632
       1P1))/DM)                                                              A 633
        A30=ABS(BU/DM)                                                         A 634
        A31=ABS((1.-BU-BU-S3R)-COS(LAMDA3)-BU-(1.+S3R)-SIN(LAMDA3))/DM)        A 635
        A32=ABS((-BU-(1.+S2F)-COS(PSI2-ALPHP2)-(1.-BU-BU-S2F)-SIN(PSI2-ALP     A 636
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1HP2))/DM)
A33=ABS((RU*SIN(GAMMA3)-COS(GAMMA3))/DM)                                                          A 637
A34=ABS((RU*(1.+S2R)*COS(LAMDA3)+(1.-RU-RU*S3R)*SIN(LAMDA3))/DM)                                   A 638
A35=ABS(((1.-RU-RU*S2F)*COS(PSI2+ALPHP2)-RU*(1.+S2F)*SIN(PSI2+ALPH                                 A 639
1P2))/DM)                                                                                          A 640
A36=ABS((SIN(GAMMA3)+RU*COS(GAMMA3))/DM)                                                           A 641
A37=ABS(((1.+RU-RU*S2F)*SIN(PSI2+ALPHP2)+RU*(S2F-1.)*COS(PSI2+ALPH                                 A 642
1P2))/DM)                                                                                          A 643
A38=ABS(((1.-RU-RU*S1F)*SIN(PSI1-ALPHP1)-RU*(1.+S1F)*COS(PSI1-ALPH                                 A 644
1P1))/DM)                                                                                          A 645
A39=ABS((RU*SIN(GAMMA2)+COS(GAMMA2))/DM)                                                           A 646
A40=ABS((S2F-1.)*SIN(PSI2+ALPHP2)-(1.+RU-RU*S2F)*COS(PSI2+ALPH                                     A 647
1P2))/DM)                                                                                          A 648
A41=ABS((-RU*(1.+S1F)*SIN(PSI1-ALPHP1)+(RU-RU-RU*S1F-1.)*COS(PSI1-ALP                              A 649
1HP1))/DM)                                                                                         A 650
A42=ABS((RU*COS(GAMMA2)-SIN(GAMMA2))/DM)                                                           A 651
A43=ABS(((1.+RU-RU*S2F)*SIN(PSI2+ALPHP2)-RU*(1.-S2F)*COS(PSI2+ALPH                                 A 652
1P2))/DM)                                                                                          A 653
A44=ABS((RU*(S1R-1.)*SIN(LAMDA1)-(1.-RU-RU*S1R)*COS(LAMDA1))/DM)                                   A 654
A45=ABS((RU*SIN(GAMMA2)+COS(GAMMA2))/DM)                                                           A 655
A46=ABS((RU*(S2F-1.)*SIN(PSI2+ALPHP2)-(1.+RU-RU*S2F)*COS(PSI2+ALPH                                 A 656
1P2))/DM)                                                                                          A 657
A47=ABS((-(1.+RU-RU*S1R)*SIN(LAMDA1)-RU*(1.-S1R)*COS(LAMDA1))/DM)                                  A 658
A48=ABS((-SIN(GAMMA2)-RU*COS(GAMMA2))/DM)                                                          A 659
A49=ABS(((1.-RU-RU*S3F)*SIN(PSI3-ALPHP3)-RU*(1.+S3F)*COS(PSI3-ALPH                                 A 660
1P3))/DM)                                                                                          A 661
A50=ABS((-RU*SIN(GAMMA4)-COS(GAMMA4))/DM)                                                          A 662
A51=ABS((-RU*(1.+S3F)*SIN(PSI3-ALPHP3)-(1.-RU-RU*S3F)*COS(PSI3-ALP                                 A 663
1HP3))/DM)                                                                                         A 664
A52=ABS((-SIN(GAMMA4)+RU*COS(GAMMA4))/DM)                                                          A 665
A53=ABS((-(1.+RU-RU*S3F)*SIN(PSI3-ALPHP3)+RU*(S3F-1.)*COS(PSI3-ALP                                 A 666
1HP3))/DM)                                                                                         A 667
A54=ABS(((1.-RU-RU*S2F-1.)*SIN(PSI2+ALPHP2)-RU*(1.+S2F)*COS(PSI2+ALPH                              A 668
1P2))/DM)                                                                                          A 669
A55=ABS((RU*SIN(GAMMA3)-COS(GAMMA3))/DM)                                                           A 670
A56=ABS((RU*(S3F-1.)*SIN(PSI3-ALPHP3)+(1.+RU-RU*S3F)*COS(PSI3-ALPH                                 A 671
1P3))/DM)                                                                                          A 672
A57=ABS((-RU*(1.+S2F)*SIN(PSI2+ALPHP2)+(1.-RU-RU*S2F)*COS(PSI2+ALP                                 A 673
1HP2))/DM)                                                                                          A 674
A58=ABS((-SIN(GAMMA3)+RU*COS(GAMMA3))/DM)                                                          A 675
A59=ABS((-(1.+RU-RU*S3F)*SIN(PSI3-ALPHP3)+RU*(S3F-1.)*COS(PSI3-ALP                                 A 676
1HP3))/DM)                                                                                          A 677
A60=ABS((RU*(1.+S2R)*SIN(LAMDA2)+(RU-RU*S2R-1.)*COS(LAMDA2))/DM)                                   A 678
A61=ABS((RU*SIN(GAMMA3)-COS(GAMMA3))/DM)                                                           A 679
A62=ABS((RU*(S3F-1.)*SIN(PSI3-ALPHP3)+(1.+RU-RU*S3F)*COS(PSI3-ALPH                                 A 680
1P3))/DM)                                                                                          A 681
A63=ABS(((RU-RU*S2R-1.)*SIN(LAMDA2)-RU*(1.+S2R)*COS(LAMDA2))/DM)                                   A 682
A64=ABS((-SIN(GAMMA3)-RU*COS(GAMMA3))/DM)                                                          A 683
C1=RU*RHO4-(A1+A3)                                                                                 A 684
C2=ACP3+(RU*S3R+COS(IS4)+DEC-LAMDA)-SIN(PSI3+DELP3-LAMDA3))-RU*                                    A 685
1(RHOP3*S3R+RHO4+(A2+A4))                                                                          A 686
C3=RU*RHO3-(A5+A8)-RU*S3R+RHOG3+ACG3+(SIN(PHI3+DELG3-LAMDA3)-RU*S3                                 A 687
1R*COS(PHI3+DELG3-LAMDA3))                                                                         A 688
                                                                                                  A 689
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      C4=MU*RHO3-(A7+A10)                                              A 690
      C5=MU*RHO3-(A6+A9)-MU*S2R*RHOP2*ACP2*(MU*S2R*COS(PSI2-DELP2-LAMDA2 A 691
     1)-SIN(PSI2-DELP2-LAMDA2))                                        A 692
      C6=ACG2*(SIN(PHI2-DELG2-LAMDA2)-MU*S2R*COS(PHI2-DELG2-LAMDA2))-MU* A 693
     1RHO2*(A11+A14)-MU*RHOG2*S2R                                      A 694
      C7=MU*RHO2*(A13+A16)                                             A 695
      C8=-ACP1*(SIN(PSI1+DELP1-LAMDA1)-MU*S1R*COS(PSI1+DELP1-LAMDA1))+B A 696
     1U*RHO2*(A12+A15)+MU*S1R*RHOP1)                                   A 697
      C9=MU*RHO1*(A18+A20)                                             A 698
      C10=MU*RHO1*(A17+A19)+ACG1*(SIN(PHI1+DELG1-LAMDA1)-MU*S1R*COS(PHI1 A 699
     1+DELG1-LAMDA1))-MU*S1R*RHOG1                                     A 700
      C11=ACG2*(SIN(PHI2-DELG2-LAMDA2)-MU*S2R*COS(PHI2-DELG2-LAMDA2))-MU A 701
     1RHO2*(A21+A24)-MU*RHOG2*S2R                                      A 702
      C12=MU*RHO2*(A23+A26)                                            A 703
      C13=G1-MU*RHO2*(A22+A25)                                         A 704
      C14=MU*RHO1*(A28+A30)                                            A 705
      C15=MU*RHO1*(A27+A29)+MU*S1F*RHOG1+ACG1*(MU*S1F*SIN(PHI1+DELG1-PSI A 706
     1+ALPHP1)-COS(PHI1-PSI1+ALPHP1))                                  A 707
      C16=MU*RHO3*(A31+A34)+ACG3*(SIN(PHI3+DELG3-LAMDA3)-MU*S3R*COS(PHI3 A 708
     1+DELG3-LAMDA3))-RHOG3-MU*S3R                                     A 709
      C17=MU*RHO3*(A33+A36)                                            A 710
      C18=MU*RHO3*(A32+A35)-G2                                         A 711
      C19=-MU*RHO2*(A37+A40)+ACG2*(COS(PHI2-DELG2-PSI2+ALPHP2)+MU*S2F*SI A 712
     1N(PHI2-DELG2-PSI2-ALPHP2))-MU*S2F*RHOG2                          A 713
      C20=MU*RHO2*(A39+A42)                                            A 714
      C21=-MU*RHO2*(A38+A41)+G1                                        A 715
      C22=-MU*RHO2*(A43+A46)+ACG2*(COS(PHI2-DELG2-PSI2-ALPHP2))-MU*S2F*SI A 716
     1N(PHI2-DELG2-PSI2-ALPHP2))-MU*S2F*RHOG2                          A 717
      C24=-MU*RHO2*(A44+A47)+ACP1*(MU*S1R*COS(PSI1+DELP1-LAMDA1)-SIN(PSI A 718
     1+DELP1-LAMDA1))-MU*S1R*RHOP1                                     A 719
      C25=MU*RHO4*(A50+A52)                                            A 720
      C26=G3-MU*RHO4*(A49+A51)                                         A 721
      C27=MU*RHO3*(A53+A56)+ACG3*(-COS(PHI3+DELG3-PSI3+ALPHP3)+MU*S3F*SI A 722
     1N(PHI3+DELG3-PSI3+ALPHP3))+MU*S3F*RHOG3                          A 723
      C28=MU*RHO3*(A55+A58)                                            A 724
      C29=MU*RHO3*(A54+A57)-G2                                         A 725
      C30=MU*RHO3*(A59+A62)-ACG3*(COS(PHI3+DELG3-PSI3+ALPHP3)-MU*S3F*SIN A 726
     1(PHI3+DELG3-PSI3+ALPHP3))+MU*S3F*RHOG3                           A 727
      C31=MU*RHO3*(A61+A64)                                            A 728
      C32=MU*RHO3*(A60+A63)-ACP2*(SIN(PSI2-DELP2-LAMDA2)-MU*S2R*COS(PSI2 A 729
     1-DELP2-LAMDA2))-MU*S2R*RHOP2                                     A 730
C                                                                     A 731
C     SEGMENT CONTROL                                                 A 732
C                                                                     A 733
      IF ((PHI1.GE.PHI1T).AND.(PHI2.LE.PHI2T).AND.(PHI3.GE.PHI3T)) GO TO A 734
     136                                                              A 735
      IF ((PHI1.LE.PHI1T).AND.(PHI2.LE.PHI2T).AND.(PHI3.GE.PHI3T)) GO TO A 736
     137                                                              A 737
      IF ((PHI1.LE.PHI1T).AND.(PHI2.GE.PHI2T).AND.(PHI3.GE.PHI3T)) GO TO A 738
     138                                                              A 739
      IF ((PHI1.GE.PHI1T).AND.(PHI2.GE.PHI2T).AND.(PHI3.GE.PHI3T)) GO TO A 740
     139                                                              A 741
                                                                      A 742
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        IF ((PHI1.LE.PHI1T).AND.(PHI2.GE.PHI2T).AND.(PHI3.LE.PHI3T)) GO TO      A 743
     1 40                                                                       A 744
        IF ((PHI1.GE.PHI1T).AND.(PHI2.GE.PHI2T).AND.(PHI3.LE.PHI3T)), GO TO     A 745
     1 41                                                                       A 746
        IF ((PHI1.GE.PHI1T).AND.(PHI2.LE.PHI2T).AND.(PHI3.LE.PHI3T)) GO TO      A 747
     1 42                                                                       A 748
        IF ((PHI1.LE.PHI1T).AND.(PHI2.LE.PHI2T).AND.(PHI3.LE.PHI3T)) GO TO      A 749
     1 43                                                                       A 750
C                                                                               A 751
C     MOMENT EXPRESSIONS                                                        A 752
C                                                                               A 753
  36  MD41=MIN*C2*C5*C8/(C3*C6*C10)-Q4*C1-Q2*C2*C5*C7/(C3*C6)-Q3*C2*C4/C        A 754
     13-Q1*C2*C5*C8*C9/(C3*C6*C10)                                             A 755
     MD4=MD41                                                                   A 756
     POINTEF=ABS(PSDOT3)*MD4/MIN                                               A 757
     WRITE (6.53) PHI1D.PHI2D.PHI3D.PSI1D.PSI2D.PSDOT1.PSDOT2.PSD              A 758
     1OT3.S1R.S2R.S3R.POINTEF                                                  A 759
     GO TO 44                                                                  A 760
  37  MD42=MIN*C2*C5*C13/(C3*C11*C15)-Q1*C2*C5*C13*C14/(C3*C11*C15)-Q2*C        A 761
     12*C5*C12/(C3*C11)-Q3*C2*C4/C3-Q4*C1                                      A 762
     MD4=MD42                                                                   A 763
     POINTEF=ABS(PSDOT3)*MD4/MIN                                               A 764
     WRITE (6.54) PHI1D.PHI2D.PHI3D.PSI1D.PSI2D.PSDOT1.PSDOT2.PSD              A 765
     1OT3.S2R.S3R.S1F.G1.POINTEF                                               A 766
     GO TO 44                                                                  A 767
  38  MD43=MIN*C2*C18*C21/(C15*C16*C19)-Q1*C2*C14*C18*C21/(C15*C16*C19)-        A 768
     1Q2*C2*C18*C20/(C16*C19)-Q3*C2*C17/C16-Q4*C1                             A 769
     MD4=MD43                                                                   A 770
     POINTEF=ABS(PSDOT3)*MD4/MIN                                               A 771
     WRITE (6.55) PHI1D.PHI2D.PHI3D.PSI1D.PSI2D.PSDOT1.PSDOT2.PSD              A 772
     1OT3.S3R.S1F.G1.S2F.G2.POINTEF                                            A 773
     GO TO 44                                                                  A 774
  39  MD44=MIN*C2*C18*C24/(C16*C16*C22)-Q1*C2*C9*C18*C24/(C16*C16*C22)-Q        A 775
     12*C2*C18*C23/(C16*C22)-Q3*C2*C17/C16-Q4*C1                              A 776
     MD4=MD44                                                                   A 777
     POINTEF=ABS(PSDOT3)*MD4/MIN                                               A 778
     WRITE (6.56) PHI1D.PHI2D.PHI3D.PSI1D.PSI2D.PSDOT1.PSDOT2.PSD              A 779
     1OT3.S1R.S3R.S2F.G2.POINTEF                                               A 780
     GO TO 44                                                                  A 781
  40  MD45=MIN*C21*C26*C29/(C15*C19*C27)-Q1*C14*C21*C26*C29/(C15*C19*C27)       A 782
     1)-Q2*C20*C26*C29/(C19*C27)-Q3*C26*C28/C27-Q4*C25                        A 783
     MD4=MD45                                                                   A 784
     POINTEF=ABS(PSDOT3)*MD4/MIN                                               A 785
     WRITE (6.57) PHI1D.PHI2D.PHI3D.PSI1D.PSI2D.PSDOT1.PSDOT2.PSD              A 786
     1OT3.S1F.G1.S2F.G2.S3F.G3.POINTEF                                         A 787
     GO TO 44                                                                  A 788
  41  MD46=MIN*C24*C25*C29/(C16*C22*C27)-Q1*C9*C24*C26*C29/(C16*C22*C27)        A 789
     1-Q2*C23*C26*C29/(C22*C27)-Q3*C26*C28/C27-Q4*C25                         A 790
     MD4=MD46                                                                   A 791
     POINTEF=ABS(PSDOT3)*MD4/MIN                                               A 792
     WRITE (6.58) PHI1C.PHI2D.PHI3D.PSI1D.PSI2D.PSI3D.PSDOT1.PSDOT2.PSD        A 793
     1OT3.S1R.S2F.G2.S3F.G3.POINTEF                                            A 794
     GO TO 44                                                                  A 795
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   42  ED47=ED14+C8+C26+C32/(C6+C10+C30)-Q1+C8+C9+C26-C32/((C6+C10+C30)-Q2+      A 796
       1C7+C26-C32)/(C6+C30)-Q3+C26-C31/C30-Q4+C25                              A 797
          ED4=ED47                                                             A 798
          POINTEF=ABS(PSDOT3)+ED4/MIN                                          A 799
          WRITE (6,59) PHI1D,PHI2D,PHI3D,PSI1D,PSI2D,PSI3D,PSDOT1,PSDOT2,PSD    A 800
       1OT3,S1R,S2R,S3F,G3,POINTEF                                             A 801
          GO TO 44                                                            A 802
   43  ED48=ED14+C13+C26+C32/(C11+C15+C30)-Q1+C13-C14+C26-C32/(C11+C15+C30      A 803
       1)-Q2+C12+C26+C32/(C11+C30)-Q3+C26-C31/C30-Q4+C25                       A 804
          ED4=ED48                                                            A 805
          POINTEF=ABS(PSDOT3)+ED4/MIN                                          A 806
          WRITE (6,60) PHI1D,PHI2D,PHI3D,PSI1D,PSI2D,PSI3D,PSDOT1,PSDOT2,PSD    A 807
       1OT3,S2R,S1F,G1,S3F,G3,POINTEF                                          A 808
   44  BTOT=BTOT+POINTEF                                                       A 809
          GO TO 29                                                            A 810
   45  CYCLEFF=-BTOT+DDPHI1/(PHI1F-PHI11)                                      A 811
          WRITE (6,61) CYCLEFF                                                 A 812
          BTOT=0.                                                             A 813
          IF (ISTOP.NE.0) GO TO 1                                             A 814
          STOP                                                                A 815
        C                                                                     A 816
        C                                                                     A 817
   46  FORMAT (6X,9HPSI1T1D =,F9.4,3X,8HTEST11 =,F9.4)                         A 818
   47  FORMAT (6X,9HPSI1T2D =,F9.4,3X,8HTEST12 =,F9.4//)                       A 819
   48  FORMAT (6X,9HPSI2T1D =,F9.4,3X,8HTEST21 =,F9.4)                         A 820
   49  FORMAT (6X,9HPSI2T2D =,F9.4,3X,8HTEST22 =,F9.4//)                       A 821
   50  FORMAT (6X,9HPSI3T1D =,F9.4,3X,8HTEST31 =,F9.4)                         A 822
   51  FORMAT (6X,9HPSI3T2D =,F9.4,3X,8HTEST32 =,F9.4//)                       A 823
   52  FORMAT (12OH0        PHI1   PHI2   PHI3   PSI1   PSI2   PSI3    DPSI1    DPSI2  A 824
       1  DPSI3   S1R   S2R   S3R   S1F   G1    S2F   G2    S3F   G3   POINTEF  A 825
       2/)                                                                     A 826
   53  FORMAT (6X,6(F4.0,2X),3(F3.0,2X),36X,F5.3)                              A 827
   54  FORMAT (6X,6(F4.0,2X),3(F5.0,2X),5X,3(F3.0,2X),F5.3,26X,F5.3)           A 828
   55  FORMAT (6X,6(F4.0,2X),3(F5.0,2X),10X,F3.0,2X,F3.0,2X,F5.3,2X,F3.0,     A 829
       12X,F5.3,14X,F5.3)                                                      A 830
   56  FORMAT (6X,6(F4.0,2X),3(F5.0,2X),F3.0,7X,F3.0,14X,F3.0,2X,F5.3,14X     A 831
       1,F5.3)                                                                 A 832
   57  FORMAT (6X,6(F4.0,2X),3(F5.0,2X),15X,3(F3.0,2X,F5.3,2X),F5.3)          A 833
   58  FORMAT (6X,6(F4.0,2X),3(F5.0,2X),F3.0,24X,F3.0,2X,F5.3,2X,F3.0,2X,    A 834
       1F5.3,2X,F5.3)                                                          A 835
   59  FORMAT (6X,6(F4.0,2X),3(F5.0,2X),2(F3.0,2X),29X,F3.0,2X,F5.3,2X,F5    A 836
       1.3)                                                                    A 837
   60  FORMAT (6X,6(F4.0,2X),3(F5.0,2X),5X,F3.0,7X,F3.0,2X,F5.3,14X,F3.0,    A 838
       12X,F5.3,2X,F5.3)                                                       A 839
   61  FORMAT (F10.3,F10.0/6F10.5/6F10.5/11)                                  A 840
   62  FORMAT (6F10.4)                                                        A 841
   63  FORMAT (6F10.4)                                                        A 842
   64  FORMAT (6F10.0)                                                        A 843
   65  FORMAT (4F10.4/F10.4/F10.6)                                            A 844
   66  FORMAT (1H1,5X,5HMIN =,F8.4,3X,4HMU =,F6.3,3X,5HRP2 =,F6.0//6X,8HC    A 845
       1APR1 =,F8.5,3X,8HCAPR2 =,F8.5,3X,8HCAPR3 =,F8.5//6X,5HRP2 =,F8.     A 846
       25,3X,5HRP2 =,F8.5,3X,5HRP4 =,F8.5//6X,6HACG1 =,F8.5,3X,6HACG2 =,F8  A 847
       3.5,3X,6HACG3 =,F8.5//6X,6HACP1 =,F8.5,3X,6HACP2 =,F8.5,3X,6HACP3 =  A 848
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         4,F8.5/)
   67 FORMAT (6X,7HRHOG1 =,F8.5,3X,7HRHOG2 =,F8.5,3X,7HRHOG3 =,F8.5,3X,7      A 849
      1HRHOP1 =,F8.5,3X,7HRHOP2 =,F8.5,3X,7HRHOP3 =,F8.5/)                    A 850
   68 FORMAT (6X,5HTG1 =,F8.5,3X,5HTG2 =,F8.5,3X,5HTG3 =,F8.5,3X,5HTP1 =      A 851
      1,F8.5,3X,5HTP2 =,F8.5,3X,5HTP3 =,F8.5/)                                A 852
   69 FORMAT (6X,5HWG1 =,F5.0,3X,5HWG2 =,F5.0,3X,5HWG3 =,F5.0   5HWP2 =       A 853
      1,F5.0,3X,5HWP3 =,F5.0,3X,5HWP4 =,F5.0/)                                A 854
   70 FORMAT (6X,6HRHO1 =,F6.3,3X,6HRHO2 =,F6.3,3X,6HRHO3 =,F6.3,3X,6HRH      A 855
      104 =,F6.3//6X,4HED =,E12.4//6X,3HK =,F6.1//6X,8HPHDGT1 =,F5.1/)        A 856
   71 FORMAT (6X,8HBETA1D =,F8.4,3X,8HBETA2D =,F8.4,3X,8HBETA3D =,F8.4//      A 857
      1)                                                                     A 858
   72 FORMAT (6X,30HSOMETHING IS WRONG WITH MESH 1)                          A 859
   73 FORMAT (6X,8HPHI1TD =,F8.4,3X,8HPSI1TD =,F2.4)                         A 860
   74 FORMAT (6X,8HPHI11D =,F8.4,3X,8HPSI11D =,F8.4,3X,8HPHI1FD =,F8.4.3      A 861
      1X,8HPSI1FD =,F8.4//)                                                  A 862
   75 FORMAT (6X,30HSOMETHING IS WRONG WITH MESH 2)                          A 863
   76 FORMAT (6X,8HPHI2TD =,F8.4,3X,8HPSI2TD =,F8.4)                         A 864
   77 FORMAT (6X,8HPHI21D =,F8.4,3X,8HPSI21D =,F8.4,3X,8HPHI2FD =,F8.4.3      A 865
      1X,8HPSI2FD =,F8.4//)                                                  A 866
   78 FORMAT (6X,30HSOMETHING IS WRONG WITH MESH 3)                          A 867
   79 FORMAT (6X,8HPHI3TD =,F8.4,3X,8HPSI3TD =,F8.4)                         A 868
   80 FORMAT (6X,8HPHI31D =,F8.4,3X,8HPSI31D =,F8.4,3X,8HPHI3FD =,F8.4.3      A 869
      1X,8HPSI3FD =,F8.4//)                                                  A 870
   81 FORMAT (1H0,5X,18HCYCLE EFFICIENCY =,F5.3)                             A 871
   82 FORMAT (4F10.5)                                                        A 872
   83 FORMAT (6X,4HR1 =,F8.5,3X,4HR2 =,F8.5,3X,4HR3 =,F8.5,3X,4HR4 =,F8.     A 873
      15/)                                                                   A 874
   84 FORMAT (6X,5HFP1 =,F8.5,3X,5HFP2 =,F8.5,3X,5HFP3 =,F8.5/)              A 875
   85 FORMAT (4E15.5)                                                        A 876
   86 FORMAT (6X,4HN1 =,E15.5,3X,4HN2 =,E15.5,3X,4HN3 =,E15.5,3X,4HN4 =,     A 877
      1E15.5/)                                                               A 878
      END                                                                    A 879
                                                                             A 880-
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      SUBROUTINE TRANS1 (RHOG,ALPHP,BETA,FP,ACG,B,DELG,Z,PSIT,PHIT,G)      B  1
      PI=3.14159                                                          B  2
      ST=(-RHOG*COS(PSIT-ALPHP)+B*SIN(BETA)+FP*SIN(PSIT-ALPHP))/ACG       B  3
      CT=(RHOG*SIN(PSIT-ALPHP)+B*COS(BETA)+FP*COS(PSIT-ALPHP))/ACG        B  4
      PHIT=ATAN2(ST,CT)-DELG                                              B  5
      PHINEXT=PHIT-.1*Z                                                   B  6
      AF=ACG*COS(PHINEXT+DELG+ALPHP)-B*COS(BETA+ALPHP)                    B  7
      BF=-ACG*SIN(PHINEXT+DELG+ALPHP)+B*SIN(BETA+ALPHP)                   B  8
      CF=RHOG                                                             B  9
      ROOTF=AF*AF+BF*BF-CF*CF                                             B 10
      YF1=AF+SQRT(ROOTF)                                                  B 11
      YF2=AF-SQRT(ROOTF)                                                  B 12
      XF=BF+CF                                                            B 13
      PSINEX1=2.*ATAN2(YF1,XF)                                            B 14
      PSINEX2=2.*ATAN2(YF2,XF)                                            B 15
      IF (PSINEX1.LT.0.) PSINEX1=PSINEX1+2.*PI                            B 16
      IF (PSINEX2.LT.0.) PSINEX2=PSINEX2+2.*PI                            B 17
      IF (ABS(PSINEX1-PSIT).LT.ABS(PSINEX2-PSIT)) GO TO 1                 B 18
      PSINEXT=PSINEX2                                                     B 19
      GO TO 2                                                             B 20
    1 PSINEXT=PSINEX1                                                     B 21
    2 G=(ACG*SIN(PHINEXT+DELG)+RHOG*COS(PSINEXT-ALPHP)-B*SIN(BETA))/SIN(  B 22
     1PSINEXT-ALPHP)                                                      B 23
      RETURN                                                              B 24
      END                                                                 B 25-
```

```
         SUBROUTINE TRANS2 (RHOG,ALPHP,BETA,FP,ACG,B,DELG,Z,PSIT,PHIT,G)         1
         PI=3.14159                                                             2
         ST=(RHOG*COS(PSIT+ALPHP)+B*SIN(BETA)+FP*SIN(PSIT+ALPHP))/ACG           3
         CT=(-RHOG*SIN(PSIT+ALPHP)+B*COS(BETA)+FP*COS(PSIT+ALPHP))/ACG          4
         PHIT=ATAN2(ST,CT)+DELG                                                 5
         PHINEXT=PHIT+.1*Z                                                      6
         AF=ACG*COS(PHINEXT-DELG-ALPHP)-B*COS(BETA-ALPHP)                       7
         BF=-ACG*SIN(PHINEXT-DELG-ALPHP)+B*SIN(BETA-ALPHP)                      8
         CF=-RHOG                                                               9
         ROOTF=AF*AF+BF*BF-CF*CF                                               10
         YF1=AF+SQRT(ROOTF)                                                    11
         YF2=AF-SQRT(ROOTF)                                                    12
         XF=BF+CF                                                              13
         PSINEX1=2.*ATAN2(YF1,XF)                                              14
         PSINEX2=2.*ATAN2(YF2,XF)                                              15
         IF (PSINEX1.LT.0.) PSINEX1=PSINEX1+2.*PI                              16
         IF (PSINEX2.LT.0.) PSINEX2=PSINEX2+2.*PI                              17
         IF (ABS(PSINEX1-PSIT).LT.ABS(PSINEX2-PSIT)) GO TO 1                   18
         PSINEXT=PSINEX2                                                       19
         GO TO 2                                                               20
       1 PSINEXT=PSINEX1                                                       21
       2 G=(ACG*SIN(PHINEXT-DELG)-RHOG*COS(PSINEXT+ALPHP)-B*SIN(BETA))/SIN(    22
        1PSINEXT+ALPHP)                                                        23
         RETURN                                                                24
         END                                                                 25-
```

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```
MIN =  .1645   MU =  .200   RPM = 1000.

CAPRP1 =  .47725   CAPRP2 =  .20670   CAPRP3 =  .17560

RP2 =  .09085   RP3 =  .06890   RP4 =  .05905

ACG1 =  .47725   ACG2 =  .20670   ACG3 =  .17560

ACP1 =  .09085   ACP2 =  .06890   ACP3 =  .05905

R1 =  .75000   R2 =  .75000   R3 =  .75000   R4 =  .75000

RHOG1 =  .03870   RHOG2 =  .02070   RHOG3 =  .01910   RHOP1 =  .01740   RHOP2 =  .01040   RHOP3 =  .00875

TG1 =  .03480   TG2 =  .02520   TG3 =  .02170   TP1 =  .02800   TP2 =  .02080   TP3 =  .01750

NG1 = 42.   NG2 = 27.   NG3 = 27.   NP2 = 8.   NP3 = 9.   NP4 = 9.

M1 =  .69515E-04   M2 =  .97028E-05   M3 =  .70027E-05   M4 =  .79188E-06

RHO1 =  .060   RHO2 =  .030   RHO3 =  .025   RHO4 =  .020

MD =  .1500E-04

K = 25.0

PHDOT1 = -1.0

FP1 =  .0691?   FP2 =  .0691?   FP3 =  .05840

BETA1D =112.2552   BETA2D =145.0978   BETA3D =164.6850

PSI1T1D = 305.6017   TEST11 =   4.4495
PSI1T2D = 343.6259   TEST12 =  42.4736

PHI1T1D =113.5016   PSI1T1D =305.6017
PHI1T1D =115.9930   PSI1T1D =292.4283   PHI1FD =107.4216   PSI1FD =337.4283

PSI2T1D = 286.9442   TEST21 =  29.4719
PSI2T2D = 312.0785   TEST22 =   4.3377

PHI2T1D =142.0461   PSI2T1D =312.0785
PHI2T1D =136.7828   PSI2T1D =330.9521   PHI2FD =150.1161   PSI2FD =290.9521

PSI3T1D = 357.5003   TEST31 =   4.2938
PSI3T2D =  25.1346   TEST32 =  31.9282

PHI3T1D =166.7857   PSI3T1D =357.5003
PHI3T1D =173.6791   PSI3T1D =336.8610   PHI3FD =160.3457   PSI3FD =15.8610
```

| PHI1 | PHI2 | PHI3 | PSI1 | PSI2 | PSI3 | DPSI1 | DPSI2 | DPSI3 | S1R | S2R | S3R | S1F | G1 | S2F | G2 | S3F | G3 | POINTEF |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 116. | 137. | 174. | 292. | 331. | 337. | 5. | -16. | 59. | 1. | -1. | 1. | | | | | | | .468 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 116. | 137. | 173. | 293. | 330. | 339. | 5. | -16. | 49. | 1. | .068 | .476 |
| 116. | 137. | 173. | 293. | 330. | 340. | 5. | -16. | 48. | 1. | .066 | .484 |
| 116. | 137. | 172. | 293. | 329. | 342.- | 5. | -16. | 48. | 1. | .068 | .492 |
| 116. | 137. | 172. | 293. | 329. | 343. | 5. | -16. | 47. | 1. | .067 | .499 |
| 116. | 138. | 171. | 293. | 328. | 345. | 5. | -16. | 47. | 1. | .067 | .506 |
| 116. | 138. | 170. | 294. | 328. | 347. | 5. | -16. | 47. | 1. | .067 | .512 |
| 116. | 138. | 170. | 294. | 328. | 348. | 5. | -16. | 47. | 1. | .067 | .519 |
| 116. | 138. | 169. | 294. | 327. | 350. | 5. | -16. | 47. | 1. | .067 | .526 |
| 116. | 139. | 169. | 294. | 327. | 351. | 5. | -16. | 47. | 1. | .067 | .532 |
| 116. | 139. | 168. | 294. | 326. | 353. | 5. | -16. | 47. | 1. | .066 | .539 |
| 116. | 139. | 168. | 294. | 326. | 353. | 5. | -16. | 47. | 1. | .066 | .531 |
| 116. | 139. | 167. | 295. | 325. | 355. | 5. | -16. | 47. | 1. | .066 | .529 |
| 116. | 139. | 166. | 295. | 324. | 356. | 5. | -16. | 48. | 1. | .066 | .527 |
| 115. | 140. | 166. | 295. | 323. | 358. | 5. | -16. | 48. | 1. | .066 | .525 |
| 115. | 140. | 165. | 295. | 323. | 359. | 5. | -16. | 48. | 1. | .066 | .524 |
| 115. | 140. | 165. | 295. | 322. | 1. | 5. | -16. | 48. | 1. | .066 | .523 |
| 115. | 140. | 164. | 296. | 322. | 3. | 5. | -16. | 48. | 1. | .066 | .521 |
| 115. | 140. | 164. | 296. | 321. | 4. | 5. | -16. | 48. | 1. | .065 | .520 |
| 115. | 140. | 163. | 296. | 321. | 6. | 5. | -16. | 47. | 1. | .066 | .519 |
| 115. | 141. | 162. | 296. | 320. | 8. | 5. | -16. | 47. | 1. | .066 | .517 |
| 115. | 141. | 162. | 296. | 320. | 9. | 5. | -16. | 46. | 1. | .066 | .515 |
| 115. | 141. | 161. | 297. | 319. | 11. | 5. | -16. | 45. | 1. | .066 | .514 |
| 115. | 141. | 161. | 297. | 319. | 12. | 5. | -16. | 49. | 1. | .066 | .512 |
| 115. | 141. | 161. | 297. | 318. | 14. | 5. | -16. | 48. | 1. | .066 | .510 |
| 115. | 141. | 173. | 297. | 318. | 15. | 5. | -16. | 48. | 1. | .066 | .521 |
| 115. | 142. | 173. | 297. | 317. | 337. | 5. | -16. | 48. | 1. | .066 | .529 |
| 115. | 142. | 72. | 297. | 316. | 339. | 5. | -16. | 47. | 1. | .066 | .537 |
| 115. | 142. | 172. | 298. | 316. | 340. | 5. | -16. | 47. | 1. | .066 | .551 |
| 115. | 142. | 171. | 298. | 315. | 342. | 5. | -16. | 47. | 1. | .066 | .554 |
| 115. | 142. | 170. | 298. | 315. | 343. | 5. | -16. | 47. | 1. | .066 | .558 |
| 115. | 143. | 170. | 298. | 314. | 345. | 5. | -16. | 47. | 1. | .065 | .562 |
| 115. | 143. | 169. | 298. | 314. | 347. | 5. | -16. | 48. | 1. | .066 | .565 |
| 115. | 143. | 169. | 299. | 313. | 348. | 5. | -16. | 48. | 1. | .066 | .569 |
| 115. | 143. | 168. | 299. | 313. | 350. | 5. | -16. | 49. | 1. | .066 | .572 |
| 115. | 143. | 168. | 299. | 312. | 351. | 5. | -16. | 49. | 1. | .068 | .575 |
| 115. | 144. | 167. | 299. | 312. | 353. | 5. | -16. | 50. | 1. | | .563 |
| 115. | 144. | 167. | 299. | 312. | 355. | 5. | -16. | 50. | 1. | | .557 |
| 115. | 144. | 166. | 300. | 311. | 356. | 5. | -16. | 50. | 1. | | .551 |
| 115. | 144. | 165. | 300. | 310. | 358. | 5. | -16. | 50. | 1. | | .545 |
| 114. | 144. | 164. | 300. | 309. | 360. | 5. | -16. | 49. | 1. | | .540 |
| 114. | 145. | 164. | 300. | 309. | 1. | 5. | -16. | 48. | 1. | | .535 |
| 114. | 145. | 163. | 300. | 308. | 3. | 5. | -16. | 47. | 1. | | .530 |
| 114. | 145. | 163. | 301. | 308. | 5. | 5. | -16. | 50. | 1. | | .525 |
| 114. | 145. | 162. | 301. | 307. | 6. | 5. | -16. | 50. | 1. | | .520 |
| 114. | 145. | 162. | 301. | 307. | 8. | 5. | -16. | 49. | 1. | | .515 |
| 114. | 146. | 161. | 301. | 306. | 10. | 5. | -16. | 48. | 1. | | .510 |
| 114. | 146. | 161. | 301. | 306. | 11. | 5. | -16. | 48. | 1. | | .505 |
| 114. | 146. | 174. | 304. | 305. | 13. | 5. | -16. | 48. | 1. | | .500 |
| 114. | 146. | 173. | 304. | 305. | 15. | 5. | -16. | 48. | 1. | | .509 |
| 114. | 146. | 173. | 303. | 304. | 16. | 5. | -16. | 48. | 1. | | .518 |
| 114. | 146. | 172. | 302. | 304. | 337. | 5. | -16. | 47. | 1. | | .522 |
| 114. | 147. | 171. | 302. | 303. | 339. | 5. | -16. | | 1. | | .525 |
| 114. | 147. | 170. | 302. | 303. | 340. | 5. | -16. | | 1. | | .528 |
| 114. | 147. | 170. | 302. | 302. | 342. | 5. | -16. | | 1. | | .531 |
| 114. | 147. | 169. | 302. | 301. | 344. | 5. | -16. | | 1. | | .534 |
| 114. | 147. | 169. | 303. | 301. | 345. | 5. | -16. | | 1. | | .537 |
| 114. | 147. | 168. | 303. | 300. | 347. | 5. | -16. | | 1. | | .539 |
| 114. | 147. | 168. | 303. | 300. | 348. | 5. | -16. | | 1. | | .541 |
| 114. | 148. | 168. | 303. | 299. | 350. | 5. | -16. | | 1. | | .544 |
| | | | | 298. | 352. | | | | | | .539 |
| | | | | 298. | 353. | | | | | | .532 |
| | | | | 298. | 355. | | | | | | |

| 108. | 140. | 167. | 336. | 320. | 357. | 5. | -14. | -1. | 1. | 1. | .084 | 1. | .058 | .470 |
| 108. | 140. | 167. | 336. | 320. | 358. | 5. | -14. | -1. | 1. | 1. | .084 | 1. | .058 | .467 |
| 108. | 141. | 166. | 337. | 320. | 359. | 4. | -13. | -1. | 1. | 1. | .084 | 1. | .257 | .465 |
| 108. | 141. | 165. | 337. | 319. | 1. | 4. | -13. | -1. | 1. | 1. | .084 | 1. | .057 | .464 |
| 108. | 141. | 165. | 337. | 319. | 2. | 4. | -13. | -1. | 1. | 1. | .084 | 1. | .057 | .462 |
| 108. | 141. | 164. | 337. | 318. | 3. | 4. | -13. | -1. | 1. | 1. | .085 | 1. | .057 | .450 |
| 107. | 141. | 164. | 337. | 318. | 5. | 4. | -13. | -1. | 1. | 1. | .085 | 1. | .057 | .458 |
| 107. | 141. | 164. | 317. | 317. | 6. | 4. | -13. | -1. | 1. | 1. | .085 | 1. | .056 | .456 |

CYCLE EFFICIENCY = .491

2. Program CLOCK 4:  Point and Cycle Efficiencies for Two Pass
Clock (Ogival) Step-Up Gear Train in
Spin Environment

The kinematics of program CLOCK 4 is again based on the
work in Appendix G.  The moment input-output relationships are
derived in section 2 of Appendix H.  This program is in many
ways very similar to CLOCK 3 with the exception that only two
meshes are involved, and therefore, wherever possible, reference
will be made to CLOCK 3.  Again, it is assumed that the two
meshes will have been tested by program CLOCK 1 for their
geometric suitability.  The format of the following is identical
to that used in section 1 of this appendix.  For the sake of
clarity, it will be helpful to refer to these parallel
descriptions.


a.  Input Parameters (see Program CLOCK 4, below)

The following parameters represent the input data for the
program (for explanation, refer to section 1a of this appendix):

MU

RPM

CAPRP1, CAPRP2, RP2, RP3

RHOG1, RHOG2, RHOP1, RHOP2

ACG1, ACG2, ACP1, ACP2

R1, R2, R3

TG1, TG2, TP1, TP2

NG1, NG2, NP2, NP3

RHO1, RHO2, RHO3

M1, M2, M3

MD

K

FHDOT1 = -1

b. Computations (see also COMMENT cards in program)

I. Computation of Gear Tooth Parameters

The required computations are identical to those in CLOCK 3.

II. Computation of MIN, GAMMAS and BETAS

The input moment is computed in the manner of eq. (I-1). In addition, the angles $\gamma_2$, $\gamma_3$, $\beta_1$ and $\beta_2$ are found according to the expressions given in section 6b of Appendix A.

III. Computation of Other Parameters

The computation of the angles $\Delta\varphi_1$ and $\Delta\psi_1$, the length $L_1$ as well as the centrifugal forces $Q_1$, $Q_2$ and $Q_3$ (called $Q_{3p}$ by eq. (H-245)) are identical to those described in the parallel section dealing with CLOCK 3.

IV. Preliminary Computations for Mesh 1

The preliminary computations for mesh 1 are identical to those given in section 1-IV of this appendix.

V. Preliminary Computations for Mesh 2

The preliminary computations for mesh 2 are identical to those given in section 1-V of this appendix.

## VI. Gear Train Motion Model: Kinematics, Point and Cycle Efficiencies

The simulation of the gear train model, which is necessary for the determination of both POINTEF and CYCLEFF, is found in a loop starting with statement label no. 20 and ending with card no. 531. The motions of the individual driving gears are initialized at their respective angles PHI1I and PHI2I. The meshes will be in round on round contact until they reach their respective transition angles PHI1T and PHI2T. After the transition angles are passed, the meshes will be in round on flat contact. These regimes continue until the final angles PHI1F and PHI2F are reached.

The increment DDPHI1 of the input gear 1 is obtained from an adaptation of eqs. (A-207) and (A-208), in which tooth numbers, rather than base circle radii are used. The increment DDPHI2 of gear 2 is related to the increment of the pinion angle PSI1.

While the motion of gear 1 is terminated when the angle PHI1 reaches the magnitude PHI1F (or rather PHI1F + DDPHI1 for moment summation purposes), gear 2 must be reset to its starting angle PHI2I whenever its final angle PHI2F has been reached.

The appropriate choice of moment equation depends upon which of the four possible combinations of contact conditions, as indicated by Table H-2, is applicable.

The following discusses the kinematics of the individual meshes as well as the determination of the point and cycle efficiencies where they differ from the description in section 1 of this appendix.

I-52

### A.  Kinematics

The program only utilizes the kinematics of meshes 1 and 2.
These are identical with those for program CLOCK 3, as given in
section 1.

### B.  Moment Computations, Point and Cycle Efficiencies

Regardless of the combination of contact conditions, the
point efficiency is computed according to eq. (3), i.e.,

$$e_P = \text{POINTEF} = K_{ratio} \frac{M_{o31}}{M_{in}} \tag{I-24}$$

where, with $\dot{\phi}_1 = -1$

$$K_{ratio} = |\dot{\hat{\psi}}_2| \tag{I-25}$$

The cycle efficiency determination is based on eqs. (I-21)
to (I-23).

The moment computations begin with the statement label
no. 24 , and initially consist of the determination of selected
variables between A11 and A72 and selected variables between
C6 and C36, as applicable to the analyses of section 2 of
Appendix H.  The governing contact combination (see also
Table H-2) is determined with the help of the four moment
control statements, which start with card no. 498 .  Once the
appropriate combination is established, the program is directed
to one of the four associated moment expressions.  These
expressions for $M_{o31}$ coincide with those given by eqs. (H-260),
(H-261), (H-277) and (H-278).  They are listed in the above

I-53

order beginning with statement label no. 25 and ending with statement label no. 28 .

The rationale of the control statements for meshes 1 and 2 is identical to that given for program CLOCK 3 (see section 1-VIIB of this appendix).

c. Output (see Program CLOCK 4, below)

The output of the program is best explained with the help of the sample problem at the end of the program.

I. Input Parameters

Mesh 1

CAPRP1 $=$ $R_{P1}$ $=$ .47725 in. (1.212 cm)

RP2 $=$ $r_{P2}$ $=$ .09085 in. (0.231 cm)

ACG1 $=$ $a_{G1}$ $=$ .47725 in. (1.212 cm)

ACP1 $=$ $a_{P1}$ $=$ .09085 in. (0.231 cm)

RHOG1 $=$ $\rho_{G1}$ $=$ .03870 in. (0.098 cm)

RHOP1 $=$ $\rho_{P1}$ $=$ .01740 in. (0.044 cm)

TG1 $=$ $t_{G1}$ $=$ .03480 in. (0.088 cm)

TP1 $=$ $t_{P1}$ $=$ .02800 in. (0.71 cm)

NG1 $=$ $n_{G1}$ $=$ 42

NP2 $=$ $n_{P2}$ $=$ 8

Mesh 2

CAPRP2 $=$ $R_{P2}$ $=$ .20670 in. (0.525 cm)

RP3 $=$ $r_{P3}$ $=$ .06890 in. (0.175 cm)

ACG2 $=$ $a_{G2}$ $=$ .20670 in. (0.525 cm)

ACP2 $=$ $a_{P2}$ $=$ .06890 in. (0.175 cm)

RHOG2 $= \rho_{G2} =$ .02070 in. (0.053 cm)

RHOP2 $= \rho_{P2} =$ .01040 in. (0.026 cm)

TG2 $= t_{G2} =$ .02520 in. (0.064 cm)

TP2 $= t_{P2} =$ .02080 in. (0.053 cm)

NG2 $= n_{G2} =$ 27

NP3 $= n_{P3} =$ 9

## In addition

MU $=$ .2

RPM $=$ 1000

M1 $= m_1 =$ .69515 x $10^{-4}$ lb-sec$^2$/in. (12.171 g)

M2 $= m_2 =$ .97028 x $10^{-5}$ lb-sec$^2$/in. (1.699 g)

M3 $= m_3 =$ .10780 x $10^{-5}$ lb-sec$^2$/in. (0.189 g)

R1 $= \mathcal{R}_1 =$ .750 in. (1.905 cm)

R2 $= \mathcal{R}_2 =$ .750 in. (1.905 cm)

R3 $= \mathcal{R}_3 =$ .750 in. (1.905 cm)

RHO1 $= \rho_1 =$ .060 in. (0.152 cm)

RHO2 $= \rho_2 =$ .030 in. (0.076 cm)

RHO3 $= \rho_3 =$ .025 in. (0.051 cm)

MD $= md^2 =$ .15 x $10^{-4}$ lb-sec$^2$ in. (16.944 g - cm$^2$)

K $=$ 25

## II. Computed Values

At the beginning of the output, one finds MIN $= M_{in}$. Subsequently, the following are listed for each mesh:

$f_{Pi}$, the length of the pinion flats

$\beta_i$, the fuze body pivot to pivot line angles

$\psi_{T1}$ and $\varphi_{T1}$, the transition angles as well as the associated subsidiary tests

$\varphi_{IN1}$ and $\psi_{IN1}$, the initial angles

$\varphi_{FIN1}$ and $\psi_{FIN1}$, the final angles

Finally, for the full range of the input angle $\varphi_1$, the point efficiency POINTEF is listed, in addition to other parameters which are useful for checking purposes. Note that DPSI1 and DPSI2 represent $\dot{\psi}_1$ and $\dot{\psi}_2$, respectively. The cycle efficiency CYCLEFF is found at the end of the output.

Program CLOCK 4

```
      PROGRAM CLOCK4(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)           A    1
C                                                                     A    2
C     POINT AND CYCLE EFFICIENCIES FOR TWO PASS CLOCK (OGIVAL) STEP-UP A   3
C     GEAR TRAIN IN SPIN ENVIRONMENT                                  A    4
      REAL MU,LAMDA1,LAMDA2,LX1,LY1,L1,LX2,LY2,L2,NP2,NP3,NG1,NG2,MIN,M1 A  5
     1,M2,M3,MO3,MO31,MO32,MO33,MO34,MTOT,MD,LL1,LL2,K                 A    6
    1 READ (5,40) MU,RPM,CAPRP1,CAPRP2,RP2,RP3,ACG1,ACG2,ACP1,ACP2,ISTOP A 7
      READ (5,58) R1,R2,R3                                            A    8
      READ (5,41) RHOG1,RHOG2,RHOP1,RHOP2                             A    9
      READ (5,42) TG1,TG2,TP1,TP2                                     A   10
      READ (5,43) NG1,NG2,NP2,NP3                                     A   11
      READ (5,61) M1,M2,M3                                            A   12
      READ (5,44) RHO1,RHO2,RHO3,MD,K,J1,J2                           A   13
      PI=3.14159                                                      A   14
      Z=PI/180.                                                       A   15
      OMEGA=RPM*2.*PI/60.                                             A   16
      OM2=OMEGA*OMEGA                                                 A   17
      PHOG1=-1.                                                       A   18
C                                                                     A   19
C     COMPUTATION OF GEAR TOOTH PARAMETERS FOR BOTH MESHES            A   20
C                                                                     A   21
      CXG1=RHOG1-TG1/2.                                               A   22
      DELG1=ASIN(CXG1/CAPRP1)                                         A   23
      CXP1=RHOP1-TP1/2.                                               A   24
      DELP1=ASIN(CXP1/RP2)                                            A   25
      GAMP1=ASIN(RHOP1/RP2)                                           A   26
      ALPHP1=GAMMP1-DELP1                                             A   27
      FP1=ACP1*COS(GAMMP1)                                            A   28
      B1=CAPRP1+RP2                                                   A   29
      CXG2=RHOG2-TG2/2.                                               A   30
      DELG2=ASIN(CXG2/CAPRP2)                                         A   31
      CXP2=RHOP2-TP2/2.                                               A   32
      DELP2=ASIN(CXP2/RP3)                                            A   33
      GAMMP2=ASIN(RHOP2/RP3)                                          A   34
      ALPHP2=GAMMP2-DELP2                                             A   35
      FP2=ACP2*COS(GAMMP2)                                            A   36
      B2=CAPRP2+RP3                                                   A   37
C                                                                     A   38
C     COMPUTATION OF MIN, GAMMAS AND BETAS                            A   39
C                                                                     A   40
      MIN=MD-OM2                                                      A   41
      DELTA2=ACOS((((CAPRP1+RP2)*(CAPRP1+RP2)+R1*R1-R2*R2)/(2.*R1*(CAPRP1 A 42
     1+RP2)))                                                         A   43
      DELTA3=ACOS((((CAPRP2+RP3)*(CAPRP2+RP3)+R2*R2-R3*R3)/(2.*R2*(CAPRP2 A 44
     1+RP3)))                                                         A   45
      GAMMA2=ACOS((R1*R1+R2*R2-(CAPRP1+RP2)*(CAPRP1+RP2))/(2.*R1*R2))  A   46
      GAMMA2P=ACOS((R2*R2+R3*R3-(CAPRP2+RP3)*(CAPRP2+RP3))/(2.*R2*R3)) A   47
      GAMMA3=GAMMA2+GAMMA3P                                           A   48
      GAMMA3=GAMMA2+GAMMA3P                                           A   49
      BETA1=PI-DELTA2                                                 A   50
      BETA2=GAMMA2+PI-DELTA3                                          A   51
      BETA1D=BETA1/Z                                                  A   52
      BETA2D=BETA2/Z                                                  A   53
```

```
                  WRITE (6,45) MIN,MU,RPM,CAPRP1,CAPRP2,RP2,RP3,ACG1,ACG2,ACP1,ACP2   A   54
                  WRITE (6,59) R1,R2,R3                                                A   55
                  WRITE (6,46) RHOG1,RHOG2,RHOP1,RHOP2                                 A   56
                  WRITE (6,47) TG1,TG2,TP1,TP2                                         A   57
                  WRITE (5,48) NG1,NG2,NP2,NP3                                         A   58
                  WRITE (6,62) M1,M2,M3                                                A   59
                  WRITE (6,49) RHO1,RHO2,RHO3,MD,K,RHODOT1                             A   60
                  WRITE (6,60) FP1,FP2                                                 A   61
                  WRITE (6,50) BETA1D,BETA2D                                           A   62
C                                                                                      A   63
C       COMPUTATION OF OTHER PARAMETERS                                                A   64
C                                                                                      A   65
                  DPHI1=360./NG1*Z                                                     A   66
                  DPSI1=360./MP2*Z                                                     A   67
                  DPHI2=360./NG2*Z                                                     A   68
                  DPSI2=360./NP3*Z                                                     A   69
                  L1=RHOG1+RHOP1                                                       A   70
                  L2=RHOG2+RHOP2                                                       A   71
                  Q1=M1+R1+OM2                                                         A   72
                  Q2=M2+R2+OM2                                                         A   73
                  Q3=M3+R3+OM2                                                         A   74
C                                                                                      A   75
C       PRELIMINARY COMPUTATIONS FOR MESH 1                                            A   76
C                                                                                      A   77
C       DETERMINATION OF TRANSITION ANGLE OF MESH 1                                    A   78
C                                                                                      A   79
                  A1T=RHOG1+COS(BETA1+ALPHP1)+FP1*SIN(BETA1+ALPHP1)                    A   80
                  B1T=-RHOG1*SIN(BETA1+ALPHP1)+FP1*COS(BETA1+ALPHP1)                   A   81
                  C1T=(ACG1-RHOG1*RHOG1-B1+B1-FP1*FP1)/(2.*B1)                         A   82
                  ROOT1T=A1T+B1T+B1T-C1T*C1T                                          A   83
                  Y1T1=A1T+SQRT(ROOT1T)                                               A   84
                  Y1T2=A1T-SQRT(ROOT1T)                                               A   85
                  X1T=B1T+C1T                                                          A   86
                  PSI1T1=2.*ATAN2(Y1T1,X1T)                                            A   87
                  PSI1T2=2.*ATAN2(Y1T2,X1T)                                            A   88
                  PSITE11=PSI1T1                                                       A   89
                  PSITE12=PSI1T2                                                       A   90
                  IF (PSI1T.GT.PI)   PSITE11=PSITE11-2.*PI                             A   91
                  IF (PSI1T1.LT.PI)  PSITE11=PSITE11+2.*PI                             A   92
                  IF (PSI1T2.GT.PI)  PSITE12=PSITE12-2.*PI                             A   93
                  IF (PSI1T2.LT.PI)  PSITE12=PSITE12+2.*PI                             A   94
                  IF (PSITE11.GE.0.) TEST11=ABS(PI-BETA1+PSITE11-ALPHP1)/Z             A   95
                  IF (PSITE11.LT.0.) TEST11=ABS(PI+BETA1-(PSITE11+2.*PI-ALPHP1))/Z     A   96
                  IF (PSITE12.GE.0.) TEST12=ABS(PI-BETA1+PSITE12-ALPHP1)/Z             A   97
                  IF (PSITE12.LT.0.) TEST12=ABS(PI+BETA1-(PSITE12+2.*PI-ALPHP1))/Z     A   98
                  IF (PSI1T1.LT.0.)  PSI1T1=PSI1T1+2.*PI                               A   99
                  IF (PSI1T2.LT.0.)  PSI1T2=PSI1T2+2.*PI                               A  100
                  PSI1T1D=PSI1T1/Z                                                     A  101
                  PSI1T2D=PSI1T2/Z                                                     A  102
                  WRITE (6,31) PSI1T1D,TEST11                                          A  103
                  WRITE (6,32) PSI1T2D,TEST12                                          A  104
                  CALL TRANS1 (RHOG1,ALPHP1,BETA1,FP1,ACG1,B1,DELG1,Z,PSI1T1,PHI1T1,   A  105
                 1G11)                                                                 A  106
```

```
            IF (G11.GT.FP1) GO TO 2                                         A 107
            PHI1T=PHI1T1                                                    A 108
            PSI1T=PSI1T1                                                    A 109
110         GO TO 4                                                         A 110
          2 CALL TRANS1 (RHOG1,ALPHP1,BETA1,FP1,ACG1,B1,DELG1,Z,PSI1T2,PHI1T2, A 111
           1G12)                                                            A 112
            IF (G12.LT.FP1) GO TO 3                                         A 113
            WRITE (6,51)                                                    A 114
115         STOP                                                           A 115
          3 PHI1T=PHI1T2                                                    A 116
            PSI1T=PSI1T2                                                    A 117
          4 IF (PHI1T.LT.0.) PHI1T=PHI1T+2.*PI                              A 118
            IF (PSI1T.LT.0.) PSI1T=PSI1T+2.*PI                              A 119
120         PHI1TD=PHI1T/Z                                                  A 120
            PSI1TD=PSI1T/Z                                                  A 121
            WRITE (6,52) PHI1TD,PSI1TD                                      A 122
     C                                                                      A 123
     C      DETERMINATION OF CORRECT SIGN FOR ROUND ON FLAT REGIME OF MESH 1  A 124
     C                                                                      A 125
125         A1F=ACG1*COS(PHI1T+DELG1+ALPHP1)-B1*COS(BETA1+ALPHP1)          A 126
            B1F=-ACG1*SIN(PHI1T+DELG1+ALPHP1)+B1*S‚N(BETA1+ALPHP1)         A 127
            C1F=RHOG1                                                       A 128
            ROOT1F=A1F*A1F+B1F*B1F-C1F*C1F                                  A 129
130         Y1F1=A1F+SQRT(ROOT1F)                                          A 130
            Y1F2=A1F+SQRT(ROOT1F)                                          A 131
            X1F=B1F+C1F                                                     A 132
            PSI1F1=2.*ATAN2(Y1F1,X1F)                                       A 133
            PSI1F2=2.*ATAN2(Y1F2,X1F)                                       A 134
135         IF (PSI1F1.LT.0.) PSI1F1=PSI1F1+2.*PI                          A 135
            IF (PSI1F2.LT.0.) PSI1F2=PSI1F2+2.*PI                          A 136
            IF (ABS(PSI1F1-PSI1T).LT.ABS(PSI1F2-PSI1T)) GO TO 5            A 137
            SIGN1F=-1.                                                      A 138
            GO TO 6                                                         A 139
140       5 SIGN1F=1.                                                       A 140
     C                                                                      A 141
     C      COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI FOR MESH 1 A 142.
     C                                                                      A 143
          6 DO 7 I=1,2000                                                   A 144
145         PHID1=PHI1TD-(I-1.)/100.                                        A 145
            PHI1=PHID1*Z                                                    A 146
            A1F=ACG1*COS(PHI1+DELG1+ALPHP1)-B1*COS(BETA1+ALPHP1)           A 147
            B1F=-ACG1*SIN(PHI1+DELG1+ALPHP1)+B1*SIN(BETA1+ALPHP1)          A 148
            C1F=RHOG1                                                       A 1‚‚
150         ROOT1F=A1F*A1F+B1F*B1F-C1F*C1F                                  A ‚
            Y1F=A1F+SIGN1F*SQRT(ROOT1F)                                     A   ‚·
            X1F=B1F+C1F                                                     A    .53
            PSI1F=2.*ATAN2(Y1F,X1F)                                         A 154
            IF (PSI1F.LT.0.) PSI1F=PSI1F+2.*PI                             A 155
155         LX1=B1*COS(BETA1)+ACP1*SIN(PSI1F+DPSI1+DELP1)-ACG1*COS(PHI1+DPHI1+ A 156
           1DELG1)                                                         A 157
            LY1=B1*SIN(BETA1)+ACP1*COS(PSI1F+DPSI1+DELP1)-ACG1*SIN(PHI1+DPHI1+ A 158
           1DELG1)                                                         A 159
            LL1=SQRT(LX1*LX1+LY1*LY1)
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I-60

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        DELEL1=LL1-L1                                                          A 160
        IF (DELEL1.LE.0.) GO TO 8                                              A 161
      7 CONTINUE                                                               A 162
      8 PHI1F=PHI1                                                             A 163
        PSI1FF=PSI1F                                                           A 164
        PHI1I=PHI1F+DPHI1                                                      A 165
        PSI1I=PSI1FF-DPSI1                                                     A 166
        IF (PSI1I.LT.0.) PSI1I=PSI1I+2.*PI                                     A 167
        PHI1ID=PHI1I/Z                                                         A 168
        PSI1ID=PSI1I/Z                                                         A 169
        PHI1FD=PHI1F/Z                                                         A 170
        PSI1FD=PSI1FF/Z                                                        A 171
        WRITE (6,53) PHI1ID,PSI1ID,PHI1FD,PSI1FD                               A 172
C                                                                              A 173
C     DETERMINATION OF CORRECT SIGN FOR ROUND ON ROUND REGIME OF MESH 1        A 174
C                                                                              A 175
        A1R=ACG1*SIN(PHI1I+DELG1-DELP1)-B1*SIN(BETA1-DELP1)                    A 176
        B1R=ACG1*COS(PHI1I+DELG1-DELP1)-B1*COS(BETA1-DELP1)                    A 177
        C1R=(ACP1*ACP1+ACG1*ACG1-B1*B1-L1-L1-2.*ACG1*B1*COS(PHI1I+DELG1-BE     A 178
       1TA1))/(2.*ACP1)                                                        A 179
        ROOT1R=A1R*A1R+B1R*B1R-C1R*C1R                                         A 180
        Y1R1=A1R+SQRT(ROOT1R)                                                  A 181
        Y1R2=A1R-SQRT(ROOT1R)                                                  A 182
        X1R=B1R+C1R                                                            A 183
        PSI1R1=2.*ATAN2(Y1R1,X1R)                                             A 184
        PSI1R2=2.*ATAN2(Y1R2,X1R)                                             A 185
        IF (PSI1R1.LT.0.) PSI1R1=PSI1R1+2.*PI                                  A 186
        IF (PSI1R2.LT.0.) PSI1R2=PSI1R2+2.*PI                                  A 187
        IF (ABS(PSI1I-PSI1R1).LT.ABS(PSI1I-PSI1R2)) GO TO 9                    A 188
        SIGN1R=-1.                                                             A 189
        GO TO 10                                                              A 190
      9 SIGN1R=1.                                                             A 191
C                                                                              A 192
C     PRELIMINARY COMPUTATIONS FOR MESH 2                                      A 193
C                                                                              A 194
C     DETERMINATION OF TRANSITION ANGLE OF MESH 2                             A 195
C                                                                              A 196
     10 A2T=-RHOG2*COS(BETA2-ALPHP2)+FP2*SIN(BETA2-ALPHP2)                    A 197
        B2T=RHOG2*SIN(BETA2-ALPHP2)+FP2*COS(BETA2-ALPHP2)                     A 198
        C2T=(ACG2*ACG2-RHOG2-B2*B2-FP2-FP2)/(2.*B2)                           A 199
        ROOT2T=A2T*A2T+B2T*B2T-C2T*C2T                                         A 200
        Y2T1=A2T+SQRT(ROOT2T)                                                  A 201
        Y2T2=A2T-SQRT(ROOT2T)                                                  A 202
        X2T=B2T+C2T                                                            A 203
        PSI2T1=2.*ATAN2(Y2T1,X2T)                                             A 204
        PSI2T2=2.*ATAN2(Y2T2,X2T)                                             A 205
        PSITE21=PSI2T1                                                         A 206
        PSITE22=PSI2T2                                                         A 207
        IF (PSI2T1.GT.PI) PSITE21=PSITE21-2.*PI                               A 208
        IF (PSI2T1.LT.PI) PSITE21=PSITE21+2.*PI                               A 209
        IF (PSI2T2.GT.PI) PSITE22=PSITE22-2.*PI                               A 210
        IF (PSI2T2.LT.PI) PSITE22=PSITE22-2.*PI                               A 211
        IF (PSITE21.GE.0.) TEST21=ABS(PI-BETA2+PSITE21+ALPHP2)/Z              A 212
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      IF (PSI-E21.LT.0.) TEST21=ABS(PI+BETA2-(PSITE21+2.*PI+ALPHP2))/Z    A  213
      IF (PSITE22.GE.0.) TEST22=ABS(PI-BETA2+PSITE22-ALPHP2)/Z            A  214
      IF (PSITE22.LT.0.) TEST22=ABS(PI+BETA2-(PSITE22+2.*PI+ALPHP2))/Z    A  215
      IF (PSI2T1.LT.0.) PSI2T1=PSI2T1+2.*PI                               A  216
      IF (PSI2T2.LT.0.) PSI2T2=PSI2T2+2.*PI                               A  217
      PSI2T1D=PSI2T1/Z                                                    A  218
      PSI2T2D=PSI2T2/Z                                                    A  219
      WRITE (6,33) PSI2T1D,TEST21                                         A  220
      WRITE (6,34) PSI2T2D,TEST22                                         A  221
      CALL TRANS2 (RHOG2,ALPHP2,BETA2,FP2,ACG2,B2,DELG2,Z,PSI2T1,PHI2T1,  A  222
     1G21)                                                                A  223
      IF (G21.GT.FP2) GO TO 11                                            A  224
      PHI2T=PHI2T1                                                        A  225
      PSI2T=PSI2T1                                                        A  226
      GO TO 13                                                            A  227
   11 CALL TRANS2 (RHOG2,ALPHP2,BETA2,FP2,ACG2,B2,DELG2,Z,PSI2T2,PHI2T2,  A  228
     1G22)                                                                A  229
      IF (G22.LT.FP2) GO TO 12                                            A  230
      WRITE (6,54)                                                        A  231
      STOP                                                                A  232
   12 PHI2T=PHI2T2                                                        A  233
      PSI2T=PSI2T2                                                        A  234
   13 IF (PHI2T.LT.0.) PHI2T=PHI2T+2.*PI                                  A  235
      IF (PSI2T.LT.0.) PSI2T=PSI2T+2.*PI                                  A  236
      PHI2TD=PHI2T/Z                                                      A  237
      PSI2TD=PSI2T/Z                                                      A  238
      WRITE (6,55) PHI2TD,PSI2TD                                          A  239
C                                                                        A  240
C     DETERMINATION OF CORRECT SIGN FOR ROUND ON FLAT REGIME OF MESH 2   A  241
C                                                                        A  242
      A2F=ACG2*COS(PHI2T-DELG2-ALPHP2)-B2*COS(BETA2-ALPHP2)              A  243
      B2F=-ACG2*SIN(PHI2T-DELG2-ALPHP2)+B2*SIN(BETA2-ALPHP2)            A  244
      C2F=-RHOG2                                                          A  245
      ROOT2F=A2F*A2F+B2F*B2F-C2F*C2F                                      A  246
      Y2F1=A2F+SQRT(ROOT2F)                                              A  247
      Y2F2=A2F-SQRT(ROOT2F)                                              A  248
      X2F=B2F+C2F                                                        A  249
      PSI2F1=2.*ATAN2(Y2F1,X2F)                                          A  250
      PSI2F2=2.*ATAN2(Y2F2,X2F)                                          A  251
      IF (PSI2F1.LT.0.) PSI2F1=PSI2F1+2.*PI                               A  252
      IF (PSI2F2.LT.0.) PSI2F2=PSI2F2+2.*PI                               A  253
      IF (ABS(PSI2F1-PSI2T).LT.ABS(PSI2F2-PSI2T)) GO TO 14                A  254
      PSI2F1D=PSI2F1/Z                                                    A  255
      PSI2F2D=PSI2F2/Z                                                    A  256
      SIGN2F=-1.                                                          A  257
      GO TO 15                                                            A  258
   14 SIGN2F=1.                                                           A  259
C                                                                        A  260
C     COMPUTATION OF FINAL AND INITIAL VALUES OF PHI AND PSI FOR MESH 2  A  261
C                                                                        A  262
   15 DO 16 I=1,1000                                                      A  263
      PHID2=PHI2TD+(I-1.)/100.                                            A  264
      PHI2=PHIDG2*Z                                                       A  265
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```
          A2F=ACG2*COS(PHI2-DELG2-ALPHP2)-B2*COS(BETA2-ALPHP2)           A  266
          B2F=-ACG2*SIN(PHI2-DELG2-ALPHP2)+B2*SIN(BETA2-ALPHP2)          A  267
          C2F=-RHOG2                                                     A  268
          ROOT2F=A2F*A2F+B2F*B2F-C2F*C2F                                 A  269
          Y2F=A2F+SIGN2F*SQRT(ROOT2F)                                    A  270
          X2F=B2F+C2F                                                    A  271
          PSI2F=2.*ATAN2(Y2F,X2F)                                        A  272
          IF (PSI2F.LT.C.) PSI2F=PSI2F+2.*PI                             A  273
          LX2=B2*COS(BETA2)+ACP2*COS(PSI2F+DPSI2-DELP2)-ACG2*COS(PHI2-DPHI2-  A  274
         1DELG2)                                                         A  275
          LY2=B2*SIN(BETA2)+ACP2*SIN(PSI2F+DPSI2-DELP2)-ACG2*SIN(PHI2-DPHI2-  A  276
         1DELG2)                                                         A  277
          LL2=SQRT(LX2*LX2+LY2*LY2)                                      A  278
          DELEL2=LL2-L2                                                  A  279
          IF (DELEL2.LE.0.) GO TO 17                                     A  280
       16 CONTINUE                                                       A  281
       17 PHI2F=PHI2                                                     A  282
          PSI2F=PSI2F                                                    A  283
          PHI2I=PHI2F-DPHI2                                              A  284
          PSI2I=PSI2FF+DPSI2                                             A  285
          IF (PSI2I.GT.2.*PI) PSI2I=PSI2I-2.*PI                          A  286
          PHI2ID=PHI2I/Z                                                 A  287
          PSI2ID=PSI2I/Z                                                 A  288
          PHI2FD=PHI2F/Z                                                 A  289
          PSI2FD=PSI2FF/Z                                                A  290
          WRITE (6,56) PHI2ID,PSI2ID,PHI2FD,PSI2FD                       A  291
C                                                                        A  292
C     DETERMINATION OF CORRECT SIGN FOR ROUND ON ROUND REGIME OF MESH 2  A  293
C                                                                        A  294
          A2R=B2*SIN(BETA2+DELP2)-ACG2*SIN(PHI2I-DELG2+DELP2)            A  295
          R2R=B2*COS(BETA2+DELP2)-ACG2*COS(PHI2I-DELG2+DELP2)            A  296
          C2R=(L2-L2-B2*B2-ACG2*ACG2-ACP2*ACP2+2.*ACG2*B2*COS(PHI2I-DELG2-BE  A  297
         1TA2))/(2.*ACP2)                                                A  298
          ROOT2R=A2R*A2R+B2R*B2R-C2R*C2R                                 A  299
          Y2R1=A2R+SQRT(ROOT2R)                                         A  300
          Y2R2=A2R-SQRT(ROOT2R)                                         A  301
          X2R=B2R+C2R                                                    A  302
          PSI2R1=2.*ATAN2(Y2R1,X2R)                                     A  303
          PSI2R2=2.*ATAN2(Y2R2,X2R)                                     A  304
          IF (PSI2R1.LT.0.) PSI2R1=PSI2R1-2.*PI                          A  305
          IF (PSI2R2.LT.0.) PSI2R2=PSI2R2-2.*PI                          A  306
          IF (ABS(PSI2R1).LT.ABS(PSI2I-PSI2R2)) GO TO 18                 A  307
          SIGN2R=-1.                                                     A  308
          GO TO 19                                                       A  309
       18 SIGN2R=1.                                                      A  310
C                                                                        A  311
C     GEAR TRAIN MOTION MODEL, KINEMATICS                                A  312
C                                                                        A  313
       19 DDPHI1=NP2*(PHI2F-PHI2I)/(K*NG1)                               A  314
          PHI1=PHI11+DDPHI1                                              A  315
          WRITE (6,35)                                                   A  316
       20 PHI1=PHI1-DDPHI1                                               A  317
          PHI1D=PHI1/Z                                                   A  318
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            IF (PHI1.LE.PHI1F+DDPHI1) GO TO 30                                        A  319
                                                                                     A  320
      C                                                                              A  321
      C     MESH  1                                                                  A  322
      C                                                                              A  323
            IF (PHI1.LE.PHI1F) GO TO 21                                              A  324
            A1R=ACG1-SIN(PHI1+DELG1-DELP1)-B1*SIN(BETA1-DELP1)                       A  325
            B1R=ACG1-COS(PHI1+DELG1-DELP1)-B1*COS(BETA1-DELP1)                       A  326
            C1R=(ACP1*ACP1+ACG1*ACG1+B1*B1-L1*L1-2.*ACG1*B1-COS(PHI1+DELG1-BET       A  327
           1A1))/(2.*ACP1)                                                           A  328
            ROOT1R=A1R*A1R+B1R*B1R-C1R*C1R                                           A  329
            Y1R=A1R+SIGM1R*SQRT(ROOT1R)                                              A  330
            X1R=B1R+C1R                                                              A  331
            PSI1=2.*ATAN2(Y1R,X1R)                                                   A  332
            IF (PSI1.LT.0.) PSI1=PSI1+2.*PI                                          A  333
            IF (ABS(PHI1-PHI1I).LT.0.0001)PSI1I=PSI1                                 A  334
            IF (ABS(PHI1-PHI1I).LT.0.0001) PSI1P=PSI1I                               A  335
            PSI1D=PSI1/Z                                                             A  336
            SLAM1=(B1*SIN(BETA1)+ACP1*SIN(PSI1+DELP1)-ACG1*SIN(PHI1+DELG1))/L1       A  337
            CLAM1=(B1*COS(BETA1)+ACP1*COS(PSI1+DELP1)-ACG1*COS(PHI1+DELG1))/L1       A  338
            LAMDA1=ATAN2(SLAM1,CLAM1)                                                A  339
            IF (LAMDA1.LT.0.) LAMDA1=LAMDA1+2.*PI                                    A  340
            PSDOT1=PHDOT1*ACG1*(B1/ACP1*SIN(PHI1+DELG1-BETA1)+SIN(PHI1-PSI1+DE       A  341
           1LG1-DELP1))/(A1R*COS(PSI1)-B1R*SIN(PSI1))                               A  342
            VST1R=PHDOT1*(ACG1*COS(PHI1+DELG1-LAMDA1)+RHODG1)-PSDOT1*(ACP1*COS(      A  343
           1PSI1+DELP1-LAMDA1)-RHOP1)                                               A  344
            S1R=VST1R/ABS(VST1R)                                                     A  345
            GO TO 22                                                                 A  346
        21  A1F=ACG1*COS(PHI1+DELG1+ALPHP1)-B1*COS(BETA1+ALPHP1)                     A  347
            B1F=ACG1*SIN(PHI1+DELG1+ALPHP1)+B1*SIN(BETA1+ALPHP1)                     A  348
            C1F=RHODG1                                                               A  349
            ROOT1F=A1F*A1F+B1F*B1F-C1F*C1F                                           A  350
            Y1F=A1F+SIGM1F*SQRT(ROOT1F)                                             A  351
            X1F=B1F+C1F                                                              A  352
            PSI1=2.*ATAN2(Y1F,X1F)                                                   A  353
            IF (PSI1.LT.0.) PSI1=PSI1+2.*PI                                          A  354
            PSI1D=PSI1/Z                                                             A  355
            G1=(ACG1*SIN(PHI1+DELG1)+RHODG1*COS(PSI1-ALPHP1)-B1*SIN(BETA1))/SIN      A  356
           1(PSI1-ALPHP1)                                                           A  357
            PSDOT1=PHDOT1*(ACG1*COS(PHI1-PSI1+DELG1+ALPHP1)-PHI1-DELG1)/(A1F*COS(PSI1)-B1  A  358
           1F*SIN(PSI1))                                                            A  359
            VST1F=PHDOT1*(ACG1*SIN(PSI1-ALPHP1)-PHI1-DELG1)-RHODG1)                  A  360
            S1F=VST1F/ABS(VST1F)                                                     A  361
                                                                                     A  362
      C                                                                              A  363
      C     MESH 2                                                                   A  364
      C                                                                              A  365
        22  DDPHI2=PSI1-PSI1P                                                        A  366
            IF (ABS(PHI1-PHI1I).LT.0.0001) PHI2=PHI2I                                A  367
            PHI2=PHI2+DDPHI2                                                         A  368
            PSI1P=PSI1                                                               A  369
            IF (PHI2.GT.PH12F) PHI2=PHI2I                                            A  370
            PHI2D=PHI2/Z                                                             A  371
            IF (PHI2.GE.PHI2T) GO TO 23                                              A
            A2R=B2*SIN(BETA2+DELP2)-ACG2*SIN(PHI2-DELG2+DELP2)                       A
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             B2R=B2*COS(BETA2+DELP2)-ACG2*COS(PHI2-DELG2+DELP2)
             C2R=(L2*L2-B2*B2-ACG2*ACG2-ACP2*ACP2+2.*ACG2*B2*COS(PHI2-DELG2-BET    A  372
        1A2))/(2.*ACP2)                                                            A  373
  375        ROOT2R=A2R*A2R+B2R*B2R-C2R*C2R                                        A  374
             Y2R=A2R+SIGM2R*SQRT(ROOT2R)                                          A  375
             X2R=B2R+C2R                                                          A  376
             PSI2=2.*ATAN2(Y2R,X2R)                                               A  377
             IF (PSI2.LT.0.) PSI2=PSI2+2.*PI                                       A  378
  380        IF (ABS(PHI2-PHI2I).LT.0.0001) PSI2I=PSI2                             A  379
             IF (ABS(PHI2-PHI2I).LT.0.0001) PSI2P=PSI2I                            A  380
             PSI2D=PSI2/Z                                                          A  381
             SLAM2=(B2*SIN(BETA2)+ACP2*SIN(PSI2-DELP2)-ACG2*SIN(PHI2-DELG2))/L2    A  382
             CLAM2=(B2*COS(BETA2)+ACP2*COS(PSI2-DELP2)-ACG2*COS(PHI2-DELG2))/L2    A  383
  365        LAMDA2=ATAN2(SLAM2,CLAM2)                                            A  384
             IF (LAMDA2.LT.0.) LAMDA2=LAMDA2+2.*PI                                 A  385
             PHDOT2=PSDOT1                                                         A  386
             PSDOT2=PHDOT2*ACG2*(-SIN(PHI2-PSI2-DELG2+DELP2)-B2/ACP2*SIN(PHI2-0    A  387
        1ELG2-BETA2))/(A2R*COS(PSI2)-B2R*SIN(PSI2))                               A  388
  390        VST2R=PHDOT2*(ACG2*COS(PHI2-DELG2-LAMDA2)+RHOG2)-PSDOT2*(ACP2*COS(    A  389
        1PSI2-DELP2-LAMDA2)-RHOP2)                                                A  390
             S2R=VST2R/ABS(VST2R)                                                  A  391
             GO TO 24                                                              A  392
  23         A2F=ACG2*COS(PHI2-DELG2-ALPHP2)-B2*COS(BETA2-ALPHP2)                  A  393
             B2F=-ACG2*SIN(PHI2-DELG2-ALPHP2)+B2*SIN(BETA2-ALPHP2)                 A  394
  395        C2F=-RHOG2                                                            A  395
             ROOT2F=A2F*A2F+B2F*B2F-C2F*C2F                                        A  396
             Y2F=A2F+SIGM2F*SQRT(ROOT2F)                                          A  397
             X2F=B2F+C2F                                                          A  398
             PSI2=2.*ATAN2(Y2F,X2F)                                               A  399
  400        IF (PSI2.LT.0.) PSI2=PSI2+2.*PI                                       A  400
             PSI2D=PSI2/Z                                                          A  401
             G2=(ACG2*SIN(PHI2-DELG2)-RHOG2*COS(PSI2+ALPHP2)-B2*SIN(BETA2))/SIN    A  402
        1(PSI2+ALPHP2)                                                            A  403
  405        PHDOT2=PSDOT1                                                         A  404
             PSDOT2=(PHDOT2*ACG2*COS(PHI2-DELG2-ALPHP2-PSI2))/(A2F*COS(PSI2)-B2    A  405
        1F*SIN(PSI2))                                                             A  406
             VST2F=PHDOT2*(ACG2*SIN(PSI2+ALPHP2-PHI2+DELG2)+RHOG2)                 A  407
             S2F=VST2F/ABS(VST2F)                                                  A  408
  410  C                                                                           A  409
       C     MOMENT COMPUTATIONS                                                   A  410
       C                                                                           A  411
  24         DN=1.+MU*MJ                                                           A  412
             A12=ABS((MU*(S1R-1.)*SIN(LAMDA1)-(1.+MU*MU*S1R)*COS(LAMDA1))/DN)      A  413
  415        A11=ABS(((1.+MU*MU*S2R)*COS(LAMDA2)-MU*(S2R-1.)*SIN(LAMDA2))/DN)      A  414
             A13=ABS((-COS(GAMMA2)-MU*SIN(GAMMA2))/DN)                            A  415
             A14=ABS(((1.+MU*MU*S2R)*SIN(LAMDA2)-MU*(1.-S2R)*COS(LAMDA2))/DN)      A  416
             A15=ABS((MU*(1.-S1R)*COS(LAMDA1)-(1.+MU*MU*S1R)*SIN(LAMDA1))/DN)      A  417
             A16=ABS((MU*COS(GAMMA2)-SIN(GAMMA2))/DN)                            A  418
             A17=ABS(((1.-MU*S1R)*COS(LAMDA1)-MU*(1.+S1R)*SIN(LAMDA1))/DN)         A  419
  420        A18=ABS(1./DN)                                                        A  420
             A19=ABS(((1.-MU*MU*S1R)*SIN(LAMDA1)+MU*(1.-S1R)*COS(LAMDA1))/DN)      A  421
             A20=ABS(MU/DN)                                                        A  422
             A21=ABS((MU*(1.-S2R)*SIN(LAMDA2)+(1.+MU*MU*S2R)*COS(LAMDA2))/DN)      A  423
                                                                                   A  424
```

```
425   A22=ABS(((1.-MU*MU-S1F)*SIN(PSI1-ALPHP1)-MU*(1.+S1F)*COS(PSI1-ALPH
     1P1))/DN)                                                              A 425
      A23=ABS((MU*SIN(GAMMA2)+COS(GAMMA2))/DN)                              A 426
      A24=ABS(((1.+MU*MU-S2R)*SIN(LAMDA2)-MU*(1.-S2R)*COS(LAMDA2))/DN)      A 427
      A25=ABS((-MU*(1.+S2F)*SIN(PSI1-ALPHP1)+(MU-MU*S1F-1.)*COS(PSI1-ALP   A 428
     1HP1))/DN)                                                            A 429
430   A26=ABS((-SIN(GAMMA2)+MU*COS(GAMMA2))/DN)                            A 430
      A27=ABS((-(1.+MU-MU*S1F)*SIN(PSI1-ALPHP1)+MU*(S1F-1.)*COS(PSI1-ALP   A 431
     1HP1))/DN)                                                            A 432
      A28=ABS(1./DN)                                                        A 433
      A29=ABS((MU*(S1F-1.)*SIN(PSI1-ALPHP1)+(1.+MU*MU*S1F)*COS(PSI1-ALPH   A 434
     1P1))/DN)                                                             A 435
435   A30=ABS(MU/DN)                                                        A 436
      A37=ABS(((1.+MU*MU-S2F)*SIN(PSI2+ALPHP2)+MU*(S2F-1.)*COS(PSI2+ALPH   A 437
     1P2))/DN)                                                             A 438
      A38=ABS(((1.-MU*MU-S1F)*SIN(PSI1-ALPHP1)-MU*(1.+S1F)*COS(PSI1-ALPH   A 439
     1P1)/DN)                                                              A 440
      A35=ABS((MU*SIN(GAMMA2)+COS(GAMMA2))/DN)                              A 441
440   A40=ABS(MU*(S2F-1.)*SIN(PSI2+ALPHP2)-(1.+MU*MU*S2F)*COS(PSI2+ALPH    A 442
     1P2)/DN)                                                              A 443
      A41=ABS(MU*(1.+S1F)*SIN(PSI1-ALPHP1)+(MU*MU*S1F-1.)*COS(PSI1-ALP     A 444
     1HP1))/DN)                                                            A 445
      A42=ABS((MU*COS(GAMMA2)-SIN(GAMMA2))/DN)                              A 446
      A43=ABS(((1.+MU*MU-S2F)*SIN(PSI2+ALPHP2)-MU*(1.-S2F)*COS(PSI2+ALPH   A 447
     1P2))/DN)                                                             A 448
445   A44=ABS((MU*(S1R-1.)*SIN(LAMDA1)-(1.+MU*MU-S1R)*COS(LAMDA1))/DN)     A 449
      A45=ABS((MU*SIN(GAMMA2)+COS(GAMMA2))/DN)                             A 450
      A46=ABS((MU*(S2R-1.)*SIN(LAMDA2)-MU*(1.+MU*MU-S2R)*COS(LAMDA2))/DN)  A 451
      A47=ABS((-(1.+MU-MU*S1R)*SIN(LAMDA1)+MU*(1.-S1R)*COS(LAMDA1))/DN)    A 452
      A48=ABS((-SIN(GAMMA2)+MU*COS(GAMMA2))/DN)                            A 453
450   A65=ABS((-(1.+S2R)*SIN(LAMDA2)+(MU-MU*S2R-1.)*COS(LAMDA2))/DN)       A 454
      A66=ABS((MU*SIN(GAMMA3)-COS(GAMMA3))/DN)                             A 455
      A67=ABS((MU*MU*S2R-1.)*SIN(LAMDA2)-MU*(1.-S2R)*COS(LAMDA2))/DN)      A 456
      A68=ABS((-SIN(GAMMA3)-MU*COS(GAMMA3))/DN)                            A 457
      A69=ABS((MU*MU*S2F-1.)*SIN(PSI2+ALPHP2)-MU*(1.+S2F)*COS(PSI2+ALPH    A 458
     1P2))/DN)                                                             A 459
455   A70=ABS((MU*SIN(GAMMA3)-COS(GAMMA3))/DN)                             A 460
      A71=ABS((-MU*(1.+S2F)*SIN(PSI2+ALPHP2)+(1.-MU*MU*S2F)*COS(PSI2+ALP   A 461
     1HP2)/DN)                                                             A 462
      A72=ABS((-SIN(GAMMA3)-MU*COS(GAMMA3))/DN)                            A 463
      C6=ACG2*(SIN(PHI2-DELG2-LAMDA2)-MU*S2R*COS(PHI2-DELG2-LAMDA2))-MU*   A 464
     1RHOG2*S2R                                                            A 465
460   C7=MU*RHOG2*(A13+A14)-MU*RHOG2*S2R                                   A 466
      C8=-(ACP1*(SIN(PSI1-DELP1-LAMDA1)-MU*S1R*COS(PSI1+DELP1-LAMDA1))+MU  A 467
     1U*RHOG2*(A12+A15)+MU*S1R*RHOP1                                       A 468
      C9=MU*RHO1*(A18+A20)                                                 A 469
      C10=MU*RHO1*(A17+A19)+ACG1*(SIN(PHI1+DELG1-LAMDA1)-MU*S1R*COS(PHI1   A 470
465   1+DELG1-LAMDA1))-MU*S1R*RHOG1                                        A 471
      C11=ACG2*(SIN(PHI2-DELG2-LAMDA2)-MU*S2R*COS(PHI2-DELG2-LAMDA2))-MU   A 472
     1*RHOG2*(A21+A24)-MU*RHOG2*S2R                                        A 473
      C12=MU*RHO2*(A23+A26)                                                A 474
470   C13=G1-MU*RHO2*(A22+A25)                                             A 475
                                                                          A 476
                                                                          A 477
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      C14=MU*RHO1*(A28+A30)                                                        A  478
      C15=MU*RHO1*(A27+A29)+MU*S1F*RHOG1+ACG1*(MU*S1F*SIN(PHI1+DELG1-PSI            A  479
     1I+A1+HP1)-COS(PHI1+DELG1-PSI1+ALPHP1))                                       A  480
      C19=-MU*RHO2*(A37-A40)+ACG2*(COS(PHI2-DELG2-PSI2-ALPHP2)+MU*S2F*SI            A  481
     1N(PHI2-DELG2-PSI2-ALPHP2))-MU*S2F*RHOG2                                      A  482
      C20=MU*RHO2*(A39+A42)                                                        A  483
      C21=-MU*RHO2*(A38+A41)+G1                                                    A  484
      C22=-MU*RHO2*(A43+A46)+ACG2*(COS(PHI2-DELG2-PSI2-ALPHP2)+MU*S2F*SI           A  485
     1N(PHI2-DELG2-PSI2-ALPHP2))-MU*S2F*RHOG2                                      A  486
      C23=MU*RHO2*(A45+A48)                                                        A  487
      C24=-DELP1-LAMDA1))-MU*S1R*RHOP1                                             A  488
     11+DELP1-LAMDA1))-MU*S1R*RHOP1                                                A  489
      C33=-MU*2RHO3*(A65+A67)-MU*S2R*RHOP2+ACP2*(MU*S2R*COS(PSI2-DELP2-LAM         A  490
      C34=-MU*RHO3*(A65+A67)-MU*S2R*RHOP2+ACP2*(MU*S2R*COS(PSI2-DELP2-LAM          A  491
     1DA2)-SIN(PSI2-DELP2-LAMDA2))                                                 A  492
      C35=-MU*RHO3*(A70+A72)                                                       A  493
      C36=MU*RHO3*(A69+A71)-G2                                                     A  494
                                                                                   A  495
   C  MOMENT CONTROL                                                               A  496
   C                                                                               A  497
   C                                                                               A  498
      IF ((PHI1.GE.PHI1T).AND.(PHI2.LE.PHI2T)) GO TO 25                            A  499
      IF ((PHI1.LE.PHI1T).AND.(PHI2.LE.PHI2T)) GO TO 26                            A  500
      IF ((PHI1.LE.PHI1T).AND.(PHI2.GE.PHI2T)) GO TO 27                            A  501
      IF ((PHI1.GE.PHI1T).AND.(PHI2.GE.PHI2T)) GO TO 28                            A  502
                                                                                   A  503
   C  MOMENT EXPRESSIONS                                                           A  504
   C                                                                               A  505
   25 MO31=MIN*C8*C34/(C6*C10)-Q1*C8*C9*C34/(C6*C10)-Q2*C7*C34/C6-Q3*C33           A  506
      MO3=MO31                                                                     A  507
      POINTEF=ABS(PSDOT2)*MO3/MIN                                                  A  508
      WRITE (6,36) PHI1D,PHI2D,PSI1D,PSI2D,PSDOT1,PSDOT2,S1R,S2R,POINTEF           A  509
      GO TO 29                                                                     A  510
   26 MO32=MIN*C13*C34/(C11*C15)-Q1*C13*C14*C34/(C11*C15)-Q2*C12*C34/C11           A  511
     1-Q3*C33                                                                      A  512
      MO3=MO32                                                                     A  513
      POINTEF=ABS(PSDOT2)*MO3/MIN                                                  A  514
      WRITE (6,37) PHI1D,PHI2D,PSI1D,PSI2D,PSDOT1,PSDOT2,S2R,S1F,G1,POIN           A  515
     1TEF                                                                          A  516
      GO TO 29                                                                     A  517
   27 MO33=MIN*C21*C36/(C15*C19)-Q1*C14*C21*C36/(C15*C19)-Q2*C20*C36/C19           A  518
     1-G3*C35                                                                      A  519
      MO3=MO33                                                                     A  520
      POINTEF=ABS(PSDOT2)*MO3/MIN                                                  A  521
      WRITE (6,38) PHI1D,PHI2D,PSI1D,PSI2D,PSDOT1,PSDOT2,S1F,G1,S2F,G2,P           A  522
     1OINTEF                                                                       A  523
      GO TO 29                                                                     A  524
   28 MO34=MIN*C24*C36/(C10*C22)-Q1*C9*C24*C36/(C10*C22)-Q2*C23*C35/C22-           A  525
     1Q3*C35                                                                       A  526
      MO3=MO34                                                                     A  527
      POINTEF=ABS(PSDOT2)*MO3/MIN                                                  A  528
      WRITE (6,39) PHI1D,PHI2D,PSI1D,PSI2D,PSDOT1,PSDOT2,S1R,S2F,G2,POIN           A  529
     1TEF                                                                          A  530
   29 MTOT=MTOT+POINTEF
```

```
        GO TO 20
     30 CYCLEFF=-DTOT*DDPHI1/(PHI1F-PHI1I)
        WRITE (6,57) CYCLEFF
        DTOT=0.
        IF (ISTOP.NE.0) GO TO 1
        STOP
C
C
     31 FORMAT (6X,9HPSI1T1D =,F9.4,3X,8HTEST11 =,F9.4)
     32 FORMAT (6X,9HPSI1T2D =,F9.4,3X,8HTEST12 =,F9.4/)
     33 FORMAT (6X,9HPSI2T1D =,F9.4,3X,8HTEST21 =,F9.4)
     34 FORMAT (6X,9HPSI2T2D =,F9.4,3X,8HTEST22 =,F9.4/)
     35 FORMAT (8SH     PHI1  PHI2  PSI1  PSI2  DPSI1   DPSI2   S1R   S2R   S
       11F  G1      G2     POINTEF/)
     36 FORMAT (6X,4(F4.0,2X),2(F5.0,2X),2(F3.0,2X),24X,F5.3)
     37 FORMAT (6X,4(F4.0,2X),2(F5.0,2X),5X,F3.0,2X,F5.3,14X,F5.3)
     38 FORMAT (6X,4(F4.0,2X),2(F5.0,2X),10X,2(F3.0,2X,F5.3,2X),F5.3)
     39 FORMAT (6X,4(F4.0,2X),2(F5.0,2X),2(F5.0,2X),F3.0,19X,F3.0,2X,F5.3,2X,F5.3)
     40 FORMAT (F10.3,F10.0/4F10.5/4F10.5/I1)
     41 FORMAT (4F10.4)
     42 FORMAT (4F10.4)
     43 FORMAT (4F10.0)
     44 FORMAT (3F10.4/F10.6/F10.4)
     45 FORMAT (1H1,5X,5HMRIN =,F8.4,3X,4HMRU =,F6.3,3X,5HRPM =,F6.0//6X,8HC
       1APRP1 =,F8.5,3X,8HCAPRP2 =,F8.5/6X,5HRP2 =,F8.5,3X,5HRP3 =,F8.5//
       26X,6HACG1 =,F8.5,3X,6HACG2 =,F8.5,3X,6HACP1 =,F8.5,3X,6HACP2 =,F8.
       35/)
     46 FORMAT (6X,7HRHOXG1 =,F8.5,3X,7HRHOXG2 =,F8.5,3X,7HRHOXP1 =,F8.5,3X,
       1HRHOXP2 =,F8.5/)
     47 FORMAT (6X,5HTG1 =,F8.5,3X,5HTG2 =,F8.5,3X,5HTP1 =,F8.5,3X,5HTP2 =,
       1,F8.5,/)
     48 FORMAT (6X,5HRG1 =,F5.0,3X,5HRG2 =,F5.0,3X,5HRP2 =,F5.0,3X,5HRP3 =,
       1,F5.0/)
     49 FORMAT (6X,6HRHOP01 =,F6.3,3X,6HRHOP02 =,F6.3,3X,6HRHOP03 =,F6.3//6X,4HM
       1D =,F10.6,/6X,3HK =,F6.1//6X,8HPHIDOT1 =,F5.1/)
     50 FORMAT (6X,8HBETA1D =,F8.4,3X,8HBETA2D =,F8.4/)
     51 FORMAT (6X,30HSOMETHING IS WRONG WITH MESH 1)
     52 FORMAT (6X,8HPHI1T1D =,F8.4,3X,8HPSI1T1D =,F8.4,3X,8HPHI1IFD =,F8.4.3
       1X,8HPSI1FD =,F8.4//)
     53 FORMAT (6X,8HPHI1T1D =,F8.4,3X,8HPSI1T1D =,F8.4,3X,8HPHI1IFD =,F8.4,3
       1X,8HPSI1FD =,F8.4//)
     54 FORMAT (6X,30HSOMETHING IS WRONG WITH MESH 2)
     55 FORMAT (6X,8HPHI1T2D =,F8.4,3X,8HPSI12T1D =,F8.4-3X,8HPHI12FD =,F8.4.3
       1X,8HSI2FD =,F8.4//)
     56 FORMAT (6X,8HPHI12ID =,F8.4,3X,8HPSI12ID =,F8.4,3X,8HPHI12FD =,F8.4.3
       1X,8H-SI2FD =,F8.4//)
     57 FORMAT (1H0,5X,18HCYCLE EFFICIENCY =,F5.3)
     58 FORMAT (3F10.5)
     59 FORMAT (6X,3HR1 =,F8.5,3X,4HR2 =,F8.5,3X,4HR3 =,F8.5/)
     60 FORMAT (6X,5HFP1 =,F8.5,3X,5HFP2 =,F8.5/)
     61 FORMAT (3E15.5)
     62 FORMAT (6X,4HB1 =,E15.5,3X,4HB2 =,E15.5,3X,4HB3 =,E15.5/)
        END
```

```
         SUBROUTINE TRANS1 (RHOG,ALPHP,BETA,FP,ACG,B,DELG,Z,PSIT,PHIT,G)    B     1
         PI=3.14159                                                         B     2
         SI=(-RHOG*COS(PSIT-ALPHP)+B*SIN(BETA)+FP*SIN(PSIT-ALPHP))/ACG      B     3
         CT=(RHOG*SIN(PSIT-ALPHP)+B*COS(BETA)+FP*COS(PSIT-ALPHP))/ACG       B     4
         PHIT=ATAN2(ST,CT)-DELG                                             B     5
         PHINEXT=PHIT-.1*Z                                                  B     6
         AF=ACG*COS(PHINEXT+DELG+ALPHP)-B*COS(BETA+ALPHP)                   B     7
         BF=-ACG*SIN(PHINEXT+DELG+ALPHP)+B*SIN(BETA+ALPHP)                  B     8
         CF=RHOG                                                            B     9
         ROOTF=AF*AF+BF*BF-CF*CF                                            B    10
         YF1=AF+SQRT(ROOTF)                                                 B    11
         YF2=AF-SQRT(ROOTF)                                                 B    12
         XF=BF+CF                                                           B    13
         PSINEX1=2.*ATAN2(YF1,XF)                                           B    14
         PSINEX2=2.*ATAN2(YF2,XF)                                           B    15
         IF (PSINEX1.LT.0.) PSINEX1=PSINEX1+2.*PI                           B    16
         IF (PSINEX2.LT.0.) PSINEX2=PSINEX2+2.*PI                           B    17
         IF (ABS(PSINEX1-PSIT).LT.ABS(PSINEX2-PSIT)) GO TO 1                B    18
         PSINEXT=PSINEX2                                                    B    19
         GO TO 2                                                            B    20
    1    PSINEXT=PSINEX1                                                    B    21
    2    G=(ACG*SIN(PHINEXT+DELG)+RHOG*COS(PSINEXT-ALPHP)-B*SIN(BETA))/SIN( B    22
        1PSINEXT-ALPHP)                                                     B    23
         RETURN                                                             B    24
         END                                                               B    25-
```

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```
                  SUBROUTINE TRANS2 (RHOG,ALPHP,BETA,FP,ACG,B,DELG,Z,PSIT,PHIT,G)      C    1
                  PI=3.14159                                                           C    2
                  ST=(RHOG*COS(PSIT+ALPHP)+B*SIN(BETA)+FP*SIN(PSIT+ALPHP))/ACG         C    3
                  CT=(-RHOG*SIN(PSIT+ALPHP)+B*COS(BETA)+FP*COS(PSIT+ALPHP))/ACG        C    4
                  PHIT=ATAN2(ST,CT)+DELG                                               C    5
                  PHINEXT=PHIT+.1*Z                                                    C    6
                  AF=ACG*COS(PHINEXT-DELG-ALPHP)-B*COS(BETA-ALPHP)                     C    7
                  BF=-ACG*SIN(PHINEXT-DELG-ALPHP)+B*SIN(BETA-ALPHP)                    C    8
                  CF=-RHOG                                                             C    9
                  ROOTF=AF*AF+BF*BF-CF*CF                                              C   10
                  YF1=AF+SQRT(ROOTF)                                                   C   11
                  YF2=AF-SQRT(ROOTF)                                                   C   12
                  XF=BF+CF                                                             C   13
                  PSINEX1=2.*ATAN2(YF1,XF)                                             C   14
                  PSINEX2=2.*ATAN2(YF2,XF)                                             C   15
                  IF (PSINEX1.LT.0.) PSINEX1=PSINEX1+2.*PI                             C   16
                  IF (PSINEX2.LT.0.) PSINEX2=PSINEX2+2.*PI                             C   17
                  IF (ABS(PSINEX1-PSIT).LT.ABS(PSINEX2-PSIT)) GO TO 1                  C   18
                  PSINEXT=PSINEX2                                                      C   19
                  GO TO 2                                                              C   20
                1 PSINEXT=PSINEX1                                                      C   21
                2 G=(ACG*SIN(PHINEXT-DELG)-RHOG*COS(PSINEXT+ALPHP)-B*SIN(BETA))/SIN(   C   22
                 1PSINEXT+ALPHP)                                                       C   23
                  RETURN                                                              C   24
                  END                                                                 C   25-
```

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MIN = .1645  MU = .200  RPM = 1000.

CAPRP1 = .47725  CAPRP2 = .20670

RP2 = .09085  RP3 = .06890

ACG1 = .47725  ACG2 = .20670  ACP1 = .09085  ACP2 = .06890

R1 = .75000  R2 = .75000  R3 = .75000

RHOG1 = .03870  RHOG2 = .02070  RHOP1 = .01740  RHOP2 = .01040

TG1 = .03480  TG2 = .02520  TP1 = .02800  TP2 = .02060

NG1 = 42.  NG2 = 27.  NP2 = 8.  NP3 = 9.

M1 = .69514E-04  M2 = .97028E-05  M3 = .10780E-05

RHO1 = .060  RHOC2 = .030  RHO3 = .025

MD = .000015

K = 25.0

PHDOT1 = -1.0

FP1 = .08917  FP2 = .06811

BETA1D =112.2552  BETA2D =145.0978

PSI1T1D = 305.6017  TEST11 = 4.4495
PSI1T2D = 343.6259  TEST12 = 42.4736

PHI1TD =113.5016  PSI1TD =305.6017  PHI1FD =107.4216  PSI1FD =337.4283
PHI1ID =115.9930  PSI1ID =292.4283

PSI2T2D = 286.9442  TEST21 = 29.4719
PSI2T2D = 312.0785  TEST22 = 4.3377

PHI2TD =143.0461  PSI2TD =312.0785  PHI2FD =150.1161  PSI2FD =290.9521
PHI2ID =136.7828  PSI2ID =330.9521

| PHI1 | PHI2 | PSI1 | PSI2 | DPSI1 | DPSI2 | S1R | S2R | S1F | G1 | S2F | G2 | POINTER |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 116. | 137. | 292. | 331. | 5. | -16. | 1. | -1. | | | | | .617 |
| 116. | 137. | 293. | 329. | 5. | -16. | 1. | -1. | | | | | .626 |
| 116. | 138. | 294. | 328. | 5. | -16. | 1. | -1. | | | | | .634 |
| 116. | 138. | 294. | 326. | 5. | -16. | 1. | -1. | | | | | .642 |
| 116. | 139. | 295. | 324. | 5. | -16. | 1. | -1. | | | | | .650 |
| 115. | 139. | 295. | 323. | 5. | -16. | 1. | -1. | | | | | .657 |
| 115. | 140. | 296. | 321. | 5. | -16. | 1. | -1. | | | | | .665 |
| 115. | 141. | 296. | 320. | 5. | -16. | 1. | -1. | | | | | .672 |
| 115. | 141. | 297. | 318. | 5. | -16. | 1. | -1. | | | | | .679 |
| 115. | 142. | 297. | 316. | 5. | -16. | 1. | -1. | | | | | .686 |
| 115. | 142. | 298. | 315. | 5. | -16. | 1. | -1. | | | | | .693 |
| 115. | 143. | 298. | 313. | 5. | -16. | 1. | 1. | | | | | .689 |
| 115. | 143. | 299. | 312. | 5. | -16. | 1. | 1. | | | | .068 | .685 |

| | | | | | | | | | 1. | .067 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 108. | 150. | 333. | 291. | 5. | -13. | -1. | 1. | .083 | | | .531 |
| 108. | 137. | 333. | 331. | 5. | -15. | -1. | 1. | .083 | | | .551 |
| 108. | 137. | 334. | 329. | 5. | -15. | -1. | 1. | .083 | | | .555 |
| 108. | 138. | 334. | 328. | 5. | -14. | -1. | 1. | .083 | | | .559 |
| 108. | 138. | 334. | 327. | 5. | -14. | -1. | 1. | .084 | | | .563 |
| 108. | 139. | 335. | 325. | 5. | -14. | -1. | 1. | .084 | | | .566 |
| 108. | 140. | 335. | 324. | 5. | -14. | -1. | 1. | .084 | | | .569 |
| 108. | 140. | 336. | 322. | 5. | -14. | -1. | 1. | .084 | | | .572 |
| 108. | 141. | 336. | 321. | 5. | -14. | -1. | 1. | .084 | | | .575 |
| 108. | | 337. | 320. | 4. | -13. | -1. | 1. | .084 | | | .578 |

CYCLE EFFICIENCY = .620